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Routh-Hurwitz tuning method for stable/unstable time-delay MIMO processes

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Abstract. This paper presents a new method of tuning decentralized Proportional Integral Derivative (PID) control system for an $n \times n$ Multi Input Multi Output (MIMO) processes which can be either open-loop stable or unstable with time-delays. The tuning method has been established via the classical Routh-Hurwitz criteria with the aid of PID stability theorem. This new tuning method requires only 3 common tuning parameters, thus simplifying the conventional multi-loop PID tuning task to obtain the values for *3n* PID parameters. Two numerical examples are presented to demonstrate the effectiveness of the proposed tuning procedure over some of the existing multi-loop PID tuning procedures. The proposed tuning procedure shows a good performance as well as able to simplify the tuning task as compare to the existing tuning method.

1. Introduction

Most real processes in industries are multi-input and multi-output (MIMO) in nature and possess significant time-delays or deadtimes. These criteria lead to an unstable process behaviour. Decentralized (or multi-loop) PID control systems are widely used to control process plants. Controlling a MIMO process is not as simple as controlling a single-input and single-output (SISO) process because of the existence of process interactions or couplings. Process interactions and deadtimes impose an upper bound on the achievable maximum control performance. If a given MIMO process open-loop is unstable, then the unstable poles in the system will impose a lower bound on the allowable minimum control performance. To control an unstable process, the PID controller parameter values must be well tuned within the maximum lower bound and minimum upper bound of the control performance for a stable operation.

For an $n \times n$ MIMO system, there are n PID controllers involved with a total of $3n$ tuning parameters. The decentralized PID tuning methods can be categorized into: (1) detuning, (2) sequential loop closing, (3) iterative or trial-and-error, (4) simultaneous equation solving and (5) independent tuning; see [1]. A significant innovation in the decentralized PID control tuning was proposed in [2], known as the Simultaneous Multi-Loop Multi-Scale Control (SML-MSC) tuning, based on the Multi-Scale Control theory in [3], [4]. The salient feature of the SML-MSC method is that it only requires 2 to 3 scaling parameters in order to tune an arbitrary $n \times n$ multi-loop PID system. However, the technique is restricted to open-loop stable MIMO processes, in which the diagonal processes can be represented by first-order plus deadtime (FOPDT) models. The SML-MSC tuning is easy to apply compared to other tuning methods.

For SISO systems, the closed-loop stability for the standard PID control system has been well studied. Several stability methods have been developed for assessing the closed-loop stability of PID controllers, e.g., the stability analysis based on the Hermite-Biehler theorem [5], the Gain-Phase Margin stability analysis [6], and the classical Routh-Hurwitz criteria [7]. On the contrary, the closed-loop stability analysis that is applicable to the multi-loop PID system remains an open problem.

The present work attempts to address the multi-loop PID control system design via the recently published SISO PID stability theorem, based on the classical Routh-Hurwitz stability criteria [7]. An advantage of the reported PID stability theorem is that, it can be conveniently used to find a stabilizing region of PID parameters both for open-loop stable or unstable processes. Through this PID stability theorem, we can reduce the tuning of multi-loop PID control controllers to just finding the values of 3 common tuning parameters for an arbitrary $n \times n$ MIMO system. Unlike the SML-MSC tuning method in [7], the new simultaneous tuning technique developed in this work is applicable to both open-loop stable and unstable processes.

The main contributions of this work can be summarized as follows:

- Decentralized PID control tuning for a class of stable MIMO systems (Section 4).
- Decentralized PID control tuning for a class of unstable MIMO systems (Section 4).

The rest of this paper is organized as follows. Section 2 presents some preliminaries. Then, Section 3 details the derivations of tuning relations. Next, Section 4 provides two new tuning algorithms followed by Section 5 which demonstrates the applicability of the PID tuning algorithms. Finally, some concluding remarks and future works are highlighted in Section 6.

2. Preliminaries

Consider the ideal form of Proportional-Integral-Derivative (PID) controller

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \quad (1)$$

where K_c, τ_I and τ_D represent the controller gain, reset time and derivative time respectively.

In the industry, many processes which are open-loop stable can be expressed as a first-order plus deadtime (FOPDT) model given by

$$G_p(s) = \frac{K_p \exp(-\theta s)}{\tau_p s + 1} \quad (2)$$

where K_p, τ_p and θ are the process gain, time constant and deadtime respectively.

Take note that, some of the processes which are open-loop unstable can be expressed by an unstable first-order plus deadtime (UFOPDT) model given as follows

$$G_p(s) = \frac{K_p \exp(-\theta s)}{\tau_p s - 1} \quad (3)$$

Figure 1 shows the standard single-loop feedback control. Here, F_r denotes the setpoint pre-filter, G_c the controller and P the process transfer functions; R, D_i, D_o and Y denote the setpoint, input disturbance, output disturbance and controlled variable signals respectively.

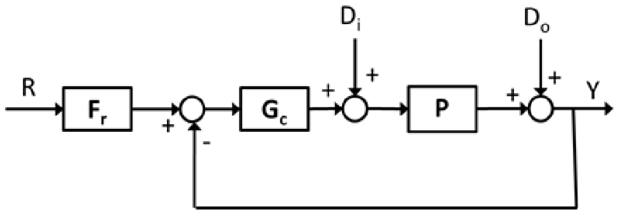

Figure 1. Standard single-loop feedback control structure.

For the standard single-loop control structure in figure 1, the closed-loop setpoint tracking and output disturbance transfer functions are given by (4) and (5), respectively.

$$H_r(s) = \frac{F_r(s)G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \tag{4}$$

$$H_d(s) = \frac{1}{1+G_c(s)G_p(s)} \tag{5}$$

The closed-loop characteristic equation for both transfer functions are similar, i.e.:

$$1+G_c(s)G_p(s) = 0 \tag{6}$$

Since the closed-loop transfer functions have a similar characteristic equation, this means that if the controller is stable for the setpoint tracking, then it should be stable also for the output disturbance rejection.

To perform the stability analysis on the closed-loop characteristic equation, it is convenient to first approximate the deadtime term as a rational transfer function, e.g., by using the 1/1 Padé approximation given as follows

$$\exp(-\theta s) \approx \frac{1-\alpha s}{1+\alpha s} \tag{7}$$

where $\alpha = 0.5\theta$.

3. SISO PID stability analysis

3.1. Stability region determination

Let us consider the stabilizing regions of PID controller for the two process types given as FOPDT and UFOPDT as Cases A and B respectively.

3.1.1. Case A: FOPDT. Based on the first-order Padé approximation of the deadtime, the closed-loop characteristic equation can be simplified to a polynomial as follows:

$$\left[\alpha\tau_I\left(\tau_p - K\tau_D\right)\right]s^3 + \tau_I\left[\alpha+\tau_p + K\left(\tau_D-\alpha\right)\right]s^2 + \left[\tau_I + K\left(\tau_I-\alpha\right)\right]s + K = 0 \tag{8}$$

Please note that the loop gain is $K = K_c K_p$. To establish a stabilizing region of PID controller, let us apply the two PID stability theorems established by Seer and Nandong [7]. By using the first part of PID stability theorem, we can construct a region of PID parameters which can meet the necessary criterion of Routh-Hurwitz stability.

It can be shown that via the first stability theorem, the ranges of derivative time, reset time and loop gain are given as follows

$$\tau_D > \alpha \tag{9}$$

$$\tau_I > \alpha \tag{10}$$

$$0 < K < \tau_p / \tau_D \tag{11}$$

To meet the sufficient criterion of Routh-Hurwitz stability, the second part of PID stability theorem [7] is applied. The sufficient condition for the closed-loop stability is given by

$$\tau_I > \max(\alpha, \tau_{I,\min A}) \tag{12}$$

with the assumption that the derivative time and loop gain values are first set to be within the region established by the first part of stability theorem, i.e., as in (9) and (11). Note that $\tau_{I,min}$ is the lower limit on the reset time established via the sufficient criterion of Routh-Hurwitz stability – the second part of PID stability theorem.

$$\tau_{I,\min_A} = \frac{K\alpha}{1+K}\left[1 + \frac{\tau_p - K\tau_D}{\alpha + \tau_p + K(\tau_D - \alpha)}\right] \tag{13}$$

3.1.2. Case B: UFOPDT. By applying the first PID stability theorem; see [7], a region of PID parameter values fulfilling the necessary criterion of Routh-Hurwitz stability is established as follows:

$$\alpha < \tau_D < \tau_p \tag{14}$$

$$\tau_I > \tau_{I,\min 1_B} = \frac{\alpha\tau_p}{\tau_p - \tau_D} \tag{15}$$

$$\frac{\tau_I}{\tau_I - \alpha} < K < \frac{\tau_p}{\tau_D} \tag{16}$$

Provided that the derivative time and loop gain are within the region defined by the first stability theorem, to ensure closed-loop stability the second PID stability theorem is applied. This results in the following range for the reset time

$$\tau_I > \max\left(\tau_{I,\min 1_B}, \tau_{I,\min 2_B}\right) \tag{17}$$

Another lower limit of the reset time is given by the sufficient criterion of Routh-Hurwitz stability:

$$\tau_{I,\min 2_B} = \frac{K\alpha}{K-1}\left[1 + \frac{\tau_p - K\tau_D}{\tau_p - \alpha + K(\tau_D - \alpha)}\right] \tag{18}$$

4. Decentralized PID tuning procedure

4.1. Tuning relations

Let us introduce three dimensionless scaling parameters denoted as r_p, r_d and r_i. These parameters are used in the decentralized PID controller tuning, i.e., to obtain the values of K_c, τ_I and τ_D. All the tuning relations required for cases A and B are summarized in the following sections.

4.1.1. Tuning relations for case A. The following relations are used for tuning the decentralized PID control for Case A.

$$\tau_D = r_d\alpha, \quad r_d > 1 \tag{19}$$

$$K_c = \frac{r_p}{K_p}\left(\frac{\tau_p}{\tau_D}\right), \quad 0 < r_p < 1 \tag{20}$$

$$\tau_I = r_i\left(\frac{\tau_p}{\theta}\right)\left[\max\left(\alpha, \tau_{I,\min_C}\right)\right], \quad r_i > 1 \tag{21}$$

4.1.2. Tuning relations for case B. The proposed tuning relations for a class of open-loop unstable MIMO system are given as follows:

$$\tau_D = r_d\left(\tau_p - \alpha\right) + \alpha, \quad 0 < r_d < 1 \tag{22}$$

$$K_c = \frac{r_p}{K_p}\left[\frac{\tau_p}{\tau_D} - \frac{\tau_I}{\tau_I - \alpha}\left(1 - \frac{1}{r_p}\right)\right], \quad 0 < r_p < 1 \tag{23}$$

$$\tau_I = r_i\left(\frac{\tau_p}{\theta}\right)\left[\max\left(\tau_{I,\min 1_B}, \tau_{I,\min 2_B}\right)\right], \quad r_i > 1 \qquad (24$$

4.2. MIMO model assumptions
In order to use the case A tuning relations, we make the following assumptions.

- **Assumption A.1**: A class of $n \times n$ MIMO system P.
- **Assumption A.2**: All diagonal transfer functions $g_{ii}, i = 1, 2, \dots n$ can be approximated by the FOPDT model in (2) – consider direct controller pairings are used.
- **Assumption A.3**: All transfer function in the plant matrix (**P**) are open-loop stable, i.e., none of them with an unstable or integrating pole.
- **Assumption A.4**: The PID controller for any given *i-th* control loop is designed by using the corresponding diagonal transfer function g_{ii}, i.e., direct controller pairings are adopted.
- **Assumption A.5**: The time-constant to deadtime ratio of a diagonal transfer function $(\tau_{p_{ii}}/\theta_{ii}\ ) > 1$.

Meanwhile, to use *Case B* tuning relations, the following assumptions are applied:

- **Assumption B.1**: A class of $n \times n$ MIMO system as represented by (25).
- **Assumption B.2**: All diagonal transfer functions $g_{ii}, i = 1, 2, \dots n$ can be approximated by the UFOPDT model in (3).
- **Assumption B.3**: All transfer functions in the plant matrix (P) are open-loop stable except for the diagonal transfer functions.
- **Assumption B.4**: The PID controller for any given *i-th* control loop is designed by using the diagonal transfer function g_{ii}, i.e., direct controller pairings are adopted.
- **Assumption B.5**: The ratio of time-constant to deadtime of a diagonal transfer function $(\tau_{p_{ii}}/\theta_{ii}\ ) > 1$.

4.3. Tuning algorithm 1
This tuning algorithm is applied to Case A.
*Step 1: Arrange the plant matrix (**P**) so that the Relative Gain Array (RGA) gives direct controller pairings.*
Step 2: Use initial settings ($r_p = 0.2, r_i = 5, r_d = 2$).
Step 3:Performance Evaluation

- Method 1: Time-domain plots. From the step responses for setpoint tracking and output disturbance rejection, observe whether desired responses are achieved or not. If not, go back to step 2, and readjust the parameters.
- Method 2: Optimization-based technique to obtain optimal values of r_p, r_i and r_d that minimizes the total Integral Absolute Error (IAE) value.

The recommended ranges for scaling parameters are as follows:

$$0.1 \le r_p \le 0.6,\ 2 < r_i \le 8,\ 1.5 \le r_d \le 5 \tag{25}$$

4.4. Tuning algorithm 2
This tuning algorithm is applied to Case B.
*Step 1: Arrange the plant matrix (**P**) so that the Relative Gain Array (RGA) suggests direct controller pairings.*
Step 2: Use initial settings ($r_p = 0.2, r_i = 2, r_d = 0.1$).
Step 3:Performance Evaluation
Can either use Method 1 or Method 2 as mentioned in the algorithm 1.

Recommended ranges for the scaling parameters are given by

$$0.2 \le r_p \le 0.6, 1.5 < r_i \le 6,\ 0.01 \le r_d \le 0.25 \tag{26}$$

5. Numerical examples

5.1. Example 1
Consider an industrial distillation process (case A) reported in [8]. In a decentralized control design, it is crucial to first determine the right controller pairings. This can be done by applying the Relative Gain Array (RGA) analysis; see [9]. The diagonal RGA values are 1.0926, 0.1039 and 0.0983. These suggest that the direct controller pairings should be adopted, i.e., $U_1 - Y_1, U_2 - Y_2, U_3 - Y_3$ pairings where U_i and Y_i denote i-th input and output respectively.

Two running values obtained via the proposed Routh-Hurwitz (RH) method are compared with the SML-MSC tuning [2]. Here, we consider two sets of RH tuning: set 1 with $r_p = 0.15$, $r_i = 6$ and $r_d = 4$ while set 2 with $r_p = 0.3$, $r_i = 6$ and $r_d = 4$. The set 1 tuning leads to $K_c = [-7.092, -5.428, -0.109]$, $\tau_I = [200, 24.3, 27.2]$, and $\tau_D = [1.42, 1.36, 3.18]$. Meanwhile, the set 2 yields $K_c = [-14.18, -10.86, -0.2185]$, $\tau_I = [200, 24.2, 29.4]$ and $\tau_D = [1.42, 1.36, 3.18]$. To evaluate the closed-loop performances corresponding to the different tuning values, sequential step changes of 1 unit each in the setpoints of Y_1,Y_2 and Y_3 are applied. Figure 2 shows the comparative closed-loop responses under the different controller tunings. The proposed R-H tuning can provide improved performance in term of the total Integral Absolute Error (IAE) value over the SML-MSC tuning. Note that, the closer the loop gain to its upper limit (or as the tuning parameter r_p increases), the faster the closed-loop responses become. However, due to process interactions and deadtimes, there is an upper limit on the r_p above which the system becomes unstable. To ensure closed-loop stability, we recommend the value of r_p to be between 0.1 and 0.5. When the interactions among the control loops become more severe, a smaller value of r_p should be used to ensure closed-loop stability.

5.2. Example 2
In this example, we consider a 2x2 unstable system

$$\mathbf{P} = \begin{bmatrix} \dfrac{1.5e^{-3s}}{20s-1} & \dfrac{1.2e^{-2s}}{20s+1} \\ \dfrac{1.1e^{-5s}}{38s+1} & \dfrac{1.6e^{-5s}}{22s-1} \end{bmatrix} \tag{27}$$

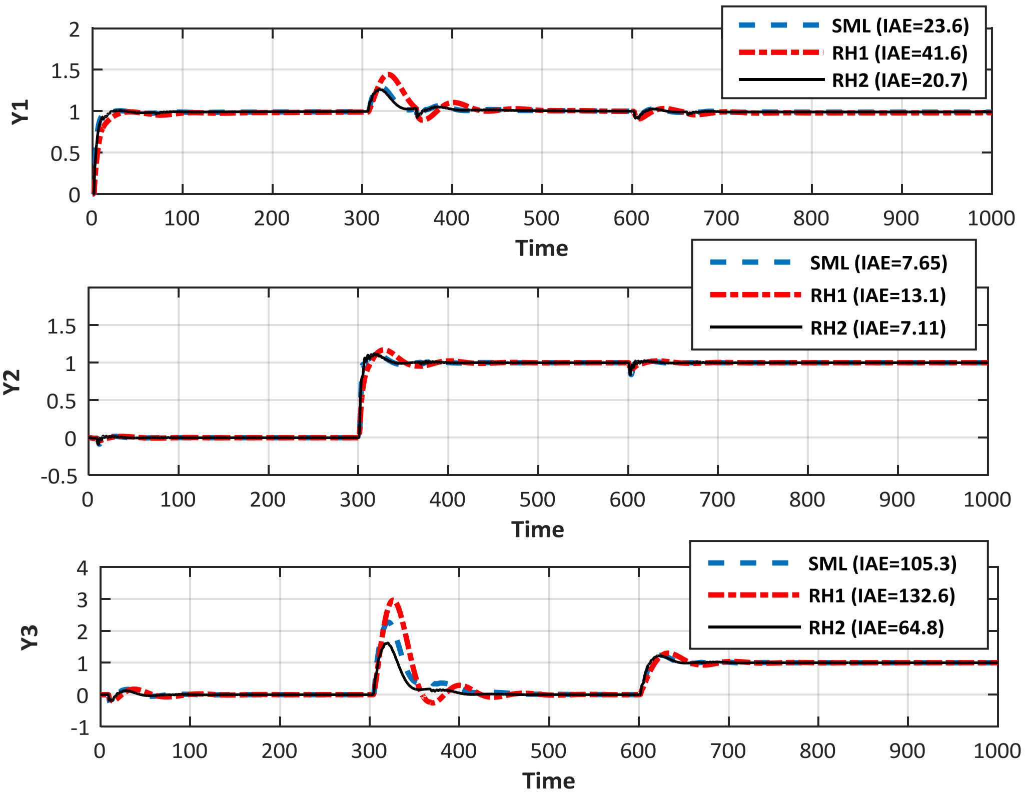

Figure 2. Setpoint responses for example 1.

For this unstable process (27), the R-H tuning (Case B) is adopted. Note that, many of the existing methods for the decentralized PID control tuning are not applicable to this type of unstable MIMO system. For an unstable system under PID controller, it is quite common to encounter a large overshoot in response to a setpoint change. To reduce this large overshoot, it is recommended to use a setpoint pre-filter which takes the form of

$$F_r = \frac{(\tau_I/\varepsilon)s+1}{\tau_I s+1}, \quad 1.5 \le \varepsilon \le 5 \tag{28}$$

where τ_I is the reset time of the given PID controller, and ε is a positive scaling parameter. The recommended range of the scaling parameter is between 1.5 and 5. The control performances are evaluated based on sequential step changes of 1 unit each in the setpoints of Y_1 and Y_2. Figure 3 shows the comparative closed-loop responses corresponding to two different tuning values: set 1 with $r_p = 0.6, r_i = 2, r_d = 0.05$ and set 2 with $r_p = 0.4, r_i = 2, r_d = 0.05$. Set 1 tuning leads to $K_c = [3.579, 2.642]$, $\tau_I = [31.9, 37.6]$ and $\tau_D = [2.425, 3.475]$ while set 2 tuning leads to $K_c = [2.615, 1.979]$, $\tau_I = [39.4, 47.4]$ and $\tau_D = [2.425, 3.475]$. For this system, it is not possible to increase the loop gain all the way close to its upper limit because of the process interactions. Also note that, reducing the loop gain so that it becomes too close to its maximum lower limit can cause the system to exhibit high oscillation. The closed-system can become unstable if the loop gain becomes too near to either its minimum upper limit or its maximum lower limit.

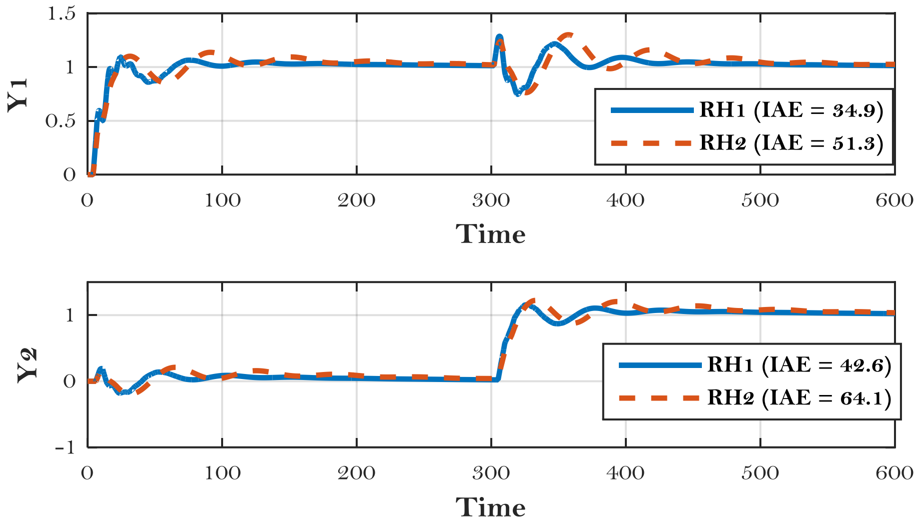

Figure 3. Responses for example 2.

6. Conclusion

A tuning method has been proposed via which we can simplify the tuning task from finding $3n$ PID parameters to finding only 3 common tuning parameters, namely r_p, r_i and r_d. This represents a great simplification for an otherwise very complicated tuning problem.

References

[1] Huang H, Jeng J, Chiang C and Pan W 2003 A direct method for multi-loop PI/PID controller design *Journal Process Contol* **13** pp 769- 86

[2] Nandong J 2015 Heuristic-based multi-scale control procedure of simultaneous multi-loop PID tuning for multivariable processes *Journal of Process Control* **35** pp 101-12

[3] Nandong J and Zang Z 2013 High-performance multi-scale control scheme for stable, integrating and unstable time-delay processes, *Journal of Process Control* **23** pp 1333-43

[4] Nandong J and Zang Z 2014 Inter-Communicative Decentralized Multi-Scale Control (ICD-MSC) Scheme: A New Approach to Overcome MIMO Process Interactions *Chemical Product and Process Modeling* **9** pp 165-78

[5] Silva G, Datta A and Bhattacharyya S 2002 New results on the synthesis of PID controllers," *IEEE Transactions of Automaic Control* **47** pp 241-52

[6] Ho W, Hang C and Cao L 1995 Tuning of PID controllers based on gain and phase margin specifications *Automatica* **31** pp 497-502

[7] Seer Q and Nandong J 2017 Stabilization and PID tuning algorithms for second-order unstable processes with time-delays *ISA Transactions* **67** pp 233-45

[8] Tyreus B Multivariable control system design for an industrial distillation column 1979 *Industrial & Engineering Chemistry Process Design and Development.* **18** pp 177-82

[9] Liceaga-Castro E, Liceaga-Castro J and Ugalde-Loo C 2005 Beyond the existence of diagonal controllers: from the relative gain array to the multivariable structure function *CDC-ECC'05, 44th IEEE Conf* (Seville Spain)