

The stability of cylindrical tanks filled with liquid under the influence of impulse loads

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Abstract. The paper presents a mathematical model of oscillations of a cylindrical shell, filled with liquid after exposure to a shock wave created by an air explosion. Two models are applied to the description of behavior of a shell depending on duration of external loading, namely the model of the tangent modulus for pulse action and the model of the elastic modulus for quasi-static influence. Levels of the maximum pressure and impulse are determined for a real-life tank using these models at which a loss of stability occurs for one of the shape of flexural deformations. Critical curves of stability in the «maximum pressure–impulse of pressure» plane are also developed.

1. Introduction

The analysis of accident of fuel storage shows that there is a possibility of contingency occurrence that arise when external impulse loads are applied to these facilities. Such loads can lead to the loss of stability of a tank shell.

This research is devoted to the problem of the cylindrical fluid tanks stability loss under the action of external impulse.

2. Literature survey

There is a number of works, for example [1-6], in which questions of stability of shells are considered. The novelty of this research is that the stability of fuel storage under action of an air explosion shock wave is calculated by constructing the critical curves of stability, which are obtained by numerical integration the equation of movement of smooth cylindrical shell.

In addition, the mass of fuel is taken into account in oscillatory process by inclusion the attached fluid mass in the governing equations. The attached fluid mass arises if the tank is filled with liquid. It increases lag effect of the system that, undoubtedly, has to be taken into account.

3. Method description

It has been experimentally established that the character of the tank shell stability loss at the influence of an air explosion shock wave depends on the maximum pressure and the duration of loading action. Depending on these parameters, three areas of impact are allocated in the “maximum pressure – impulse of pressure” ($P - I$) plane and a loading area (Figure 1):

- Impulse function (loading of a small duration);



- Quasi-impulsive function (loading of an intermediate duration);
- Quasi-static function (a long loading).

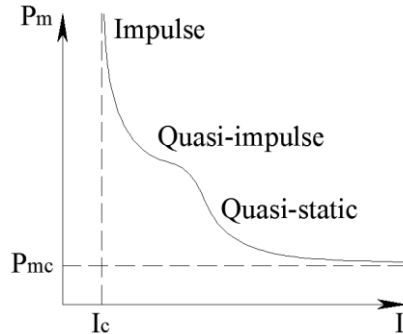


Figure 1. Critical curve of shell stability loss for various impulse durations.

The possibility of stability loss of the shell of cylindrical tank at impulse and quasi-static loadings is studied in this work. In Figure 1, it can be seen that the quasi-impulsive section occupies rather small range therefore it hasn't been examined by the authors.

To describe the behavior of the cylindrical tank shell at impulse function, the model of the tangent modulus is applied and at quasi-impulsive – an elastic deformation modulus.

The settlement scheme of the tank at the influence of an air explosion shock wave is a smooth cylindrical shell. Therefore the equation of the movement of smooth cylindrical shell can be used in both cases. The fluid in the tank is supposed to be ideal and incompressible.

3.1. Modelling of load

The considered load is a shock wave produced by an air explosion. The most essential characteristics of the load are the maximum pressure P_m and the pressure impulse I referred to the unit of the shell surface. Thus, we use just these values for assessment of stability of the shell.

Change of shock wave pressure P in time t depends on the distance to the explosion point and is expressed by exponential dependence. In this case, the area of the graph under the exponential is the pressure impulse. In calculations, the exhibitor of pressure can be replaced with a triangle (Figure 2) where P is pressure in the shock wave, P_m is the maximum pressure, t is current time, τ is total time of the shock wave action, I is the total pressure impulse.

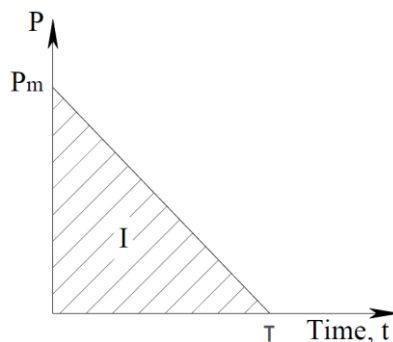


Figure 2. The shape of applied load.

In the case of triangular dependence of the applied load, the relationship between the composite impulse of pressure and the maximum pressure is represented by the following formula:

$$I = \frac{1}{2} P_m \tau \quad (1)$$

The most dangerous emergency situation for the constructions of the type considered is the case of shell loading by the shock wave of air explosion. This impact is of very short duration but with a considerable peak of the maximum pressure. Therefore the main settlement case is described by the model of the tangent modulus.

3.2. The Tangent modulus model

It is experimentally established that at pulse loading the influence of the shell length on stability loss is insignificant. Therefore, in the model of the tangent modulus it is enough to consider the loss of stability of a ring without taking into account the effect of the liquid.

We will use the equation of the movement of the ring given in [6]:

$$\ddot{w} + \frac{h^2}{12R^2E} \frac{\partial^4 w}{\partial \theta^4} + \left(\frac{h^2}{12R^2} \frac{E_t}{E} + \frac{\sigma}{E} \right) \frac{\partial^2 w}{\partial \theta^2} + \frac{\sigma}{E} (1 + w) = p - \frac{\sigma}{E} \left(u_i + \frac{\partial^2 w_i}{\partial \theta^2} \right) \quad (2)$$

where $w = u/R$, $w_i = u_i/R$ are the dimensionless movements; $u(\theta, t)$ is the radial movement of the shell (positive at inside movement); $u_i(\theta, t)$ is the deviation of initial shape of the shell from cylindrical one; θ is the angular coordinate; R is the shell radius; h is the shell thickness; E is the elasticity module of the shell material; E_t is tangent modulus of elasticity of the material beyond a proportional limit; σ is tangential membrane tension; $p = (R/Eh)p'(\theta, t)$ is dimensionless pressure.

Dimensionless time is defined as

$$t = t' R \sqrt{E/\rho},$$

where ρ is the shell material density. We will set a form of dynamic and initial deflections as well as external pressure by the following dependences:

$$\begin{cases} w_i(\theta) = \sum_{n=1}^{\infty} \delta_i \cos(n\theta) \\ w(\theta, t) = w_0(t) + \sum_{n=0}^{\infty} w_n(t) \cos(n\theta) \\ p(\theta, t) = p_0(t) + \sum_{n=0}^{\infty} p_n(t) \cos(n\theta) \end{cases} \quad (3)$$

where δ_i is the initial deviations of the shell from the correct geometrical shape.

Designating $h^2/(12R^2) = \alpha^2$, then substituting lines (3) in the equation (2), we receive the system of equations:

$$\ddot{w}_n + (n^2 - 1) \left(\alpha^2 n^2 E_t/E - \sigma/E \right) w_n = p_n + \sigma/E (n^2 - 1) \delta_i \quad (4)$$

$$\ddot{w}_0 + \sigma/E = p_0 \quad (5)$$

Dimensionless movements w_0 coincide with the ring deformation, therefore their values are small in comparison with one, and in the equation (5) they can be neglected.

At the initial moment, the shell is in a rest therefore the zero initial conditions are accepted.

The coefficient in expression (4) at w_n becomes negative at rather big deformations. Consequently, the solution loses an oscillatory view and has the nature of hyperbolic function. Therefore there is an exponential increase of the solution that makes it possible to consider as the indication on the

possibility of residual flexural deformations existence, i.e. on the possibility of stability loss. Thus, the condition of stability is defined by a positive value of the coefficient before deflection w_n in the expression (4), i.e.:

$$\alpha^2 n^2 E_t / E - \sigma / E > 0 \quad (6)$$

To determine the value of the tangent modulus E_t , data of the "tension-deformations" charts are used. The ratio σ/E_t outside the plasticity zone linearly depends on deformation. As a result, it is possible to use in calculations such expressions:

$$\frac{\sigma}{E_t} = \begin{cases} \varepsilon & \text{at } \varepsilon \leq \varepsilon_s \\ k(\varepsilon - \varepsilon_s) + \varepsilon_s & \text{at } \varepsilon > \varepsilon_s \end{cases} \quad (7)$$

where ε is the flowing deformation; $\varepsilon_s = \sigma_s / E$ is the deformation of the beginning of flowing; σ_s is the cover material fluidity limit.

In this case expressions for the tangent modulus are as follows:

$$E_t = \begin{cases} E & \text{at } w_0 \leq \varepsilon_s \\ \sigma / [k(w_0 - \varepsilon_s) + \varepsilon_s] & \text{at } w_0 > \varepsilon_s \end{cases} \quad (8)$$

where k is the tangent tilt angle to the chart "tension deformation".

To change parameters of a shock wave (to make it "softer") is possible by covering the shell with easily compressible material, for example, rubber. Passing through this covering the shock wave is transformed as follows. The peak of the pressure decreases but the period of its action increases, keeping an initial impulse of the shock wave.

In this case oscillatory process of a cylindrical shell becomes slower. This explains the necessity of the transition to the model of stability loss at an elastic stage of deformations.

3.3. Elastic modulus

We suppose that the shell has some unevenness of the surface coinciding in shape with the deflections at stability loss. At uniform load, loss of stability does not occur without this assumption. The assumption made doesn't change the basic regularities of the process under study, but significantly simplifies its theoretical analysis. It allows using the equation of the movement of a smooth cylindrical shell within a linear substitution [7]:

$$\frac{D}{h} \nabla^8 u + \frac{E}{R^2} \frac{\partial^4 u}{\partial x^4} + p \frac{R}{h} \nabla^4 \left[\frac{1}{R^2} \frac{\partial^2 (u + u_i)}{\partial \theta^2} \right] + \left(\rho + \frac{m_a}{h} \right) \frac{\partial^2}{\partial t^2} \nabla^4 u = 0, \quad (9)$$

where $D = Eh^3 / [12(1 - \mu^2)]$ is cylindrical stiffness; μ is the Poisson's ratio; x is a linear coordinate; m_a is the attached fluid mass at the movement of the shell by stability loss forms. According to the theory of long waves we have

$$m_a = (R/n) \rho_l,$$

where ρ_l is a density of a liquid filler.

Shapes of dynamic and initial deflections are given by following dependences:

$$\begin{cases} u(x, \theta, t') = u_0(t') + \sum_{n_{st}}^{\infty} u_n(t') \sin(kx) \cos(n\theta) \\ u_i(x, \theta) = \sum_{n_{st}}^{\infty} \delta_i \sin(kx) \cos(n\theta) \end{cases}, \quad (10)$$

where $k = \pi/L$, L is a shell length. We express the external pressure $p(t)$ in (9) through the radial compression of the shell (excluding axial forces):

$$p'(t') = Ehu_0/R^2$$

Substituting the series (10) in (9) and equating coefficients in each member to zero, we receive:

$$a_n u_n + (c_n - b_n u_0) u_n = b_n u_0 \delta_i, \quad (11)$$

$$a_0 u_0 + c_0 u_0 = p'(t'), \quad (12)$$

where coefficients at variable deflections of the shell are as follows:

$$\begin{cases} a_n = \left(\rho + \frac{R\rho_l}{nh} \right) \left(k^2 + \frac{n^2}{R^2} \right)^2 \\ b_n = \frac{E}{R^3} n^2 \left(k^2 + \frac{n^2}{R^2} \right)^2 \\ c_n = \frac{Eh^2}{12(1-\mu^2)} \left(k^2 + \frac{n^2}{R^2} \right)^4 + \frac{E}{R^2} k^4 \end{cases} \quad (13)$$

$$\begin{cases} a_0 = \rho h + m_l^* \\ c_0 = \frac{Eh}{R^2} \end{cases} \quad (14)$$

The coefficient a_0 in (14) is shown for the tank filled with fuel considering the attached mass of a liquid filler m_l^* . The impact of the attached mass of a liquid filler on fluctuations of a cylindrical shell under the influence of dynamic load is described in details in the research [8] according to which:

$$m_l^* = (8/3)\rho_l(L/2R)^2.$$

As well as in the models of the tangent modulus, the condition of stability is also defined by a positive value of coefficient at w_n in the expression (11), i.e.:

$$c_n - b_n w_0 > 0 \quad (15)$$

In a dimensionless structure, taken for the main sizes E, ρ, R , elastic model is expressed as:

$$\begin{aligned} a_n \ddot{w}_n + (c_n - b_n w_0) w_n &= b_n w_0 \frac{\delta_i}{R} \\ a_0 \ddot{w}_0 + c_0 w_0 &= p(t), \end{aligned}$$

where the coefficients are defined by formulas

$$\begin{cases} a_n = \left(1 + \frac{R\rho_l}{nh\rho}\right)(k^2 R^2 + n^2)^2 \\ b_n = n^2(k^2 R^2 + n^2)^2 \\ c_n = \frac{h^2}{R^2 12(1-\mu^2)}(k^2 R^2 + n^2)^4 + k^4 R^4 \end{cases} \quad \begin{cases} a_0 = \frac{h}{R} + \frac{m_l^*}{\rho} \\ c_0 = \frac{h}{R} \end{cases}$$

To use an elastic modulus, it is necessary to be convinced that the maximum hoop stress doesn't exceed the fluidity limit of the shell material. If this condition is not met, then it is possible to use the model of the tangent modulus.

4. Research results

The general problem of the assessment of shell stability both for impulse, and for quasi-static load consists in determining the maximum pressure levels P_m and the impulse I at which for any 'one form of flexural deformations there will be such growth of displacements that characteristic the stability conditions (6, 15) are violated.

The research of shell stability is conducted on the example of a vertical steel cylindrical tank for diesel fuel storage with a stationary roof with a capacity of 1000 m^3 , located in Sevastopol, the Crimean Republic. This object has the following characteristics:

$$E = 2.1 \cdot 10^5 \text{ MPa}, h = 0.006 \text{ m}, \mu = 0.3, R = 5.2 \text{ m}, \rho = 7800 \text{ kg/m}^3, \rho_l = 820 \text{ kg/m}^3, L = 12 \text{ m}.$$

The tank shell is exposed by the shock wave of air explosion. This load is applied in the form of a triangular impulse (Section 3.1 of this work).

As a result of numerical integration of the equations of shell movement (4, 5, 11, 12), the pair of maximum pressure P_m and impulse I were found (points in Figure 3), at which the coefficients of movements in expressions (4, 11) becomes negative for a number of values forms of flexural deformations n (characteristic condition of stability (6, 15) becomes violated). As a result, by connecting the obtained points on the graph, we get a critical curve of stability for the tank under study, which is illustrated in Figure 3:

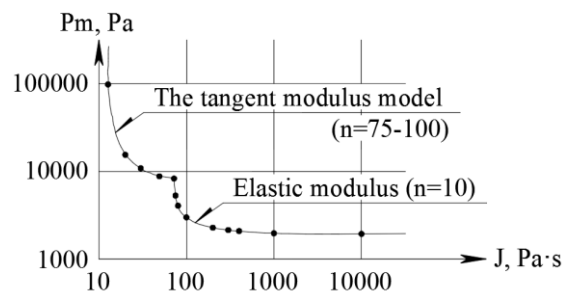


Figure 3. Critical curves of stability loss.

5. Conclusion

In this work, mathematical models for determining the critical curves of stability loss in the maximum pressure-impulse of pressure plane for a smooth cylindrical shell for loading areas of different duration are proposed.

By numerical integration, an example of a critical curve for a real-life vertical steel cylindrical tank in the field of plastic and elastic deformations was calculated.

The critical curve of stability loss in the field of impulse loads was calculated using the tangent modulus model (the upper part of the curve in Figure 3).

The critical curve of stability loss for quasi-static loads was calculated from the elastic modulus (the lower curve in Figure 3).

The curves obtained are well coordinated with results of the experiments which are described in many sources, for example, in [6].

6. Prospects of further researches

The equations (4, 5, 11, 12) of calculating and constructing critical curves of stability presented in this research cannot be solved analytically. However, the numerical integration of these equations gives sufficiently accurate solution consistent with the experimental results which are described in many papers, for example, in [6].

These methods can be applied to any structures whose design scheme can be represented as a smooth cylindrical shell. From the formulas given in this research it can be seen that these curves depend on many geometric parameters (thickness, diameter, shell length) of the structure under consideration. By adjusting these parameters (for example, the ratio of the radius to the thickness or the length ratio to the diameter), one can achieve an extension of the stability area of the shells under consideration.

These researches will help to make the design of constructions of the type considered more reliable and effective.

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