To the approximation of rectangular and complex cross-sections of reinforced concrete structures under the action torsion with bending

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Abstract. Special analytical method for determining tangential tension stresses in reinforced concrete structures operating in conditions of complex resistance –torsion with bending is proposed in this paper. Its peculiarity consists in the approximation of rectangular and any complicated cross-sections of reinforced concrete structures with the help of their division into squares with the circles inscribed there in, connected together into a single monolithic figure. The dependence of tangential torsion stresses becomes valid on the distance to the centre of the circle under consideration within each j-th circle. The further the circle from the centre of the rectangle is located, the greater its moment of inertia becomes and the maximum stresses are reached in the middle of the rectangle the long sides. Such model makes it possible to remove the question of the necessity using of special tables also for their calculation in the elastic stage. Also it makes possible to separate the stress-strain state in a whole set of circular sections from the additional field associated with the deformation of the rectangular section. The authors corrected and significantly supplemented the dependencies for taking into account the deplanation of a rectangular cross-section rod. Attention is focused on the physical essence of longitudinal displacements caused by deplanation, an analogy with elementary movements caused by shearing forces is carried out. In the study, the classification of spatial cracks for reinforced concrete rod structures under the action torsion with bending was generalized; while the process of spatial cracks formation of the first, second, and third types is tied to the proposed method for determining tangential stresses (angular deformations) for complex cross-sections. The proposed dependencies allow us to search for the values of the model design parameters at of the stress-strain state stages of the reinforced concrete rod structure, including in the plastic and in the limiting stages. The components of the torsion stress (angular deformations) are again synthesized and separated by the proposed method for the convenience of analysis in principal stresses tensors (main strains). Transformational transitions from a cylindrical to a Cartesian coordinate system and the attraction of local coordinate systems made it possible to simplify the equations as much as possible. Moreover, the equations are constructed in such a way that the resolving system does not turn into a decaying system. The physical interpretation of the solution obtained, with respect to the problem of crack resistance, is that it allows us to search for the minimum generalized load that corresponds to the formation of the first, second or third spatial crack types and the coordinates of their formation point. As a result, the effectiveness of the proposed method is shown with approximating rectangular and complex cross sections of reinforced concrete structures under the action

torsion with bending and taking into account physical nonlinearity, deplanation of cross-sections, pre stressing in longitudinal and transverse reinforcement and the influence of local stress fields.

1. Introduction

The deepening and improvement of theoretical studies of reinforced concrete structures work with their complex resistance – torsion with bending becomes more and more actual [1–3]. Due to the fact that such studies are carried out relatively, and due to the economic conditionality and the need to take into account the vast majority spatial work of the reinforced concrete structures (beams of monolithic overlapping, onboard elements and the reference contour of ribbed carvings and bridges, substructure and crane beams, contour beams of buildings with a monolithic frame, supports of power lines, reinforced concrete pylons, etc.). If we take into account at the same time, the inaccuracies admitted during all reinforced concrete structures installation and manufacturing [4–6] are practically working in conditions of complex resistance.

It should be noted that circular cross-sections have only a certain proportion of the variety of reinforced concrete structures used in the practice of modern construction, as a rule, the shape of their cross-section is more complicated.

2. Purpose and objectives of the research

Therefore, the purpose of this research is to develop methods for assessing the resistance of reinforced concrete structures under the action torsion with bending of rectangular and complex cross-sections (consisting of a set of rectangles), without resorting to complicated formulas and methods of the elasticity and plasticity theory.

3. The main part of the research

The proposed method for evaluating the reinforced concrete structures resistance of rectangular and complex cross-sections (consisting of a set of rectangles) is based on the fact that the rectangular section is divided into a number of squares, which are subsequently replaced by the rings marked in them circles, Figure 1.

Moreover, if the integer number of squares does not fit into a rectangle, in this case, the division into squares also occurs in the opposite direction.

Then one of the squares superimposition areas is excluded from different directions of breakdown and from the simulated section. Moreover, the squares beneath, in the middle and at the cross section, fit organically into the various types of cracks occurring precisely in the marked zones. Thus, the analysis of performed experiments [7–11] shows that in the case of torsion with bending, the following spatial cracks types occur: the first type (which intersects only longitudinal reinforcements for $M > M_{crc}$ and $Q \geq Q_{crc}$), the second or third type (intersecting only the transverse reinforcement for $M < M_{crc}$, $M_t > M_{t,crc}$ and $Q > Q_{crc}$ and oriented with further development in the direction of the point of concentrated force application or with an arbitrary orientation, respectively).

Let's consider the proposed approach implementation, for example, with respect to the fracture resistance problem of reinforced concrete structures with complex resistance – torsion with bending.

The formation of the first spatial crack occurs at an arbitrary point A, located on the lower or lateral faces or at an arbitrary point of the cross-section complex figure (T-shaped, I-shaped, box-shaped, etc.). Thus, each type of spatial cracks [12–15] is formed in a specific circle, and the use of the calculated formulas for a circular cross-section allows us to find an arbitrary the point of their formation.

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Now it becomes clear that the proposed approach allows the well-known formulas use for circular cross-sections for the rectangular sections simulation, as well as any complex cross-sections consisting of rectangles.
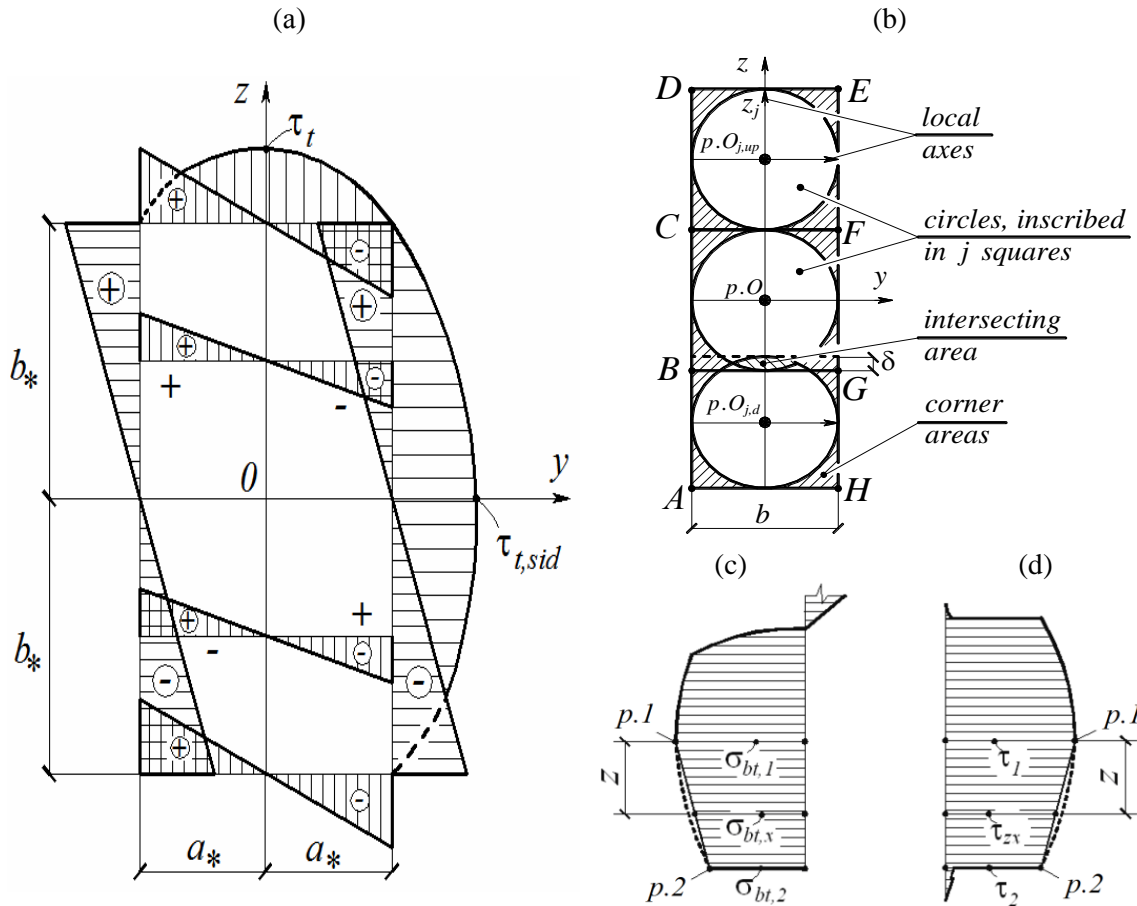

Figure 1. Diagrams of tangential tensions in torsion τ_t, positive and negative zones of transverse rectangular sections deformation (a); approximation of the *ABCDEFGH* figure of the cross-section with the help of squares and inscribed circles (b), and the diagrams of the normal $\sigma_{bt,i}$ and tangential τ_{zx} stresses in the cross-section, passing through an arbitrary point A (b, c), respectively.

The equations located at a distance x from the support for determining the tangential torsion stresses τ_t in the corresponding cross-sectional circle and are recorded in a cylindrical and Cartesian coordinate system, accordance with Figure 1:

$$\tau_t = \tau_{t,j} = \frac{M_{t,j}}{I_{t,j}} \cdot \rho = \frac{M_{t,j}}{I_{t,j}} \left((\zeta z)^2 + y^2 \right)^{1/2} \leq \tau_{t,u}, \qquad (1)$$

where ρ is the distance from the center of the j-th circle to the point at which tangential torsional stresses are determined τ_t, ζ are the coefficient of transition to local axes; $\tau_{t,u}$ is the limiting tangential stress caused by torsion.

Here, the torsion inertia moment in the general case of a complex cross-section consisting of rectangles is equal to the sum of the squares inertia moments where the rectangles are broken down with their subsequent approximation by the circles inscribed in these squares (in this case, one of the overlapping parts of the intersecting sections in the summation is included with the sign "minus", and

the angular sections in view of their insignificant influence on the values of tangential stresses are not taken into account, Figure 1, b),

$$I_t = I_{t,1} + I_{t,2} + \ldots + I_{t,j} = \sum I_{t,j} , \tag{2}$$

and each of the torsion moments incident on the inscribed circles are respectively determined, –

$$M_{t,1} = M_t \frac{I_{t,1}}{I_t} ; \qquad M_{t,2} = M_t \frac{I_{t,2}}{I_t} ; \ldots M_{t,j} = M_t \frac{I_{t,j}}{I_t} ; \tag{3}$$

$I_{t,j}$ is the inertia moment of the circle used in formula (2), inscribed in the corresponding square (the lower circle is used, as a rule, for the first type cracks, the middle circle is used for the second and third types, Figure 1, b); ζ is coefficient of transition to local axes.

Method for determining the tangential tension stresses by approximating rectangular and any complicated cross-sections of reinforced concrete structures by their division into squares and the circles inserted into these squares, connected together into a single monolithic figure is proposed. It also allows us to remove questions about the choice of their two twisting stresses τ_{zx} and τ_{yx}, the need for the addition of vectors τ_{zx} and τ_{yx} in a rectangular section (with the resulting vector coinciding with the direction of the hydrodynamic trajectories) and the necessity of use them in special tables for calculation.

Within each circle, the fairly well-known dependence of the tangential tensions from the distance to the circle center in question becomes fair. Moreover, the circle is from the rectangle center with larger its moment of inertia (except for its own, a term associated with the transfer of coordinates local axes is added to it). Thus, in the circles the tangential stresses of the torsion decrease, removed from the central circle.

It should be noted that if the proposed method closes on formula (1), which is valid only for the j-th circle, it will not consider the deplanation associated with the rectangular cross-sectional shape and this question requires separate consideration (it is given below). We should also pay attention to the calculation devicet constructing, which is advisable to write out the equations for a complicated form of the cross-section, but not in a cylindrical one in a Cartesian coordinate system. In this case $\tau_{t,j}$, sought by formula (1), we will consider as a resultant of two components $\tau_{t,xz}$ and $\tau_{t,xy}$, which are determined from the following dependences:

$$\tau_{t,xz} = \tau_{t,xz,j} = \tau_{t,j} \cdot \sin\alpha = \frac{M_{t,j}}{I_{t,j}} \left((\zeta z)^2 + y^2\right)^{1/2} \cdot \frac{\zeta z}{\left((\zeta z)^2 + y^2\right)^{1/2}} = \frac{M_{t,j}}{I_{t,j}} \cdot \zeta z \le \tau_{t,xz,u} ; \tag{4}$$

$$\tau_{t,xy} = \tau_{t,xy,j} = \tau_{t,j} \cdot \cos\alpha = \frac{M_{t,j}}{I_{t,j}} \left((\zeta z)^2 + y^2\right)^{1/2} \cdot \frac{y}{\left((\zeta z)^2 + y^2\right)^{1/2}} = \frac{M_{t,j}}{I_{t,j}} \cdot y \le \tau_{t,xy,u} , \tag{5}$$

where $\tau_{t,xz,u}$, $\tau_{t,xy,u}$, are the components of the limiting values of tangential torsional stresses.

In order to take into account plastic deformations, the moment of inertia $I_{t,j}$ and I_j is recommended to be simplified in the cracking stage:

$$I_{t,j} = 0,85 \cdot I_{t,j,red} ; \quad I_j = 0,85 \cdot I_{j,red} , \tag{6}$$

for dependencies in where the parameter $I_{t,j}$ does not belong to the same recommendation can be used with respect to the modules E and G, with respect to the stage of crack formation, and in subsequent loading stages, it is necessary to use the secant strain modulus and the variable coefficient of transverse strains for concrete [2] in the calculated dependences.

The following formulas also contain reservations regarding the inelastic resistance of concrete where it is necessary.

In addition to taking into account the physical nonlinearity of concrete with the help of a split module, the deplanation of cross-sections, pre stresses in longitudinal and transverse reinforcement, and the influence of the local stresses field are taken into account in the proposed method.

In this case, the deplanation of the rectangular cross section of the structure (Figure 1, a) will be taken into account with the use of the dependences given in [17], with some correction and necessary development for the relative angular deformations:

$$w = \frac{M_t}{G \cdot I_t} \cdot f(y, z);$$ (7)

where

$$f(y, z) = \beta_t \cdot y \cdot z,$$ (8)

$$\beta_t = \frac{a_*^2 - b_*^2}{a_*^2 + b_*^2}.$$ (9)

The physical nature of the displacements w consists in the fact that they are due to the tangential stresses of torsion and the shear deformations caused by them (here one can draw an analogy with the displacements Δ_Q, caused by the transverse force [17]). It follows that relative displacement of the deplanation w will correspond to the relative deformations of the shift γ_{zx}, which are caused by shear stresses of torsion τ_t, Figure 1. It should also be taken into account that the displacement of the deplanation w varies along the length of the reinforced concrete rod construction from zero at the pinched end to its maximum value at the free end. To take into account this circumstance, we correct the formula (6) by introducing an additional function $f_2(x)$ in the form:

$$f_2(x) = l \cdot \left(1 - \frac{x}{l}\right).$$ (10)

Indeed, for $x = 0$, $f_2(x) = l$ and for $x = l$, $f_2(x) = 0$. As a result, the displacement due to the cross-sectional deformation is written as:

$$w = \frac{M_t}{G \cdot I_t} \cdot f(y, z) \cdot f_2(x) = \frac{M_t}{G \cdot I_t} \cdot \frac{a_*^2 - b_*^2}{a_*^2 + b_*^2} \cdot y \cdot z \cdot l \cdot \left(1 - \frac{x}{l}\right).$$ (11)

Thus, the displacement w is a complex function, depending on the coordinates y, z, x. When finding the relative angular (shear) deformation of deplanation $\gamma_{d,zx}$ and $\gamma_{d,yx}$ with using the Cauchy curves, take the form:

$$\gamma_{d,zx} = \frac{\partial w}{\partial z} + \frac{\partial \omega}{\partial x} = \frac{\partial w}{\partial z} + 0 = \frac{\partial w}{\partial z},$$ (12)

$$\gamma_{d,yx} = \gamma_{yx} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial w}{\partial y} + 0 = \frac{\partial w}{\partial y},$$ (13)

where w, v, ω are the displacements in the direction of the axes, x, y and z, accordingly. With reference to the deplanation model described by formula (11), the displacements are $v = \omega = 0$. In the end we will have:

$$\gamma_{d,zx} = \frac{M_t}{G \cdot I_t} \cdot \frac{a_*^2 - b_*^2}{a_*^2 + b_*^2} \cdot l \cdot \left(1 - \frac{x}{l}\right) \cdot z \leq \gamma_{d,zx,ul},$$ (14)

$$\gamma_{d,yx} = \frac{M_t}{G \cdot I_t} \cdot \frac{a_*^2 - b_*^2}{a_*^2 + b_*^2} \cdot l \cdot \left(1 - \frac{x}{l}\right) \cdot y \leq \gamma_{d,yx,ul},$$ (15)

where $\gamma_{d,zx,ul}$ and $\gamma_{d,yx,ul}$ are the components of the limiting relative angular deformations of deplanation.

4. The main working hypotheses and assumptions

The basis of the calculated crack-forming model construction under the action torsion with bending for spatial cracks of the first type is the following calculation prerequisites [12–15, 18, 19]:

– formation of the first type spatial crack, second or third occurs at an arbitrary point A located on the lower or lateral faces or at an arbitrary point of the cross-section complex figure (including a hollow, T-shaped cross-section, etc.) after the main deformations of the concrete elongation ε_{bt} have reached their limit values $\varepsilon_{bt,ul}$.

– diagrams of tangential stresses in torsion τ_t; positive and negative zones of transverse rectangular sections deplanation and the approximation of the cross section figure using the squares of the ABCDEFGH and the inscribed circles, are executed in accordance with the diagrams in Figure 1, a, b;

– the diagrams of normal σ_x and tangential stresses $\tau_{zx,Q}$ in the cross-section, passing through an arbitrary point A are approximated between the points 1 and 2 (Figure 1, c, d) by linear dependences.

5. Derivation of calculated dependencies

The following equations are used to evaluate the resistance of rod concrete structures to the formation of the first spatial crack.

1). Equation of communication between normal stresses σ_x in a cross-section located at a distance from x the support and a generalized external load expressed through the support reaction $R_{\sup}$ at the time of the first spatial crack formation including the bending moment from the external forces $R_{\sup} \cdot x$, the bending moment $P_0 \cdot e_{0,p}$ from the pre stress force, the longitudinal force N and the force preliminary stresses P_0, as well as local influences $\dfrac{R_{\sup}}{b \cdot h} \cdot \varphi_x$ (φ_x – the calculation coefficient of local normal stresses σ_x in the direction of the x axis from the reference reactions), from which follows:

$$R_{\sup} = \frac{\sigma_x \cdot A_{red} \cdot 0.85 I_{red} - N \cdot 0.85 I_{red} - P_0 \cdot e_{0,p} \cdot A_{red} \cdot 0.5h}{x \cdot 0.5h \cdot A_{red} + \varphi_x \cdot 0.85 I_{red}} \ . \tag{16}$$

Note that in the formulas (16)–(33) below, all notations that are not deciphered correspond to the common notions of the mechanics of a solid deformable body and the theory of reinforced concrete.

2). Equation for determining tangential torsion stresses $\tau_{t,xy,j}$ in a cross section located at a distance x from the support for spatial cracks of the first type. It is written in accordance with Figure 1, equation (5). For spatial cracks of the second and third types, the unknown coordinate y from equation (5) is determined:

$$y = \frac{\tau_{t,xy} \cdot I_{t,j}}{M_{t,j}} \le 0.5b \ . \tag{17}$$

3). Equations for the determination of tangential stresses $\tau_{xz,Q}$ in a cross section located at a distance x from the support. In this case, the coupling equations between tangential stresses in the cross-section of the reinforced concrete rod and the generalized load $R_{\sup}$ τ_2 and τ_1 take into account the transverse force from the reference reaction (taking into account local stresses) and the transverse force perceived by the bent rod.

From this equation, the unknown coordinate z of the formation of spatial cracks of the second and third types is determined:

$$z = \frac{\left(R_{\sup} - Q_{inc}\right)B_1 + \dfrac{R_{\sup}}{A_{red}} \cdot \varphi_{xz} - \tau_{xz,Q}}{\left(R_{\sup} - Q_{inc}\right) \cdot B_2} \le 0.5h \ , \tag{18}$$

where φ_{xz} is the coefficient of local shear stress τ_{xz} in the z direction from the reference reactions.

With respect to cracks of the first type, there is no need in finding the coordinate z (the equation degenerates into an equality $z = -0.5h$), in this case it is expedient to use equation (18) for determination $\tau_{zx,Q}$. After algebraic transformations, with respect to cracks of the first type, we get:

$$\tau_{zx,Q} = \left(R_{\sup} - Q_{inc}\right)\left(-0.5h \cdot B_2 - B_1\right) - \frac{R_{\sup}}{A_{red}} \cdot \varphi_{zx} \leq \tau_{u,Q}. \tag{19}$$

Here the parameter B_1 is determined by the formula:

$$B_1 = \frac{S_{n.ax}}{0.85 I_{red} \cdot b}, \tag{20}$$

the parameter B_2 for the *T*-section (shelf above), and the *T*-section (the shelf from below), is determined by the formulas, respectively,

$$B_2 = \frac{1}{0.85 I_{red} \cdot b \cdot \left(h - z_d - h'_f\right)}; \tag{21}$$

$$B_2 = \frac{1}{0.85 I_{red} \cdot b \cdot \left(z_d - h_f\right)}, \tag{22}$$

z_d is the distance from the gravity center of the section to the lower face, the parameter h'_f for rectangular sections is assumed to be equal $(h - z_d)/3$, and the parameter h_f is assumed to be equal to $z_d/3$; τ_u is the limiting tangential stress caused by transverse forces.

4). The equation of the external load connection (expressed through the reference reaction Rsup) and normal stresses σ_z, is recorded taking into account local stress fields from the reference reaction and applied to the construction of the concentrated force, and also taking into account the pre stress in the clamps and bends. From this equation, with the respect to the spatial cracks of the first, second and third types, the unknown σ_z is determined:

$$\sigma_z = \frac{R_{\sup}}{A_{red}} \cdot \left(\varphi_z + k \cdot \varphi_{2,z}\right) + B_3. \tag{23}$$

Here

$$B_3 = \frac{\sigma_{sw,p} \cdot A_{sw,p}}{s_{sw,p} \cdot b} + \frac{\sigma_{inc,p} \cdot A_{inc,p}}{s_{inc,p} \cdot b} \cdot \sin\theta, \tag{24}$$

φ_z is the coefficient of local normal stresses σ_z in the direction of the z axis from the reference reactions; $k \cdot \varphi_{2,z}$ is the coefficient of local normal stresses σ_z in the direction of the z axis from the concentrated forces.

5). Using the precondition that the main deformations of the concrete elongation reach their limiting values in the formation of spatial cracks, applied to spatial cracks of the first type $\sigma_y = 0$,

$$\varepsilon_y = -\frac{\mu}{0.85E}\left(\sigma_x + \sigma_z\right); \quad \varepsilon_x = \frac{1}{0.85E}\left[\sigma_x + \sigma_{x,d} - \mu\sigma_z\right]; \quad \varepsilon_z = \frac{1}{0.85E}\left[\sigma_z - \mu\sigma_x\right]; \quad \tau_{zy} = \tau_{yz} = 0;$$

$\gamma_{yz} = 0$ after algebraic transformations, a formula is obtained for determining the axial deformations of the elongation of concrete in the direction of the axis x:

$$\varepsilon_x = \frac{\gamma_{xz}^2\left(\varepsilon_y - \varepsilon_{bt,ul}\right) + 4\varepsilon_{bt,ul}^3 - 4\varepsilon_{bt,ul}^2\left(\varepsilon_y + \varepsilon_z\right) + \varepsilon_{bt,ul}\left(4\varepsilon_z\varepsilon_y - \gamma_{xy}^2\right) + \varepsilon_z \cdot \gamma_{xy}^2}{4\left(\varepsilon_z\varepsilon_{bt,ul} - \varepsilon_{bt,ul}^2 + \varepsilon_y\varepsilon_{bt,ul} - \varepsilon_y\varepsilon_z\right)}. \tag{25}$$

Here $\gamma_{xz} = \gamma_{xz,Q} \pm \gamma_{xz,t} + \gamma_{d,xz}$; $\gamma_{xy} = \gamma_{xy,Q} \pm \gamma_{xy,t} + \gamma_{d,xy}$. The "plus" sign or "minus" sign is selected depending on the direction of the transverse force and the torque in the left or right side of construction the cross-section with respect to its vertical axis.

Then, using the connection $\sigma_x - \varepsilon_x$, we get:

$$\sigma_x = \varepsilon_x \cdot 0.85E - \sigma_{x,d} + \mu\sigma_z. \tag{26}$$

Other equations are used for fractures of the second and third types. In particular, to determine the normal stresses σ_x, the condition to achieve the values of the principal tensile stresses equal to R_{bt} is used. Then, from the equation for determining the principal tensile stresses, taking into account that $\sigma_y = 0$, $\tau_{zy} = \tau_{yz} = 0$, $\tau_{zx} = \pm\tau_{zx,t} + \tau_{zx,Q}$, $S_1 = \sigma_x + \sigma_z$, $S_2 = \sigma_x\sigma_z - \tau_{zx}^2$; $S_3 = 0$, after algebraic transformations we obtain:

$$\sigma_x = \frac{\tau_{zx}^2 + R_{bt} \cdot \sigma_z - R_{bt}^2}{\sigma_z - R_{bt}}, \tag{27}$$

Here $\tau_{zx} = \tau_{zx,Q} \pm \tau_{zx,t} + \tau_{d,zx}$; the "plus" sign or "minus" sign is selected depending on the direction of the transverse force and the torque in the left or right side of the structure cross-section with respect to its vertical axis; $\tau_{d,zx}$ – tangential torsion stresses caused by the deplanation of a rectangular cross-section rod, complementing tangential torsion stresses that occur in a rod of circular cross-section (or a complex section simulated with a square and inscribed circles in it – Figure 1, b).

From the equation for determining the main deformations of concrete elongation (taking them equal $\varepsilon_{bt,ul}$), angular deformations γ_{xz} are sought. As a result, using the dependence $\tau_{xz} - \gamma_{xz}$, for tangential stresses τ_{xz} in the cross section, the following equation is obtained:

$$\tau_{xz} = \pm\frac{0.85E}{(1+\mu)}\left(\frac{4\varepsilon_{bt,ul}\varepsilon_x\varepsilon_z - 4\varepsilon_{bt,ul}^2(\varepsilon_x + \varepsilon_z) + 4\varepsilon_{bt,ul}\varepsilon_x\varepsilon_y\varepsilon_z \cdot (\varepsilon_z + \varepsilon_x) - \varepsilon_y\left(4\varepsilon_{bt,ul}^2 + 4\varepsilon_x\varepsilon_z\right)}{\left(\varepsilon_{bt,ul} - \varepsilon_y\right)}\right)^{1/2} \leq \tau_u. \tag{28}$$

Here it is necessary to emphasize that the dependence (28) for τ_{zx} includes the influence of tangential stresses (angular deformations) caused by transverse force, torque and deplanation of a rectangular section. From the dependence (28) is sought τ_{zx}. From the dependence (4) we have tangential stresses $\tau_{zx,t}$ (in the limiting stage, before the formation of a crack, they can reach their limiting values $\tau_{zx,u}$).

Tangential stresses $\tau_{d,zx}$ are sought using a known relationship valid for an isotropic body – for this $\gamma_{d,zx}$, the value obtained from (14) is multiplied by the shear modulus.

Now, having the components $\tau_{zx,t}$, $\tau_{d,zx}$ and the resulting τ_{zx}, by their difference, one can find $\tau_{zx,Q}$ either their limiting values (in a whole series of problems, including for the considered problem of crack resistance), to have limiting values $\tau_{zx,Q}$ is extremely important.

6). Note that maximum tangential torsional stresses $\tau_{t,xz,j}$ applied to cracks of the second and third type τ_t are in the central circle and are determined from the relationship between the bending and torsion moments $M_{bend}/M_t = \eta$:

$$\tau_{t,xz} = \tau_{t,xz,j} = \tau_{t,j}\sin\alpha = \frac{M_{t,j}}{I_{t,j}}\left((\varsigma z)^2 + y^2\right)^{1/2}\frac{\varsigma z}{\left((\varsigma z)^2 + y^2\right)^{1/2}} = \frac{(R_{sup} \cdot x)_j}{I_{t,j} \cdot \eta}\varsigma z \leq \tau_{t,xz,u}. \tag{29}$$

In turn, the bending moment depends on R_{sup} and x, and with respect to the cracks of the second and third types on $\tau_{t,xz,j}$ the variable ςz also exerts a significant influence.

It should be emphasized that the maximum tangential torsion stresses $\tau_{t,xy,j}$ are sought not only with the ratio between the bending and tourque moments $M_{bend}/M_t = \eta$, but also the relationship

between the maximum tangential stresses $\tau_{t,xy,j}$ and $\tau_{t,xz,j}$, similarly to that adopted in the materials resistance [16] using the coefficient γ.

$$\tau_{t,xy} = \tau_{t,xy,j} = \tau_{t,j} \cdot \cos\alpha = \frac{(R_{\sup} \cdot x)_j}{I_{t,j} \cdot \eta} \cdot y = \frac{(R_{\sup} \cdot x)_j}{I_{t,j} \cdot \eta} \cdot \zeta z \cdot \gamma \le \tau_{t,xy,u}. \tag{30}$$

Here, according to the proposed model for approximating rectangular sections by dividing them into squares with inscribed circles, each circle has its own different from other circles, a pair of shearing stresses in the local Cartesian coordinate system, – $\tau_{t,xy} = \tau_{t,zx}$. Then in formula (30) $\gamma=1$.

It should be emphasized that, with respect to cracks of the first type, tangential torsion stresses $\tau_{t,xy,j}$ in the lower circle are taken into account, reaching their highest values in accordance with formula (5).

7). Analyzing the resolving equations, it should also be noted that in the tensors of principal stresses (principal deformations), the components of the torsion stresses (angular deformations) separated by the proposed method for the convenience of analysis are synthesized. Transformational transitions from a cylindrical to a Cartesian coordinate system and the attraction of local coordinate systems made it possible to simplify the equations as much as possible. Moreover, the equations are constructed in such a way that the resolving system does not turn into a decaying system.

Using the resulting equations with respect to the cracks of the second and third types, we form the function of several variables $F\left(R_{\sup}, y, z, \sigma_z, \sigma_x, \tau_{t,xz}, \tau_{t,xy}, x, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8\right)$, which has the following form:

$$F\left(R_{\sup}, y, z, \sigma_z, \sigma_x, \tau_{t,xz}, \tau_{t,xy}, x, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8\right) =$$

$$= \frac{\sigma_x \cdot A_{red} \cdot 0.85 I_{red} - N \cdot 0.85 I_{red} - P_0 \cdot e_{0,p} \cdot A_{red} \cdot z}{x \cdot z \cdot A_{red} + \varphi_x \cdot 0.85 I_{red}} + \left[y - \frac{\tau_{t,xy} \cdot I_{t,j}}{M_{t,j}}\right]\lambda_1 +$$

$$+ \left[z - \left(\frac{\left(R_{\sup} - Q_{inc}\right)B_1 + \frac{R_{\sup}}{A_{red}} \cdot \varphi_{xz} - \tau_{xz,Q}}{\left(R_{\sup} - Q_{inc}\right) \cdot B_2}\right)\right]\lambda_2 + \left[\sigma_z - \left(\frac{R_{\sup}}{A_{red}} \cdot \left(\varphi_z + k \cdot \varphi_{2,z}\right) + B_3\right)\right]\lambda_3 +$$

$$+ \left[\sigma_x - \frac{\tau_{zx}^2 + R_{bt} \cdot \sigma_z - R_{bt}^2}{\sigma_z - R_{bt}}\right]\lambda_4 +$$

$$+ \left[\tau_{zx} - \frac{0.85E}{(1+\mu)}\left(\frac{4\varepsilon_{bt,ul}\varepsilon_x\varepsilon_z - 4\varepsilon_{bt,ul}^2(\varepsilon_x + \varepsilon_z) + 4\varepsilon_{bt,ul}\varepsilon_x\varepsilon_y\varepsilon_z \cdot (\varepsilon_z + \varepsilon_x) - \varepsilon_y\left(4\varepsilon_{bt,ul}^2 + 4\varepsilon_x\varepsilon_z\right)}{\left(\varepsilon_{bt,ul} - \varepsilon_y\right)}\right)^{1/2}\lambda_5\right] +$$

$$+ \left[\tau_{t,xz} - \frac{(R_{\sup} \cdot x)_j \cdot \zeta z}{I_{t,j} \cdot \eta}\right]\lambda_6 + \left[R_{\sup} - \frac{\sigma_x \cdot A_{red} \cdot 0.85 I_{red} - N \cdot 0.85 I_{red} - P_0 \cdot e_{0,p} \cdot A_{red} \cdot z}{x \cdot z \cdot A_{red} + \varphi_x \cdot 0.85 I_{red}}\right]\lambda_7 +$$

$$+ \left[\tau_{t,xy} - \frac{(R_{\sup} \cdot x)_j \cdot \zeta z}{I_{t,j} \cdot \eta}\right]\lambda_8 \tag{31}$$

Performing the differentiation of the function (31) with respect to the corresponding variables and equating their derivatives with zero, an additional system of equations is obtained using the Lagrange multipliers λ_i. From the solution of the equations additional system, a formula is obtained for determining the coordinate x of the spatial crack point formation:

$$x = \frac{M_{t,j} \cdot I_{t,j} \cdot \eta \cdot \left((\zeta \cdot z)^2 + y^2\right)^{1/2} \cdot \left(\tau_{t,xz}^2 \cdot I_{t,j}^2 - (\zeta \cdot z)^2\right)^{1/2}}{\tau_{t,xz} \cdot R_{\sup} \cdot y}. \tag{32}$$

Similarly, a function of several variables F_1 is compiled, differentiation with respect to the corresponding variables is performed with the equating of the derivatives to zero, and the coordinate x of the point of the first type spatial crack formation is determined.

$$x = \frac{0.85 I_{red}\lfloor \varphi_x + \varphi_z + k \cdot \varphi_{2,z} - \mu \cdot \varphi_z - \mu \cdot k \cdot \varphi_{2,z} \rfloor}{0.5 h \cdot A_{red}}. \tag{33}$$

As a result, all the resolving equations and the parameters determined from them turn, out to be "closed" in a single solution to the problem associated with determining the minimum generalized load and the formation coordinates of the various types spatial cracks in reinforced concrete structures under the action torsion with bending.

After finding the abscissa x of the point A, a spatial crack of the first, second or third type is formed and the search for the general cracking load expressed as a function through the support reaction R_{sup}, the spatial arrangement of the main sites is determined, in the vicinity of this point and the direction of the spatial crack development.

In this case, the direction cosines l, m, n are found from the stress state equations on the principal and axial areas and the condition of equality to the unit squared direction of the cosines, taking into account that for the problem under consideration $\sigma_y = 0$, $\tau_{zy} = \tau_{yz} = 0$, $\tau_{zx} = \tau_{zx,Q} \pm \tau_{zx,t} + \tau_{d,zx}$, $\tau_{xy} = \tau_{xy,t} + \tau_{d,xy}$, but when modeling a rectangular section with inscribed circles $\tau_{xy,t} = \tau_{xz,t}$, $\sigma = \sigma_1 = \beta \cdot R_{bt}$ (β is the coefficient taking into account the reduction of the limiting principal (minimum) tensile stresses compared with normal tensile stresses $\sigma_x = R_{bt}$).

The physical interpretation of the solution obtained is that it allows us to determine the minimum generalized load, which corresponds to the formation of the first spatial crack at an arbitrary point in the structure and the corresponding coordinates of its formation.

6. Conclusions

Based on the analysis of domestic existing scientific research and foreign scientists devoted to the study of reinforced concrete beams under complex resistance conditions, torsion with bending, normative documents, and the experimental and theoretical studies performed in this work, the following conclusions should be drawn.

1. At the present time in Russia and abroad there are no sufficiently stringent recommendations and corresponding normative documents for determining the limiting states of the first and second group for reinforced concrete structures operating under conditions of a complex stress-strain bending state with torsion. The current documents rely either on too simplified models and do not reflect the actual resistance, or they do not give a clear algorithm for their calculation, and first of all in the investigation of fracture problems, since the torsion with bending the moment of crack formation and the magnitude of their inclination significantly influence the further tensile-deformed state.

2. The authors proposed a special reception determining torsion shear stresses by approximating rectangular and any complex cross-sections of reinforced concrete structures by their division into squares with inscribed circles in them, interconnected into a single monolithic piece. This model allows you to remove questions about choosing the right of the two torsion stress τ_{zx} and τ_{yx} on the uncertainty of the addition of vectors τ_{zx} and τ_{yx} rectangular cross-section (with the resulting vector, which coincides with the direction of hydrodynamic trajectories) and the need using special tables for calculating them, not only in the elastic stage. It also makes possible to separate the strain-stressed state of a circular sections set from the additional field associated with the deformation of the rectangular section. Within each j-th circle, the known dependence of the tangential torsion stresses on the distance to the center of the circle under consideration becomes valid. In this case, the further the circle from the center of the rectangle is located, the greater its moment of inertia becomes; the maximum tangential torsion stresses are still reached in the middle of the long sides of the rectangle.

3. The deplanation of a rectangular cross-section is taken into account by correcting the already known dependencies and their necessary development with respect to relative angular deformations. Attention is focused on the physical nature of the displacements in the direction of the longitudinal axis of the reinforced concrete rod structure, associated with the tangential torsion stresses and the shear deformations caused by them (an analogy is made with the displacements Δ_Q, caused by the transverse force).

The dependence proposed by the authors also takes into account the deplanation displacements w vary along the length of the reinforced concrete rod structure from zero at the clamped end to their maximum value at the free end.

Thus, relative displacement of deplanation w (or their decomposed components in Cartesian axes) due to tangential torsion stresses τ_d (or their decomposed components along the Cartesian axes) will correspond to the absolute shear displacements γ_d.

4. In the conducted research the classification of spatial cracks for reinforced concrete constructions under the action torsion with bending is generalized; while the spatial cracks of the first, second and third types' formation is related to the proposed method of determining shearing stresses (angular deformations) for complex cross sections.

5. The proposed dependencies allow us to find the values of the model design parameters (at an arbitrary point) at all stages of the stress-strain state of the reinforced concrete rod structures, including in the plastic stage and in the limiting stages. In particular, the dependences for the shear stresses τ_{zx} (angular deformations γ_{zx}) due to both the transverse force and the torque allow us to find their values in the limiting stage that occurs before spatial cracks formation. The tangential stresses $\tau_{zx,Q}$ in the plastic stage are determined by the difference between the total tangential stresses (corresponding to the achievement $\sigma_1 = R_{bt}$) and the plastic torsion stresses $\tau_{t,u}$. Similarly, angular deformations $\gamma_{zx,Q}$ in the plastic stage are determined by the difference between the total angular deformations γ_{zx} (corresponding to the achievement $\varepsilon_1 = \varepsilon_{bt,ul}$) and plastic torsion deformations $\gamma_{t,ul}$.

6. In tensors of main stresses (main deformations), the components of the torsion stress (angular deformations) separated by the proposed method for the convenience of analysis are again synthesized. Transformational transitions from the cylindrical to the Cartesian coordinate system and the attraction of local coordinate systems made it possible to simplify the equations as much as possible. Moreover, the equations are constructed in such a way that the resolving system does not turn into a decaying system. The physical interpretation of the solution obtained is that it allows us to search for the minimum generalized load that corresponds to the first spatial crack of the first, second or third types' formation and the coordinates of their formation point.

7. As a result, using the example of constructing a calculation model for the spatial cracks formation of the first, second and third types under the action torsion with bending based on the criterion for the formation of a spatial crack in the condition form for achieving the ultimate values of the concrete elongation ε_{bt} by its main deformations $\varepsilon_{bt,ul}$. The efficiency of the proposed method for approximating rectangular and complex cross-sections of reinforced concrete structures under the action torsion with bending is shown taking into account physical nonlinearity, deplanation of cross-sections, pre stressing in longitudinal and transverse reinforcement and influence of local stress fields.

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