

Automatic modelling of regular polyhedrons and visualizing their formation in AutoCad through the Autolisp language

V A Romanova

People's Friendship University of Russia, 6, Miklukho-Maklaya st., Moscow, 117198, Russia

V.A.R-Victoryna@mail.ru

Abstract. Computer modeling of polyhedral surfaces is one of the most complex and interesting areas of architectural shaping. Computer-aided design systems, such as MathCad, Mathematika, AutoCad, etc., allow solving various problems on the formation of surfaces. The AutoCad system has significant advantages, because it has a built-in functional language AutoLisp, which has functions for mathematical calculations and for drawing different objects in 3D space. In this paper, a method of forming the surfaces of regular polyhedra in an AutoCAD environment through the use of programs in the language of Autolisp is considered. To demonstrate the method, a dodecahedron (Plato's body), a large dodecahedron and a small star dodecahedron (Kepler-Poinsot bodies) are selected.

1. Introduction

The first works on the geometric modeling of regular polyhedra belong to the ancient Greek scholar Plato (360 BC), whose name is given to five regular polyhedra. Four stellated regular polyhedra were obtained by Kepler (1619) and Poinsot (1809) by elongating the edges to their intersection and by increasing the faces of the dodecahedron and icosahedron. These polyhedra are named after the mathematicians who investigated them: "Kepler-Poinsot bodies" [1].

Further discoveries of new forms of polyhedra were carried out by geometric transformations of the bodies of Plato and Kepler-Poinsot [2], scanning methods, origami [3], geometric modeling involving combinatorial geometry, graph theory, Lobachevski geometry [4].

The capabilities of the AutoCAD system, containing the functional language Autolisp, allow forming analytical surfaces by kinematic method in a dynamic mode [5].

The purpose of this work is to visualize the process of forming the surfaces of the following regular polyhedra: a dodecahedron, a small stellated dodecahedron and a large dodecahedron.

The solution of the task includes the following sections:

- creating an algorithm for constructing polyhedra;
- creating constructive forms of polyhedron surfaces to visualize the process of their formation;
- creating programs in the AutoLisp language to visualize the formation of polyhedron surfaces in the AutoCAD environment.



2. Formation of the surface of a dodecahedron

The dodecahedron is a body bounded by twelve regular pentagons. Two of them form the base of the dodecahedron, and ten are arranged in two rows, forming a lateral surface.

2.1. Framework of a dodecahedron

The dodecahedron framework is used for forming the dodecahedron surface. In Figure 1 shows the elements of the dodecahedron in the initial position. Two pentagons are installed in the horizontal cells of the dodecahedron framework. On the sides AC and DK are shown additional coordinate systems $x^* y^* z^*$, $x' y' z'$ and dihedral angles ψ .

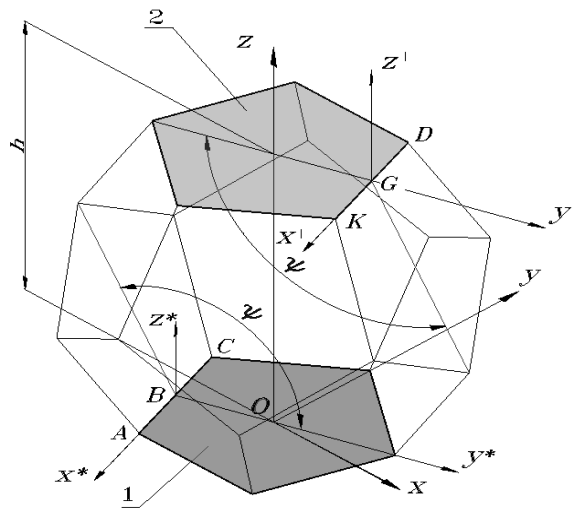


Figure 1. The initial position of the elements of the dodecahedron.

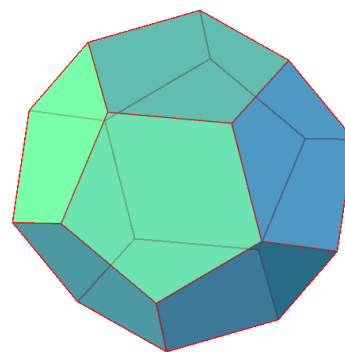


Figure 2. The constructive model of a dodecahedron.

2.2. Operations for forming a dodecahedron model

The process of forming a dodecahedron model consists of the following operations:

- Transferring the coordinate system to the point B - the middle of the side AC and the direction of the axis x^* along the side AC;
- Rotating the lower pentagon to the angle ψ around the axis x^* ;
- Forming the lower constructive form of the lateral surface of the dodecahedron by creating an array of five pentagons;
- Transferring the coordinate system to the point G - the middle of the side DK and the direction of the x' axis along the side DK;
- Rotating the upper pentagon to the angle ψ around the axis x' ;
- Forming the upper constructive form of the lateral surface of the dodecahedron by creating an array of five pentagons;
- Installing pentagons in the lower and upper cells of the framework.

The result of these operations is shown in Figure 2. The dodecahedron model consists of four constructive forms: the upper and lower bases, the upper and lower forms of the lateral surfaces. To form complex surfaces, such as polyhedra, AutoLisp has a Loft function that allows us to form a surface using the "cross-section" option. Its use makes possible to represent the formation of a surface as a sequential image on the screen of its compartments. Therefore, in order to show the process of formation of the dodecahedron surface, it is necessary to compose each constructive form of the compartments.

2.3. Forming sets of compartments

The Autolisp program includes four user-defined functions to form four sets of compartments. Compartments are formed in cycle. Each compartment is placed in the certain layer of the drawing. Layers in the drawing are downloaded in advance.

Creation of a set of compartments is carried out separately for the upper and lower bases, which is connected with the direction of their formation. The bases are divided into triangles having a common point in the center of the pentagon. On the lower base, the process of forming the surface of the compartments should go from the center to the sides of the pentagon, and on the upper base the same process must go from the contour of the pentagon to its center O . The formation of the compartments is performed in the same order (Figure 3, Figure 4).

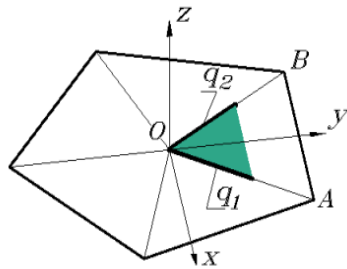


Figure 3. Formation of compartments of the lower base.

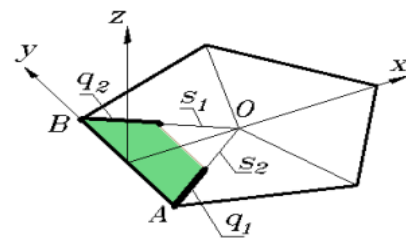


Figure 4. Formation of compartments of the upper base.

The directing curves are two sides of the triangle AO and BO , and the generating lines are the straight lines q_1 and q_2 , moving along the directing curves. Two sets obtained are constructive forms of bases. To form the lateral surface of the model, two more sets of compartments are created differing in the direction of formation (Figure 5, Figure 6).

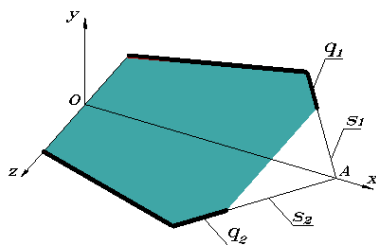


Figure 5. Formation of a set of compartments in direction 1.

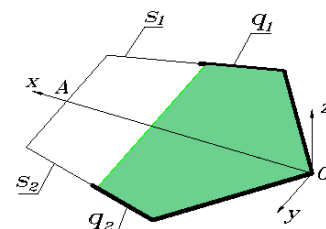


Figure 6. Formation of a set of compartments in direction 2.

From the resulting sets of compartments, two constructive forms of the lateral surface are formed by creating polar arrays (Figure 7, Figure 8). In the program, constructive forms are represented in the form of blocks.

2.4. Forming a dodecahedron surface

Stages of surface formation:

- Load the required number of layers into the drawing.
- “Freezing” layers intended for compartments expect the first.
- Insert the blocks with constructive forms of the dodecahedron model.
- Gradual “thawing” of layers with compartments and combining compartments into one surface.

The surface is formed by a kinematic method: the surface appears on the screen following the generators moving along the directing edges of the dodecahedron. The compartment of the previous step is deleted.



Figure 7. Lower constructive form.



Figure 8. Upper constructive form.

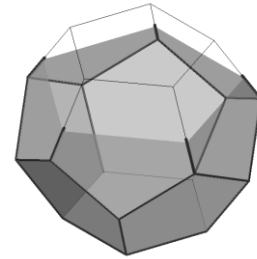


Figure 9. Dodecahedron surface formation.

Therefore, a complete surface of the dodecahedron is observed on the screen at the end of the cycle. The Figure 9 shows the formation of the dodecahedron by the motion of the surface following the generating lines moving along the guide-edges.

3. Formation of the surface of a small stellated dodecahedron

The small stellated dodecahedron (Figure 10) is obtained by extending the edges of dodecahedron to their intersection. The surface of the dodecahedron is transformed into twelve regular pentagonal pyramid surfaces, whose bases coincide with the faces of the dodecahedron.



Figure 10. Small stellated dodecahedron.

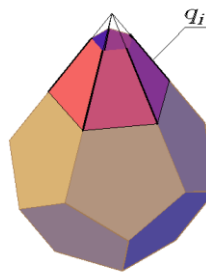


Figure 11. Formation of compartments of the pyramidal surface.

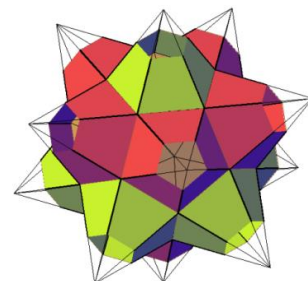


Figure 12. Formation of a small stellated dodecahedron.

3.1. Compartments of the lateral surface of a pyramid

The construction of the surface of a small stellated dodecahedron in the AutoCad system is advisable to perform by forming the lateral pyramidal surfaces on the faces of the dodecahedron. A set of compartments of the lateral surface of one pyramid is formed on the upper base of the dodecahedron by the movement of the straight generating lines q_i along the framework lines of the pyramid (Figure 11).

3.2. Constructive form of a small stellated dodecahedron

To create a constructive form of the lateral surface of a small stellated dodecahedron, the same operations are used as for the construction of the Plato dodecahedron, but with a set of compartments

of the lateral surface of the pyramid. The constructive forms of the small stellated dodecahedron are the array of compartments of 12 pyramidal surfaces and dodecahedron.

3.3. Forming the surface of a small stellated dodecahedron

When the layers with the compartments are "frozen", the surface of the dodecahedron and the framework lines of the pyramidal surfaces remain in the drawing. When these layers are "thawed", the surface of a small stellated dodecahedron is formed (Figure 12').

4. The formation of a large dodecahedron

A large dodecahedron (Figure 13) belongs to the Kepler-Poinsot bodies. Its faces are formed by the continuation of the faces of a small stellated dodecahedron to new pentagons. A large dodecahedron consists of 12 pentagonal faces with five pentagons at each apex. Two pentagons of the large dodecahedron are located horizontally at the bases of the Plato's dodecahedron. The remaining ten pentagons form two families, differing in the direction of formation, and form a lateral surface. Pentagons form stars on the faces of a large dodecahedron. In Figure 13, the surface of a large dodecahedron is represented.

4.1. Shapes of a large dodecahedron

For the construction of a large dodecahedron, the dodecahedron frame of Plato is used. The process of formation of a large dodecahedron model includes the same operations as the formation dodecahedron of Plato. Two families of pentagons are transformed into two design shapes by replacing each pentagon with set of compartments (Figure 14, Figure 15).

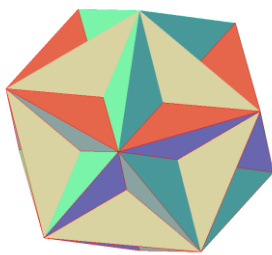


Figure 13. Small stellated dodecahedron.

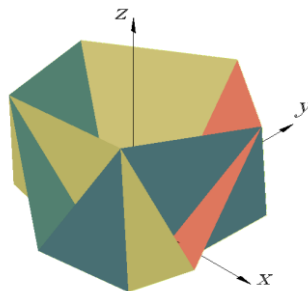


Figure 14. Lower design shape.

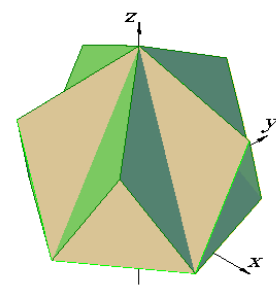


Figure 15. Upper design shape.

4.2. Four stages of surface formation

Visualizing the formation of a large dodecahedron surface is performed by the method of "unfreezing" layers with compartments. The process of surface formation consists of four stages (Figure 16, Figure 17, Figure 18, Figure 19).

In the first stage at $-h_1 < z \leq 0$ the surface is formed only by the compartments of the lower design shape. At $z = 0$ the pentagon surface is inserted into the lower horizontal cell of the framework.

In the second stage, when $0 < z \leq h_2$, the surface is formed by the compartments of both design shapes of the lateral surface. Compartments of lower design shape form part of surface, which slide along the bottom row of the cells of the framework following the generators. The part of the surface formed by the compartments of upper design shape is outside the framework, but is formed parallel to the cells of the upper row of the dodecahedron framework.

In the third stage, when $h_2 < z \leq h$, compartments of upper design shape form part of surface, which slides along the upper row of the framework following the generators. The part of the surface formed by the compartments of lower design shape at $z > h_3$ extends beyond the framework. When $z = h$, the pentagon surface is inserted into the upper horizontal cell of the framework.

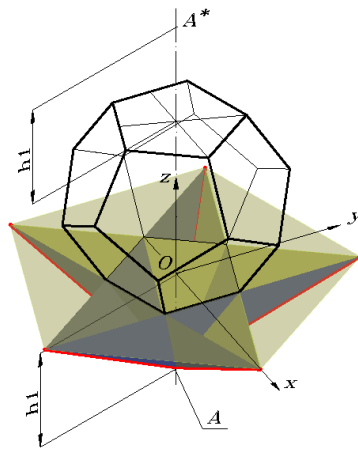


Figure 16. Stage 1.

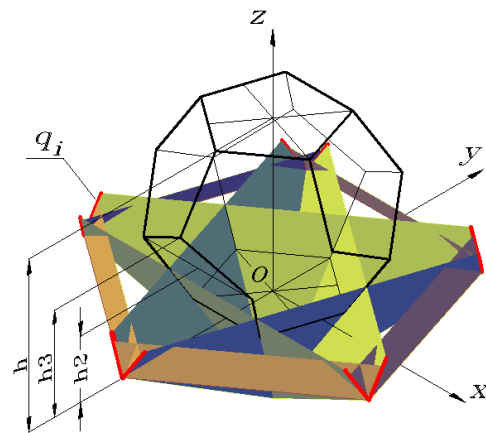


Figure 17. Stage 2.

In the fourth stage, when $h < z \leq (h + h_1)$ the surface is formed only by compartments of the upper design shape. The intersections of compartments with horizontal planes are the pentagrams.

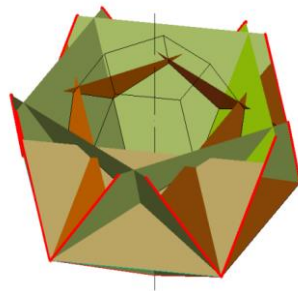


Figure 18. Stage 3.

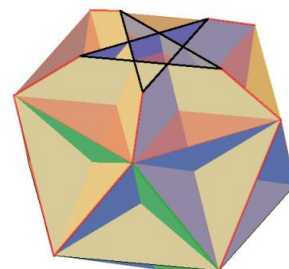


Figure 19. Stage 4.

5. Conclusion

The material presented shows that using programs in the AutoLisp language, it is possible to perform the formation of surfaces of regular polyhedra in dynamic mode in AutoCAD and to observe their cross sections. The presence of commands in AutoCAD for converting its files to files with the extension bmp, jpg, png allows creating mini-films which can be used for demonstration in Internet.

References

- [1] Gordon V O 2000 *Course Descriptive Geometry* (Moscow: Nauka) p 270
- [2] *Modeling of Platonic Solids and Stellate Polyhedra* Available at: <https://infourok.ru/modelirovanie-platonovih-tel-i-zvezdchatih-mnogogrannikov-1586026.html>
- [3] Wenninger M J 1974 *Polyhedron Models* (Moscow: Publishing house "Mir") p 238
- [4] *The list of uniform polyhedrons by the generating Schwarz Triangles*. Available at: https://en.wikipedia.org/wiki/List_of_uniform_polyhedra_by_Schwarz_triangle
- [5] Ivanov V N and Romanova V A 2016 *Constructive Forms of Space Constructions. Visualization of the Surfaces at Systems MathCAD, AutoCAD* (Moscow: ASV) p 410