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Buckling analysis of plain-woven fabric structure using shell element and a one cell-based integration scheme in smoothed finite element method

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Abstract. A one smoothing cell integration scheme in the strain smoothing technique in finite elements (referred as SFEM) was proposed to evaluate the nodal train fields of a four-node quadrilateral (Q4) shell element, which is based on the first-order shear deformation theory of plate (FSDT). A mixed interpolation of tensorial components (MITC) approaches for Q4 transverse shear strains also applied to eliminate a shear locking phenomenon that may occur when the thin plate/shell elements are geometrically distorted in curved geometries of fabric sheet. The numerical eigenvalues of buckling analysis of a plain-woven fabric sample, of which physical and mechanical parameters extracted from Kawabata evaluation system for fabrics (KES-FB), obtained a higher efficiency in numerical computation and approximated to Q4 shell element implemented in the finite element method (FEM).

1. Introduction and kinematics of shells

Analysis of buckling behaviour of textile fabric using thin plate/shell finite elements helps understand the basic practical problem of residual curvature in the fabric to understand the behaviour of fabric during garment manufacturing as well as during its use as a garment.[1, 2] An excellent review in the past development of plate/shell finite elements can be found in the works of Yang et al..[3] This paper presents an Q4 shell element based on the FSDT [4-6] and the cell-based smoothed finite element in SFEM [7, 8] for buckling analysis of thin and moderately thick plain-woven fabric structure. The numerical implementation and results indicated a better efficiency of numerical computation but accuracy results compared to the same Q4 shell elements without integrating cell-based models. The strain smoothing operator and the summarized formulation of Q4 shell plate/shell elements are respectively presented in the following sections.

1.1. A strain smoothing method in finite elements

The strain smoothing technique recently proposed by Liu et al. [9], referred as SFEM, which improved the accuracy and convergence rate of the existing conventional finite element finite element method (FEM) of elastic solid mechanics problems.[10-12] This technique avoids evaluating derivatives of mesh-free shape functions at nodes and therefore eliminates defective modes. The major techniques used in smoothed finite element methods appear summarized in references [8, 13, 14].

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The strain smoothing operator over an element Ω^e bounded by Γ^e referred as a smoothing domain (or cell) Ω_k^s, which is defined as

$$\bar{\nabla}\boldsymbol{u}(x_k) = \boldsymbol{\varepsilon}(\mathrm{x}) = \int_{\Omega_i^s} \nabla\boldsymbol{u}(x)\Phi(x - x_k)d\Omega \tag{1}$$

and has to satisfy conditions:

$$\Phi \geq 0 \; and \int_{\Omega_i^s} \Phi d\Omega = 1, \Phi(x - x_k) = \begin{cases} \dfrac{1}{A_k^s}, & x \in \Omega_k^s \\ 0, & x \notin \Omega_k^s \end{cases}, \tag{2}$$

where Φ is a smoothing or weight function in Ω_k^s and $A_k = \int_{\Omega_k^s} d\Omega$ is the area of smoothing domain.

The strain fields in Ω_k^s can be further assumed to be a constant and equals $\bar{\boldsymbol{\varepsilon}}(\mathrm{x}_k)$, which gives:

$$\bar{\boldsymbol{\varepsilon}}_k = \bar{\boldsymbol{\varepsilon}}_k(x) = \bar{\boldsymbol{\varepsilon}}(x_k) = \frac{1}{A_k^s} \int_{\Omega_k^s} \boldsymbol{\varepsilon}(x)d\Omega. \tag{3}$$

Substituting function Φ into Eq. (1), the smoothed gradient of displacement can be written as

$$\bar{\boldsymbol{\varepsilon}}(\mathrm{x}_k) = \frac{1}{A_k^s} \int_{\Gamma_k^s} \boldsymbol{n}(\mathrm{x}).\hat{\boldsymbol{u}}(\mathrm{x})d\Gamma, \tag{4}$$

in which $\boldsymbol{n}(\mathrm{x})$ is the outward unit normal matrix containing the components of the outward unit normal vector to the boundary Γ_k^s.

1.2. *Kinematics of shells*

Based on the FSDT, the displacement components $\boldsymbol{u} = [u, v, w]$ at a local coordinate system (x, y, z) within problem domain Ω bounded by Γ are defined as follows:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\theta_x(x,y) \\ v(x,y,z) &= v_0(x,y) - z\theta_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{5}$$

where u_0, v_0 and w_0 be the translation displacements, θ_x and θ_y be the rotations about the yz and xz planes in the Cartesian coordinate system.

In terms of the mid-plane deformations, the strain vector $\boldsymbol{\varepsilon}$ can be written using Eq. (5), which gives:

$$\boldsymbol{\varepsilon} = \{\varepsilon_x \quad \varepsilon_y \quad \varepsilon_{xy} \quad \varepsilon_{xz} \quad \varepsilon_{yz}\}^T = \begin{Bmatrix} \boldsymbol{\varepsilon}^m \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\boldsymbol{\varepsilon}^b \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \boldsymbol{\varepsilon}^s \end{Bmatrix} \tag{6}$$

in which superscripts m, b and s are respectively the membrane, bending and the shear terms. The generalized strain vector $\hat{\boldsymbol{\varepsilon}}$ can be written as

$$\hat{\boldsymbol{\varepsilon}} = \begin{Bmatrix} \boldsymbol{\varepsilon}^m \\ \boldsymbol{\varepsilon}^b \\ \boldsymbol{\varepsilon}^s \end{Bmatrix}, \tag{7}$$

where

$$\boldsymbol{\varepsilon}^m = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial x} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \boldsymbol{\varepsilon}^b = \begin{Bmatrix} \dfrac{\partial \theta_x}{\partial x} \\ \dfrac{\partial \theta_y}{\partial y} \\ \dfrac{\partial \theta_x}{\partial x} + \dfrac{\partial \theta_y}{\partial y} \end{Bmatrix}, \boldsymbol{\varepsilon}^s = \begin{Bmatrix} \dfrac{\partial w_0}{\partial x} - \theta_x \\ \dfrac{\partial w_0}{\partial y} - \theta_y \end{Bmatrix}. \tag{8}$$

The constitutive equations can be expressed as

$$\hat{\boldsymbol{\sigma}} = \hat{\boldsymbol{D}}\hat{\boldsymbol{\varepsilon}}, \tag{9}$$

where

$$\hat{\boldsymbol{\sigma}} = \begin{Bmatrix} \hat{\boldsymbol{N}} \\ \hat{\boldsymbol{M}} \\ \hat{\boldsymbol{Q}} \end{Bmatrix}, \hat{\boldsymbol{D}} = \begin{bmatrix} \boldsymbol{D}^m & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D}^b & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}^s \end{bmatrix}, \tag{10}$$

in which $\hat{\boldsymbol{N}} = \{N_x \quad N_y \quad N_{xy}\}^T$ is the vector of membrane force, $\hat{\boldsymbol{M}} = \{M_x \quad M_y \quad M_{xy}\}^T$ is the vector of bending moment, $\hat{\boldsymbol{Q}} = \{Q_x \quad Q_y\}^T$ is the vector of transverse shear force and $\boldsymbol{D}$ are stiffness constitutive coefficients matrix, which given as

$$\boldsymbol{D}^m = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \frac{E_1}{1-v_1 v_2} & \frac{v_2 E_1}{1-v_2 v_1} & 0 \\ \frac{v_1 E_2}{1-v_1 v_2} & \frac{E_2}{1-v_2 v_1} & 0 \\ 0 & 0 & G \end{bmatrix} dz\,, \boldsymbol{D}^b = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & H \end{bmatrix} dz, \boldsymbol{D}^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{5}{6} G \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} dz, \tag{11}$$

where h is the thickness of the plate or shell, G is shear modulus, E_* is Young's modulus and v_* is Poisson's ratio corresponding to the warp and weft direction of yarns, B_* and H stand for flexural moduli and torsional rigidity.

According to FEM procedures for buckling analysis, the problem domain Ω is discretized into a set of four-node quadrilateral flat shell elements Ω^e with boundary Γ^e, the generalized mid-plane displacement vector $\hat{\boldsymbol{u}}$ can be then defined as

$$\hat{\boldsymbol{u}} = \sum_{I=1}^{4} \begin{bmatrix} N_I(\mathbf{x}) & 0 & 0 & 0 & 0 \\ 0 & N_I(\mathbf{x}) & 0 & 0 & 0 \\ 0 & 0 & N_I(\mathbf{x}) & 0 & 0 \\ 0 & 0 & 0 & N_I(\mathbf{x}) & 0 \\ 0 & 0 & 0 & 0 & N_I(\mathbf{x}) \end{bmatrix} \begin{Bmatrix} u_I \\ v_I \\ w_I \\ \theta_{xI} \\ \theta_{yI} \end{Bmatrix} = \sum_{I=1}^{4} \boldsymbol{N}_I(\mathbf{x}) \boldsymbol{d}_I, \tag{12}$$

where $N_I(\mathbf{x})$ is the basis function associated to node I of a four-node quadrilateral shell element.

The eigenvalue equation for buckling analysis can be expressed through the direct application of variational principles and using Eq. (5 to 12), which is given as

$$\left(\boldsymbol{K} - \lambda \boldsymbol{K}_g\right)\boldsymbol{d} = 0, \tag{13}$$

where λ is the critical buckling load, $\boldsymbol{K}$ and $\boldsymbol{K}_g$ are the global stiffness matrix and the geometric stiffness matrix, respectively, which express in [7, 15]

The shear locking phenomenon may appear due to incorrect transverse forces under bending, or in the case of the thickness of the plate tends to zero. To overcome the shear locking phenomena, the approximation of the shear strain $\boldsymbol{\varepsilon}^s$ in Eq. (8) can be formulated with MITC4 element as in [4].

Applying Eq. (1, 3 and 4) with $k = 1$ in order to evaluate the membrane and bending trains of a Q4 shell element being formulated in this section using the following shape functions:

$$N_1 = (1,0,0,0), N_2 = (0,1,0,0), N_3 = (0,0,1,0), N_4 = (0,0,0,1). \tag{14}$$

2. Numerical implementation and results

The span-to-thickness ratio l/h of a square woven fabric sample was taken to be 23.5849 in the numerical example. Two types of boundary conditions, simply supported (S) and clamped (C) edges applied for meshes such as 5x5, 10x10, 15x15 and 20x20. Both the Q4 shell elements with and without being integrated smoothing cells were programmed.

Mechanical and physical parameters of a plain-woven fabric sample were measured with Kawabata evaluation system for fabrics (KES-FB) [16, 17] and derived as: elastic modulus [gf/cm], $E_1 = 3823.7993$, $E_2 = 14092.4464$ and $E_{12} = 6896.5517$, Poisson's ratio $v_1 = 0.0211$ and $v_2 = 0.0778$, bending rigidity $[gf.cm^2/cm]$ of $B_1 = 0.1237$, $B_2 = 0.1333$ and $B_{12} = 0.0880$, transverse shear modulus $[gf.cm^2]$ of $G = 217.3100$.

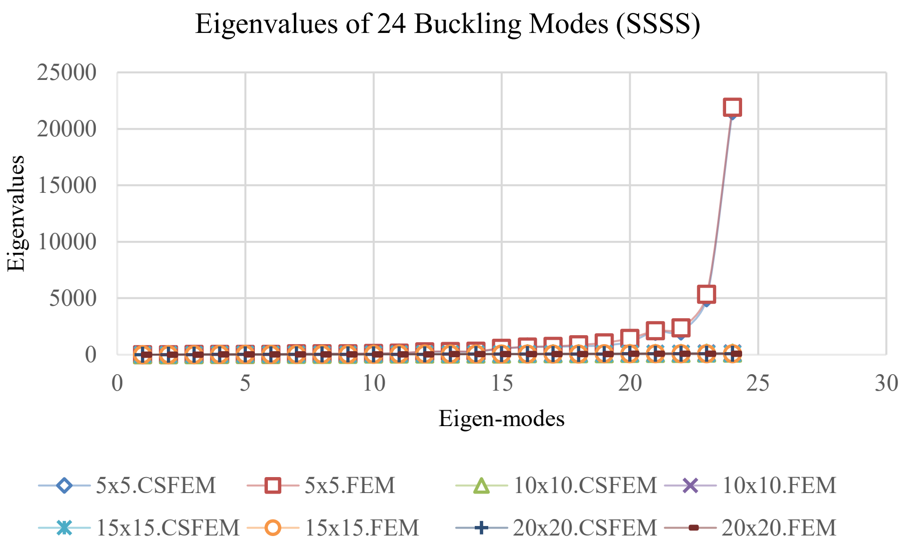

Figure 1. Compare the eigenvalues of 24 buckling modes between the implemented CS-FEM model and FEM with simply supported edges and different mesh density

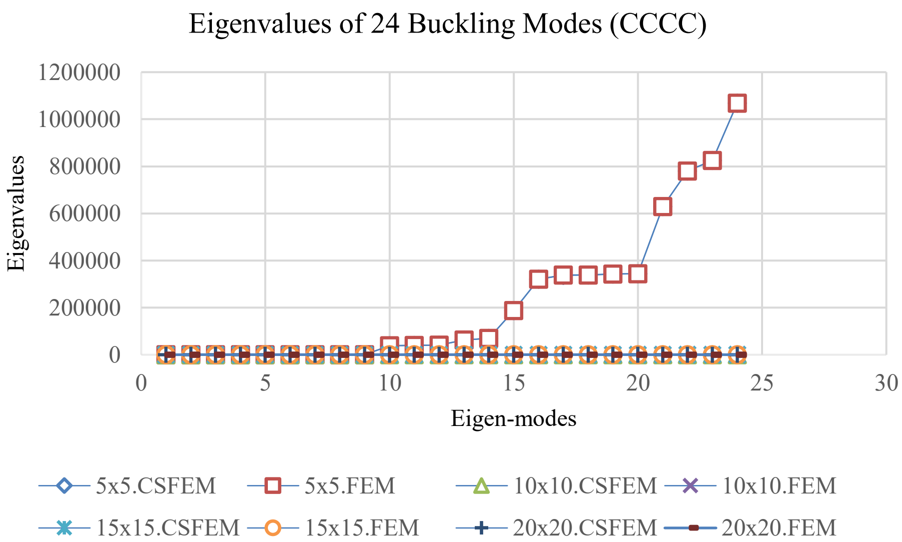

Figure 2. Compare the eigenvalues of 24 buckling modes between the implemented CS-FEM model and FEM with clamped edges and different mesh density

Figures 1 and 2 indicated that the eigenvalues of 24 modes of the critical buckling load λ computed by CSFEM model is approximate and coincided with that one of FEM on the same boundary conditions and mesh configurations. The shape functions in SFEM model are constants as presented in Eq. 14. Vice versa, the corresponding shape functions of a Q4 element in FEM are bilinear shape functions interpolated in natural coordinates (ξ, η, ζ). Thus, the strain smoothing technique reduces the numerical implementation and computation time in terms of the central processing unit (CPU) time.

3. Conclusions

A one cell-base integration scheme in SFEM for a Q4 element, which based on the FSDT, presented an effective computation in terms of CPU time for the buckling analysis of plain-woven fabric structure, produced an accurate numerical result and also reduced tasks in numerical implementation compared with FEM.

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