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About local numerical solution of boundary problems of elasticity three-dimensional theory

M L Mozgaleva[1] **and P A Akimov**[2, 3, 4, 5]

[1] Department of Applied Mathematics, National Research Moscow State University of Civil Engineering, 26, Yaroslavskoe Shosse, Moscow, 129337, Russia
[2] Russian Academy of Architecture and Construction Sciences, 24, ul. Bolshaya Dmitrovka, Moscow, 107031, Russia
[3] Research &Development Centre StaDyO, office 810, 18, 3ya Ulitsa Yamskogo Polya, Moscow, 125040, Russia
[4] Department of Applied Mathematics, Tomsk State University of Architecture and Building, 2, Solyanaya sq., Tomsk, 634003, Russia
[5] Department of Architecture and Civil Engineering, Peoples' Friendship University of Russia, 6, Miklukho-Maklaya str., Moscow, 117198, Russia

akimov@raasn.ru

Abstract. The distinctive paper is devoted to wavelet-based multilevel method of local numerical solution of boundary problems of elasticity three-dimensional theory. As it is known, effective qualitative multilevel analysis of local and structure global stress-strain states is normally required in various technical applications. Operational and variational formulations of the problem (particularly with the use of wavelet (Haar) basis) are presented. Computer-oriented algorithms of fast direct and inverse discrete Haar transforms are described. Due to special algorithms of averaging within corresponding multigrain approach, problem reduction is provided.

1. Basic formulas of fast direct and inverse discrete Haar transforms and averaging

1.1. Algorithms of fast direct and inverse discrete Haar transforms

Let us consider the three-dimensional rectangular domain

$$\Omega = \{ (x_1, x_2, x_3): \ 0 \le x_1 \le l_1, \ \ 0 \le x_2 \le l_2, \ 0 \le x_3 \le l_3 \} \tag{1}$$

where x_1, x_2, x_3 are coordinates; l_1, l_2, l_3 are dimensions along x_1, x_2, x_3. Let us divide ω into $(n-1)$ equal parts along x_1, into $(n-1)$ equal parts along x_2 and into $(n-1)$ equal parts along x_3, where $n = 2^M$, M is the number of levels in the Haar basis [1-12]. We have the following formulas for coordinates of mesh nodes:

$$x_{1,i} = (i_1 - 1)h_1, \ i_1 = 1, 2, \ldots, n; \quad x_{2,i} = (i_2 - 1)h_2, \ i_2 = 1, 2, \ldots, n; \quad x_{3,i} = (i_3 - 1)h_3, \ i_3 = 1, 2, \ldots, n, \tag{2}$$

$$h_1 = l_1/(n-1); \quad h_2 = l_2/(n-1); \quad h_3 = l_3/(n-1). \tag{3}$$

Haar mesh functions $\psi^{p}_{s_1,s_2,s_3,j_1,j_2,j_3}(i_1,i_2,i_3)$, $p=1,\dots,M$, $j_1,j_2,j_3=1,\dots,N_p$, $s_1,s_2,s_3=0,1$ (except $s_1=s_2=s_3=0$) can be defined by formulas:

$$\Psi^{p}_{s_3,s_2,s_1,j_3,j_2,j_1}(i_1,i_2,i_3)=\alpha_p^{-1}\Psi_{s_3s_2s_1}\left(\frac{i_1-1}{2^p}-(j_1-1),\frac{i_2-1}{2^p}-(j_2-1),\frac{i_3-1}{2^p}-(j_3-1)\right),\ 1\le p\le M\,; \quad (4)$$

$$\psi^{M}_{0,0,0,1,1,1}(i_1,i_2,i_3)=\alpha_M^{-1}\,; \quad (5)$$

$$\Psi_{s_3s_2s_1}(x_1,x_2,x_3)=\begin{cases}1, & 0\le x_1,x_2,x_3<1/2\\ (-1)^{s_1}, & 1/2\le x_1<1\wedge 0\le x_2<1/2\wedge 0\le x_3<1/2\\ (-1)^{s_2}, & 0\le x_1<1/2\wedge 1/2\le x_2<1\wedge 0\le x_3<1/2\\ (-1)^{s_1+s_2}, & 1/2\le x_1<1\wedge 1/2\le x_2<1\wedge 0\le x_3<1/2\\ (-1)^{s_3}, & 0\le x_1,x_2<1/2\wedge 1/2\le x_3<1\\ (-1)^{s_1+s_3}, & 1/2\le x_1<1\wedge 0\le x_2<1/2\wedge 1/2\le x_3<1\\ (-1)^{s_2+s_3}, & 0\le x_1<1/2\wedge 1/2\le x_2<1\wedge 1/2\le x_3<1\\ (-1)^{s_1+s_2+s_3}, & 1/2\le x_1<1\wedge 1/2\le x_2<1\wedge 1/2\le x_3<1\\ 0, & x_1,x_2,x_3<0\vee x_1,x_2,x_3\ge 1,\end{cases} \qquad s_1,s_2,s_3=0,1; \quad (6)$$

$$N_p=n/2^p=2^{M-p},\ 1\le p\le M\quad \alpha_p=2^p\sqrt{2^p},\ 1\le p\le M \quad (7)$$

Let $f(i_1,i_2,i_3)$ be an arbitrary mesh function. Consequently we have

$$\begin{aligned}f(i_1,i_2,i_3)=v^{M}_{0,0,0,1,1,1}\psi^{M}_{0,0,0,1,1,1}+\sum_{p=1}^{M}\sum_{j_1=1}^{N_p}\sum_{j_2=1}^{N_p}\sum_{j_3=1}^{N_p}(v^{p}_{1,0,0,j_1,j_2,j_3}\psi^{p}_{1,0,0,j_1,j_2,j_3}(i_1,i_2,i_3)+\\ +v^{p}_{0,1,0,j_1,j_2,j_3}\psi^{p}_{0,1,0,j_1,j_2,j_3}(i_1,i_2)+v^{p}_{0,0,1,j_1,j_2,j_3}\psi^{p}_{0,0,1,j_1,j_2,j_3}(i_1,i_2)+\\ +v^{p}_{1,1,0,j_1,j_2,j_3}\psi^{p}_{1,1,0,j_1,j_2,j_3}(i_1,i_2)+v^{p}_{1,0,1,j_1,j_2,j_3}\psi^{p}_{1,0,1,j_1,j_2,j_3}(i_1,i_2)+\\ +v^{p}_{0,1,1,j_1,j_2,j_3}\psi^{p}_{0,1,1,j_1,j_2,j_3}(i_1,i_2)+v^{p}_{1,1,1,j_1,j_2,j_3}\psi^{p}_{1,1,1,j_1,j_2,j_3}(i_1,i_2)),\end{aligned} \quad (8)$$

where $v^{p}_{s_1,s_2,s_3,j_1,j_2,j_3}$, $s_1,s_2,s_3=0,1$, $j_1,j_2,j_3=1,2,\dots,N_p$, $p=1,2,\dots,M$ are Haar expansion coefficients,

$$v^{p}_{s_1,s_2,s_3,j_1,j_2,j_3}=\sum_{i_1=1}^{N}\sum_{i_2=1}^{N}\sum_{i_3=1}^{N}f(i_1,i_2,i_3)\psi^{p}_{s_1,s_2,s_3,j_1,j_2,j_3}(i_1,i_2,i_3)\,. \quad (9)$$

Algorithm of fast direct discrete Haar transform is described below.

$$u^{0}_{j_1,j_2,j_3}=f(j_1,j_2,j_3),\ j_1=1,2,\dots,n,\ j_2=1,2,\dots,n,\ j_3=1,2,\dots,n\,. \quad (10)$$

We have (for all $p=1,\dots,M$, $j_1,j_2,j_3=1,\dots,N_p$, $s_1,s_2,s_3=0,1$ (except $s_1=s_2=s_3=0$)):

$$z_1=(-1)^{s_1},\quad z_2=(-1)^{s_2},\quad z_3=(-1)^{s_3}\,;\quad \alpha_{p+1}=2\sqrt{2}\cdot\alpha_p\,; \quad (11)$$

$$v^p_{s_1,s_2,s_3,j_1,j_2,j_3} = \alpha_p^{-1}(u^p_{2j_1-1,2j_2-1,2j_3-1} + z_1 u^p_{2j_1,2j_2-1,2j_3-1} + z_2 u^p_{2j_1-1,2j_2,2j_3-1} + z_3 u^p_{2j_1-1,2j_2-1,2j_3} + \\ + z_1 z_2 u^p_{2j_1,2j_2,2j_3-1} + z_1 z_3 u^p_{2j_1,2j_2-1,2j_3} + z_2 z_3 u^p_{2j_1-1,2j_2,2j_3} + z_1 z_2 z_3 u^p_{2j_1,2j_2,2j_3}); \tag{12}$$

$$u^{p+1}_{j_1,j_2,j_3} = u^p_{2j_1-1,2j_2-1,2j_3-1} + u^p_{2j_1,2j_2-1,2j_3-1} + u^p_{2j_1-1,2j_2,2j_3-1} + u^p_{2j_1,2j_2,2j_3-1} + \\ + u^p_{2j_1-1,2j_2-1,2j_3} + u^p_{2j_1,2j_2-1,2j_3} + u^p_{2j_1-1,2j_2,2j_3} + u^p_{2j_1,2j_2,2j_3}; \tag{13}$$

$$v^M_{0,0,0,1,1,1} = \alpha_M^{-1} u^M_{1,1,1}, \tag{14}$$

where $u^p_{j_1,j_2,j_3}$, $j_1, j_2, j_3 = 1, 2, \ldots, N_p$, $p = 1, 2, \ldots, M$ are auxiliary quantities.

Algorithm of fast inverse Haar transform is described below.

$$\alpha_M = n\sqrt{n}; \quad u^M_{1,1,1} = \alpha_M^{-1} v^M_{0,0,0,1,1,1}. \tag{15}$$

We have ($p = M-1, M-2, \ldots, 1, i_1, i_2, i_3 = 1, 2, \ldots, N_p$):

$$i_k = 2j_k + s_k - 1, \quad s_k = 0, 1, \quad k = 1, 2, 3; \tag{16}$$

$$z_1 = (-1)^{s_1}; \quad z_2 = (-1)^{s_2}; \quad z_3 = (-1)^{s_3}; \quad \alpha_{p-1} = \alpha_p / 2\sqrt{2}; \tag{17}$$

$$u^p_{i_1,i_2,i_3} = \alpha_p^{-1}(z_1 v^p_{1,0,0,j_1,j_2,j_3} + z_2 v^p_{0,1,0,j_1,j_2,j_3} + z_3 v^p_{0,0,1,j_1,j_2,j_3} + z_1 z_2 v^p_{1,1,0,j_1,j_2,j_3} + \\ + z_1 z_3 v^p_{1,0,1,j_1,j_2,j_3} + z_2 z_3 v^p_{0,1,1,j_1,j_2,j_3} + z_1 z_2 z_3 v^p_{1,1,1,j_1,j_2,j_3}) + u^{p+1}_{j_1,j_2,j_3}. \tag{18}$$

$$f(i_1, i_2, i_3) = u^0_{i_1,i_2,i_3}, \quad i_1 = 1, \ldots, n, \quad i_2 = 1, \ldots, n, \quad i_3 = 1, \ldots, n. \tag{19}$$

1.2. Algorithm of averaging.

Let us assume that it is necessary to make averaging at level q. For all $p = 1, 2, \ldots, q$, $j_1, j_2, j_3 = 1, 2, \ldots, N_p$, $s_1, s_2, s_3 = 0, 1$ (except $s_1 = s_2 = s_3 = 0$) we suppose

$$(D_1^+ u^p)_{2j_1-1,2j_2-1,2j_3-1} = (D_1^+ u^p)_{2j_1-1,2j_2,2j_3-1} = (D_1^+ u^p)_{2j_1,2j_2-1,2j_3-1} = (D_1^+ u^p)_{2j_1,2j_2,2j_3-1} = \\ = (D_1^+ u^p)_{2j_1-1,2j_2-1,2j_3} = (D_1^+ u^p)_{2j_1-1,2j_2,2j_3} = (D_1^+ u^p)_{2j_1,2j_2-1,2j_3} = (D_1^+ u^p)_{2j_1,2j_2,2j_3} \approx \\ \approx (D_1 \tilde{u}^p)_{2j_1-1,2j_2-1,2j_3-1}; \tag{20}$$

$$(D_1^+ u^p)_{2j_1-1,2j_2-1,2j_3-1} = (D_2 u^p)_{2j_1-1,2j_2,2j_3-1} = (D_2 u^p)_{2j_1,2j_2-1,2j_3-1} = (D_2 u^p)_{2j_1,2j_2,2j_3-1} = \\ = (D_2 u^p)_{2j_1-1,2j_2-1,2j_3} = (D_2 u^p)_{2j_1-1,2j_2,2j_3} = (D_2 u^p)_{2j_1,2j_2-1,2j_3} = (D_2 u^p)_{2j_1,2j_2,2j_3} \approx \\ \approx (D_2 \tilde{u}^p)_{2j_1-1,2j_2-1,2j_3-1}; \tag{21}$$

$$(D_3^+ u^p)_{2j_1-1,2j_2-1,2j_3-1} = (D_3^+ u^p)_{2j_1-1,2j_2,2j_3-1} = (D_3^+ u^p)_{2j_1,2j_2-1,2j_3-1} = (D_3^+ u^p)_{2j_1,2j_2,2j_3-1} = \\ = (D_3^+ u^p)_{2j_1-1,2j_2-1,2j_3} = (D_3^+ u^p)_{2j_1-1,2j_2,2j_3} = (D_3^+ u^p)_{2j_1,2j_2-1,2j_3} = (D_3^+ u^p)_{2j_1,2j_2,2j_3} \approx \\ \approx (D_3^+ \tilde{u}^p)_{2j_1-1,2j_2-1,2j_3-1}; \tag{22}$$

$$(D_{21} u^p)_{2j_1-1,2j_2-1,2j_3-1} = (D_{21} u^p)_{2j_1-1,2j_2,2j_3-1} = (D_{21} u^p)_{2j_1,2j_2-1,2j_3-1} = \\ = (D_{21} u^p)_{2j_1,2j_2,2j_3-1} = (D_{21} u^p)_{2j_1-1,2j_2-1,2j_3} = (D_2^+ D_1^+ u^p)_{2j_1-1,2j_2,2j_3} = \\ = (D_{21} u^p)_{2j_1,2j_2-1,2j_3} = (D_{21} u^p)_{2j_1,2j_2,2j_3} \approx (D_{21} \tilde{u}^p)_{2j_1-1,2j_2-1,2j_3-1}; \tag{23}$$

$$
\begin{aligned}
&(D_{31}u^p)_{2j_1-1,2j_2-1,2j_3-1} = (D_{31}u^p)_{2j_1-1,2j_2,2j_3-1} = (D_{31}u^p)_{2j_1,2j_2-1,2j_3-1} = \\
&\quad = (D_{31}u^p)_{2j_1,2j_2,2j_3-1} = (D_{31}u^p)_{2j_1-1,2j_2-1,2j_3} = (D_{31}u^p)_{2j_1-1,2j_2,2j_3} = \\
&\quad = (D_{31}u^p)_{2j_1,2j_2-1,2j_3} = (D_{31}u^p)_{2j_1,2j_2,2j_3} \approx (D_{31}\tilde{u}^p)_{2j_1-1,2j_2-1,2j_3-1};
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&(D_{32}u^p)_{2j_1-1,2j_2-1,2j_3-1} = (D_{32}u^p)_{2j_1-1,2j_2,2j_3-1} = (D_{32}u^p)_{2j_1,2j_2-1,2j_3-1} = \\
&\quad = (D_{32}u^p)_{2j_1,2j_2,2j_3-1} = (D_{32}u^p)_{2j_1-1,2j_2-1,2j_3} = (D_{32}u^p)_{2j_1-1,2j_2,2j_3} = \\
&\quad = (D_{32}u^p)_{2j_1,2j_2-1,2j_3} = (D_{32}u^p)_{2j_1,2j_2,2j_3} \approx (D_{32}\tilde{u}^p)_{2j_1-1,2j_2-1,2j_3-1};
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
&(D_3^+D_2^+D_1^+u^p)_{2j_1-1,2j_2-1,2j_3-1} = (D_3^+D_2^+D_1^+u^p)_{2j_1-1,2j_2,2j_3-1} = (D_3^+D_2^+D_1^+u^p)_{2j_1,2j_2-1,2j_3-1} = \\
&\quad = (D_3^+D_2^+D_1^+u^p)_{2j_1,2j_2,2j_3-1} = (D_3^+D_2^+D_1^+u^p)_{2j_1-1,2j_2-1,2j_3} = (D_3^+D_2^+D_1^+u^p)_{2j_1-1,2j_2,2j_3} = \\
&\quad = (D_3^+D_2^+D_1^+u^p)_{2j_1,2j_2-1,2j_3} = (D_3^+D_2^+D_1^+u^p)_{2j_1,2j_2,2j_3} \approx (D_3^+D_2^+D_1^+\tilde{u}^p)_{2j_1-1,2j_2-1,2j_3-1};
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&v^p_{s_1,s_2,s_3,2j_1-1,2j_2-1,2j_3-1} = v^p_{s_1,s_2,s_3,2j_1,2j_2-1,2j_3-1} = v^p_{s_1,s_2,s_3,2j_1-1,2j_2,2j_3-1} = v^p_{s_1,s_2,s_3,2j_1,2j_2,2j_3-1} = \\
&\quad = v^p_{s_1,s_2,s_3,2j_1-1,2j_2-1,2j_3} = v^p_{s_1,s_2,s_3,2j_1,2j_2-1,2j_3} = v^p_{s_1,s_2,s_3,2j_1-1,2j_2,2j_3} = v^p_{s_1,s_2,s_3,2j_1,2j_2,2j_3};
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
\tilde{u}^p_{j_1,j_2,j_3} = (&u^p_{j_1,j_2,j_3} + u^p_{j_1+1,j_2,j_3} + u^p_{j_1,j_2+1,j_3} + u^p_{j_1+1,j_2+1,j_3} + \\
&+ u^p_{j_1,j_2,j_3+1} + u^p_{j_1+1,j_2,j_3+1} + u^p_{j_1,j_2+1,j_3+1} + u^p_{j_1+1,j_2+1,j_3+1})/8;
\end{aligned}
\tag{28}
$$

$$u^{p+1}_{j_1,j_2,j_3} = 8\tilde{u}^p_{2j_1-1,2j_2-1,2j_3-1}; \tag{29}$$

$$(D_1^+u^p)_{j_1,j_2,j_3} = (u^p_{j_1+1,j_2,j_3} - u^p_{j_1,j_2,j_3})/(2^p h); \tag{30}$$

$$(D_2^+u^p)_{j_1,j_2,j_3} = (u^p_{j_1,j_2+1,j_3} - u^p_{j_1,j_2,j_3})/(2^p h); \tag{31}$$

$$(D_3^+u^p)_{j_1,j_2,j_3} = (u^p_{j_1,j_2,j_3+1} - u^p_{j_1,j_2,j_3})/(2^p h); \tag{32}$$

$$(T_1^+u^p)_{j_1,j_2,j_3} = u^p_{j_1+1,j_2,j_3} + u^p_{j_1,j_2,j_3}; \tag{33}$$

$$(T_2^+u^p)_{j_1,j_2,j_3} = u^p_{j_1,j_2+1,j_3} + u^p_{j_1,j_2,j_3}; \tag{34}$$

$$(T_3^+u^p)_{j_1,j_2,j_3} = u^p_{j_1,j_2,j_3+1} + u^p_{j_1,j_2,j_3}; \tag{35}$$

$$D_1 = \frac{1}{4}T_3^+T_2^+D_1^+; \quad D_2 = \frac{1}{4}T_3^+T_1^+D_2^+; \quad D_3 = \frac{1}{4}T_2^+T_1^+D_3^+; \tag{36}$$

$$D_{21} = \frac{1}{2}T_3^+D_2^+D_1^+; \quad D_{31} = \frac{1}{2}T_2^+D_3^+D_2^+; \quad D_{32} = \frac{1}{2}T_1^+D_3^+D_2^+. \tag{37}$$

Final formulas of averaging have the form:

$$
\begin{aligned}
&v^p_{1,0,0,2j_1-1,2j_2-1,2j_3-1} = v^p_{1,0,0,2j_1,2j_2-1,2j_3-1} = v^p_{1,0,0,2j_1-1,2j_2,2j_3-1} = v^p_{1,0,0,2j_1-1,2j_2-1,2j_3} = \\
&\quad = v^p_{1,0,0,2j_1,2j_2,2j_3-1} = v^p_{1,0,0,2j_1,2j_2-1,2j_3} = v^p_{1,0,0,2j_1-1,2j_2,2j_3} = v^p_{1,0,0,2j_1,2j_2,2j_3} = \beta_{1,0,0}v^{p+1}_{1,0,0,j_1,j_2,j_3}, \\
&\qquad j_1, j_2, j_3 = 1, 2, \ldots, N_{p+1};
\end{aligned}
\tag{38}
$$

$$v^{p}_{0,1,0,2j_1-1,2j_2-1,2j_3-1}=v^{p}_{0,1,0,2j_1,2j_2-1,2j_3-1}=v^{p}_{0,1,0,2j_1-1,2j_2,2j_3-1}=v^{p}_{0,1,0,2j_1-1,2j_2-1,2j_3}=$$
$$=v^{p}_{0,1,0,2j_1,2j_2,2j_3-1}=v^{p}_{0,1,0,2j_1,2j_2-1,2j_3}=v^{p}_{0,1,0,2j_1-1,2j_2,2j_3}=v^{p}_{0,1,0,2j_1,2j_2,2j_3}=\beta_{0,1,0}v^{p+1}_{0,1,0,j_1,j_2,j_3}, \tag{39}$$
$$j_1,\, j_2,\, j_3=1,2,\dots,N_{p+1};$$

$$v^{p}_{0,0,1,2j_1-1,2j_2-1,2j_3-1}=v^{p}_{0,0,1,2j_1,2j_2-1,2j_3-1}=v^{p}_{0,0,1,2j_1-1,2j_2,2j_3-1}=v^{p}_{0,0,1,2j_1-1,2j_2-1,2j_3}=$$
$$=v^{p}_{0,0,1,2j_1,2j_2,2j_3-1}=v^{p}_{0,0,1,2j_1,2j_2-1,2j_3}=v^{p}_{0,0,1,2j_1-1,2j_2,2j_3}=v^{p}_{0,0,1,2j_1,2j_2,2j_3}=\beta_{0,0,1}v^{p+1}_{0,0,1,j_1,j_2,j_3}, \tag{40}$$
$$j_1,\, j_2,\, j_3=1,2,\dots,N_{p+1};$$

$$v^{p}_{1,1,0,2j_1-1,2j_2-1,2j_3-1}=v^{p}_{1,1,0,2j_1,2j_2-1,2j_3-1}=v^{p}_{1,1,0,2j_1-1,2j_2,2j_3-1}=v^{p}_{1,1,0,2j_1-1,2j_2-1,2j_3}=$$
$$=v^{p}_{1,1,0,2j_1,2j_2,2j_3-1}=v^{p}_{1,1,0,2j_1,2j_2-1,2j_3}=v^{p}_{1,1,0,2j_1-1,2j_2,2j_3}=v^{p}_{1,1,0,2j_1,2j_2,2j_3}=\beta_{1,1,0}v^{p+1}_{1,0,0,j_1,j_2,j_3}, \tag{41}$$
$$j_1,\, j_2,\, j_3=1,2,\dots,N_{p+1};$$

$$v^{p}_{1,0,1,2j_1-1,2j_2-1,2j_3-1}=v^{p}_{1,0,1,2j_1,2j_2-1,2j_3-1}=v^{p}_{1,0,1,2j_1-1,2j_2,2j_3-1}=v^{p}_{1,0,1,2j_1-1,2j_2-1,2j_3}=$$
$$=v^{p}_{1,0,1,2j_1,2j_2,2j_3-1}=v^{p}_{1,0,1,2j_1,2j_2-1,2j_3}=v^{p}_{1,0,1,2j_1-1,2j_2,2j_3}=v^{p}_{1,0,1,2j_1,2j_2,2j_3}=\beta_{1,0,1}v^{p+1}_{1,0,1,j_1,j_2,j_3}, \tag{42}$$
$$j_1,\, j_2,\, j_3=1,2,\dots,N_{p+1};$$

$$v^{p}_{0,1,1,2j_1-1,2j_2-1,2j_3-1}=v^{p}_{0,1,1,2j_1,2j_2-1,2j_3-1}=v^{p}_{0,1,1,2j_1-1,2j_2,2j_3-1}=v^{p}_{0,1,1,2j_1-1,2j_2-1,2j_3}=$$
$$=v^{p}_{0,1,1,2j_1,2j_2,2j_3-1}=v^{p}_{0,1,1,2j_1,2j_2-1,2j_3}=v^{p}_{0,1,1,2j_1-1,2j_2,2j_3}=v^{p}_{0,1,1,2j_1,2j_2,2j_3}=\beta_{0,1,1}v^{p+1}_{0,1,1,j_1,j_2,j_3}, \tag{43}$$
$$j_1,\, j_2,\, j_3=1,2,\dots,N_{p+1};$$

$$v^{p}_{1,1,1,2j_1-1,2j_2-1,2j_3-1}=v^{p}_{1,1,1,2j_1,2j_2-1,2j_3-1}=v^{p}_{1,1,1,2j_1-1,2j_2,2j_3-1}=v^{p}_{1,1,1,2j_1-1,2j_2-1,2j_3}=$$
$$=v^{p}_{1,1,1,2j_1,2j_2,2j_3-1}=v^{p}_{1,1,1,2j_1,2j_2-1,2j_3}=v^{p}_{1,1,1,2j_1-1,2j_2,2j_3}=v^{p}_{1,1,1,2j_1,2j_2,2j_3}=\beta_{1,1,1}v^{p+1}_{1,1,1,j_1,j_2,j_3},$$
$$j_1,\, j_2,\, j_3=1,2,\dots,N_{p+1}; \tag{44}$$

$$\beta_{1,0,0}=\beta_{0,1,0}=\beta_{0,0,1}=\frac{\sqrt{2}}{8};\quad \beta_{1,1,0}=\beta_{1,0,1}=\beta_{0,1,1}=\frac{\sqrt{2}}{16};\quad \beta_{1,1,1}=\frac{\sqrt{2}}{32}. \tag{45}$$

2. Multilevel wavelet-based numerical method of boundary problems local solution of elasticity three-dimensional theory

2.1. Formulation of the problem

Effective qualitative multilevel analysis of local and structure global stress-strain states is normally required in various technical problems. As it is known, defects and failures are mostly local in nature. However total load-carrying ability of the structure, associated with the condition of limit equilibrium, is determined by the global behavior of the considering project. Therefore corresponding multilevel approach is peculiarly relevant and apparently preferable in all aspects for qualitative and quantitative analysis of calculation data.

Wavelet analysis provides effective and popular tool for such researches. After expansion of the solution with the use of local wavelet basis corresponding components are considered at each level of the basis.In accordance with the method of extended domain [13], the domain Ω, occupied by considering structure, is embroidered by extended one ω of arbitrary shape, particularly elementary. Operational formulation of the problem in domain ω normally has the form

$$Lu=F, \tag{46}$$

where L is the operator of boundary problem, which takes into account the boundary conditions; u is the unknown function; F is the given right-side function.

Directly from operational formulation we have variational formulation of the problem:

$$\Phi(u) = 0.5 \cdot (Lu, u) - (F, u), \tag{47}$$

Solution of (47) is the critical point of (41). (f, g) denotes dot product of functions f and g.

Discrete formulation of the problem has the form:

$$A\bar{u} = \bar{f}, \tag{48}$$

where $A = \{a_{i,j}\}_{i,\, j=1,2,...,n_{gl}}$ is the difference approximation of operator L; $\bar{u} = [u_1 \;\; u_2 \;\; ... \;\; u_{n_{gl}}]^T$ is the unknown mesh function; $\bar{f} = [f_1 \;\; f_2 \;\; ... \;\; f_{n_{gl}}]^T$ is the given right-side mesh function; n_{gl} is dimension of problem. Various methods can be used to form the matrix of the discrete operator. We recommend method of basis (local) variations [14]. Its major peculiarities include universality and computer orientation. We can use the following formulas for linear problems:

$$a_{i,j} = \Phi(\bar{e}^{(i)} + \bar{e}^{(j)}) - \Phi(\bar{e}^{(i)}) - \Phi(\bar{e}^{(j)}) + \Phi(\bar{0})]; \; f_i = 0.5 \cdot [\Phi(\bar{e}^{(i)}) - \Phi(-\bar{e}^{(i)})], \tag{49}$$

$$\bar{e}^{(i)} = [\, e_1^{(i)} \;\; e_2^{(i)} \;\; ... \;\; e_{n_{gl}}^{(i)} \,]^T, \;\; i = 1, 2, ..., n_{gl}; \;\; e_j^{(i)} = \delta_{i,j}, \;\; j = 1, 2, ... n_{gl}; \tag{50}$$

$\bar{e}^{(i)}$, $i = 1, 2, ..., n_{gl}$ are basis mesh vectors; $\bar{0}$ is the null function; $\delta_{i,j}$ is the Kronecker delta.

2.2. Haar-based formulation of the problem

Let us consider Haar-based formulation of the problem:

$$\Phi(\bar{u}) = 0.5 \cdot (A\bar{u}, \bar{u}) - (\bar{f}, \bar{u}) = 0.5 \cdot (LQ\bar{v}, Q\bar{v}) - (\bar{f}, Q\bar{v}) = 0.5 \cdot (Q^* LQ\bar{v}, \bar{v}) - (Q^* \bar{f}, \bar{v}), \tag{51}$$

where Q is transition matrix consisting from Haar basis vectors, located in rows. Thus,

$$\tilde{\Phi}(\bar{v}) = 0.5 \cdot (Q^* LQ\bar{v}, \bar{v}) - (Q^* \bar{f}, \bar{v}), \tag{52}$$

where $\bar{v}$ is vector of Haar expansion coefficients of the vector $\bar{u}$ [15]. Corresponding operational formulation of the problem has the form

$$\tilde{L}\bar{v} = \tilde{\bar{f}}, \quad \tilde{L} = Q^* LQ; \quad \tilde{\bar{f}} = Q^* \bar{f}. \tag{53}$$

Further reduction of the problem is based on the averaging algorithm specified above.

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