The solution of structural class optimization problems. Part 2: Numerical examples

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Abstract. Earlier, the criterion of minimum material consumption was formulated within the design of the I-shaped bar width outline and the stability constraints or restriction to the value of the first natural frequency in one principal plane of the cross-section inertia. In the distinctive paper, we formulate a criterion for the minimum material capacity of the I-shaped bar with a variation in its thickness and outline of the width, with restrictions on the critical force value or restriction to the value of the first natural frequency in two principal planes of the section inertia. Numerical examples are presented.

Let us consider *I*-shaped bar with web height $b_1 = 0.28$ m and web thickness $b_{st} = 0.006$ m. The boundary conditions in the plane $x-0-y$ are shown in Figure 1 (top). The boundary conditions in the plane $x-0-z$ are shown in Figure 1 (bottom).

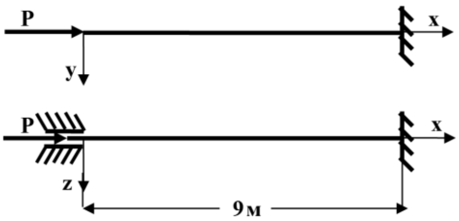

Figure 1. Considering boundary conditions.

The longitudinal force $P = 300$ kN acts on the rod. There are no external masses. Self-induced vibrations are caused by the mass of the rod. Specific mass of the bar material is equal to $\rho = 7850$ kg/m^3. Modulus of elasticity is equal to $E = 206000$ MPa. The bar can be subjected to vibrations with a frequency of $\omega_0 = 15$ sec^{-1}. The solution is realized on the basis of a discrete model which includes 30 sections.

Width of the flange ($b_2[i]$) and the thickness of the flange (δ_p) are varied. The web height (b_1) and web thickness (δ_{st}) are not varied. The values of the first critical force and the natural frequency in the two principal planes of inertia are limited. The effect of the longitudinal force on the natural frequency value and the possibility's influence of a given frequency vibrations on the magnitude of the critical force are taken into account. Target function for discrete model has the form:

$$V_0 = 2\sum_{i=1}^{n} b_2[i]\delta_p \,, \tag{1}$$

where n is the number of sections in discrete model.

Stability constraints have the form:

$$P \le P1[1]; \quad P \le P2[1], \tag{2}$$

where $P1[1]$, $P2[1]$ are the first critical forces in the planes $x-0-y$ and $x-0-z$ respectively.

Restrictions to the value of the first natural frequency have the form

$$\omega_0 = \omega 1[1] = \omega 2[1], \tag{3}$$

where $\omega 1[1]$, $\omega 2[1]$ are the lowest natural frequencies in the planes $x-0-y$ and $x-0-z$ respectively.

Criterion for the minimum material capacity of the I-shaped bar with a variation in its thickness and the width outline, with restrictions on the value of the critical force or restriction to the value of the first natural frequency in two principal planes of the section's inertia has the form:

$$\bar{\sigma}_{1\omega}[i] = \sqrt{\left[\frac{1}{3}\sigma_{1\omega}^2[i] + \frac{2}{3}\sigma_{1\omega\delta}^2[i] - (\omega_0)^2 \rho E v_{\omega}^2\right] + \mu[\sigma_{2\omega}^2[i] - (\omega_0)^2 \rho E w_{\omega}^2]} = const. \tag{4}$$

In order to find the value of the minimum material requirement, we consider the problem without using constructive constraints. The solution is implemented by random search. The results are shown in Table 1 and Table 2.

The optimum value for the selected thickness of the flange $\delta_{po} = 0.000764$ m is given, and the width $b_2[i]$ of the third column is given above the second and third columns. The second column shows the reduced stress values $\bar{\sigma}_{1\omega}[i]$. In all sections except the second and third, the value $\bar{\sigma}_{1\omega}[i]$ is close to unity, which confirms the minimum material consumption of the I-beam flange volume. In sections 2 and 3 the width of the flange is practically equal to the web thickness; it is practically merged with it. In the penultimate row below columns 2 and 3, the value of the flange material volume is given, $ft = 0.01597$ m^3. The solution obtained is unacceptable, since the dimensions of the flange are close to degeneracy, but the obtained solution gives the value of the minimum material-intensive volume of the flange when the critical force value is limited, taking into account the influence of possible vibration effects. This result will allow us to evaluate the closeness of constructively acceptable solutions to the minimum material-intensive solution.

Let us consider optimization examples of the values δ_p and $b_2[i]$ for the target function (1), constraints (2), (3) and additionally with constructive constraints on the values of the variable parameters. Introduction in the optimization of design constraints approximates the optimal solution to the practically acceptable [1,2].

Seven options for selecting constraints are considered. Out of these three, the flange thickness is limited, in other three flange widths, and in one the thickness and width of the flange are limited. The results of the decisions are given in Table 1 and Table 2.

The result of optimization in the process of the flange thickness limiting $\delta_{po} \ge 0.006$ m is shown in column 4 of Table 1.

Table 1. Results of the analysis (without using constructive constraints; with constraints for δ_{po}).

No.	Without using constructive constraints		With constraints for δ_{po}		
	$\delta_{po} = 0.000764$		$\delta_{po} \geq 0.006$ $\delta_{po} = 0.006$	$\delta_{po} \geq 0.008$ $\delta_{po} = 0.008$	$\delta_{po} \geq 0.01$ $\delta_{po} = 0.01$
	$\bar{\sigma}_{1\omega}[i]$	$b_2[i]$	$b_2[i]$	$b_2[i]$	$b_2[i]$
1	0.999	0.092	0.097	0.116	0.125
2	0.710	0.061	0.093	0.114	0.123
3	0.617	0.075	0.080	0.107	0.117
4	0.934	0.089	0.050	0.095	0.105
5	0.987	0.145	0.065	0.075	0.093
6	0.994	0.240	0.096	0.044	0.072
7	1	0.350	0.117	0.070	0.039
8	0.996	0.467	0.128	0.097	0.072
9	0.994	0.583	0.136	0.111	0.093
10	0.997	0.692	0.142	0.123	0.108
11	0.996	0.804	0.143	0.130	0.117
12	0.995	0.914	0.147	0.133	0.125
13	0.991	1.023	0.143	0.135	0.128
14	0.994	1.122	0.140	0.134	0.131
15	0.998	1.215	0.139	0.129	0.130
16	0.992	1.317	0.139	0.124	0.130
17	0.993	1.404	0.141	0.117	0.120
18	0.997	1.484	0.157	0.108	0.113
19	0.996	1.566	0.172	0.108	0.102
20	0.996	1.642	0.186	0.119	0.091
21	0.997	1.712	0.196	0.136	0.082
22	0.996	1.781	0.211	0.149	0.095
23	0.994	1.845	0.220	0.164	0.114
24	0.997	1.895	0.230	0.171	0.130
25	0.997	1.949	0.241	0.183	0.141
26	0.995	2.002	0.252	0.192	0.153
27	0.998	2.036	0.261	0.195	0.156
28	0.996	2.083	0.265	0.201	0.162
29	0.998	2.111	0.269	0.208	0.169
30	0.997	2.144	0.269	0.213	0.174
ft	0.01597		0.01773	0.01919	0.02107
eft %	0%		10.88%	20.01%	31.93%

Table 2. Results of the analysis (with constraints for $b_2[i]$).

No.	With constraints for $b_2[i]$			$b_2[i] \leq 0.14$
	$b_2[i] \leq 0.18$	$b_2[i] \leq 0.16$	$b_2[i] \leq 0.14$	$\delta_{po} \geq 0.01$
	$\delta_{po} = 0.0071$	$\delta_{po} = 0.0079$	$\delta_{po} = 0.0091$	$\delta_{po} = 0.01$
	$b_2[i]$	$b_2[i]$	$b_2[i]$	$b_2[i]$
1	0.106	0.108	0.118	0.126
2	0.098	0.102	0.113	0.124
3	0.091	0.097	0.114	0.118
4	0.076	0.084	0.099	0.106
5	0.075	0.076	0.089	0.094
6	0.078	0.079	0.093	0.072
7	0.105	0.092	0.102	0.046
8	0.117	0.113	0.097	0.073
9	0.134	0.122	0.127	0.098
10	0.147	0.140	0.129	0.109
11	0.151	0.145	0.140	0.126
12	0.151	0.151	0.140	0.129
13	0.159	0.158	0.140	0.136
14	0.157	0.160	0.140	0.138
15	0.171	0.160	0.140	0.139
16	0.171	0.160	0.140	0.138
17	0.175	0.160	0.140	0.138
18	0.177	0.160	0.140	0.132
19	0.180	0.160	0.140	0.125
20	0.180	0.160	0.140	0.117
21	0.180	0.160	0.140	0.106
22	0.180	0.160	0.140	0.112
23	0.180	0.160	0.140	0.105
24	0.180	0.160	0.140	0.119
25	0.180	0.160	0.140	0.134
26	0.180	0.160	0.140	0.140
27	0.180	0.160	0.140	0.140
28	0.180	0.160	0.140	0.140
29	0.180	0.160	0.140	0.140
30	0.180	0.160	0.140	0.140
ft	0.01910	0.01982	0.02124	0.02137
eft %	19.61%	24.11%	32.99%	33.81%

The volume of the flange is equal to $ft = 0.01773$ m^3, which is 10.88% more than the minimum material-intensive volume.

The optimization result in the process of the flange thickness limiting $\delta_{po} \geq 0.008$ m is shown in column 5 of Table 1. The volume of the flange is equal to $ft = 0.01919$ m^3, which is 20.01% more than the minimum material-consuming.

The result of optimization when limiting the thickness of the flange $\delta_{po} \geq 0.01$ m is shown in column 6 of Table 1. The volume of the flange is equal to $ft = 0.02107$ m^3, which is 31.93% more than the minimum material-consuming.

The optimization result in the process of the flange width limiting $b_2[i] \leq 0.18$ m is shown in column 2 of Table 2. The volume of the flange is equal to $ft = 0.01910$ m^3, which is 19.61% more than the minimum material-consuming.

The optimization result in the process of the flange width limiting $b_2[i] \leq 0.16$ m is shown in column 3 of Table 2. The volume of the flange is equal to $ft = 0.01982$ m^3, which is 24.11% more than the minimum material-consuming.

The optimization result in the process of the flange width limiting $b_2[i] \leq 0.14$ m is shown in column 4 of Table 2. The volume of the flange is equal to $ft = 0.02124$ m^3, which is 32.99% more than the minimum material-consuming.

The optimization result in the process of the flange thickness limiting $\delta_{po} \geq 0.01$ m and the width of the flange $b_2[i] \leq 0.14$ m is shown in column 5 of Table 2. The volume of the flange is equal to $ft = 0.02137$ m^3, which is 33.81% more than the minimum material-consuming.

Let's consider one more optimization variant of the flange with the target function (1), constraints (2), (3) and structural constraints for the thickness of the flange, its width, but under the condition of constant width along the length of the bar. We take the following constraints:

$$\delta_{po} \geq 0.01 \text{ m}, \quad b_2[i] = b_2 \leq 0.14 \text{ m}.$$

As a result of optimization, we get

$$\delta_{po} \geq 0.0104 \text{ m}, \quad b_2[i] = b_2 \leq 0.126 \text{ m}.$$

The flange volume is $ft = 0.02360$ m^3, which differs from the minimum material-intensive by 47.78%. The difference in the material consumption is much smaller according to the optimization solutions, where the width of the flange varies along the length of the bar. So the difference with the solution for a variable along the length of the flange's bar width and under constraints ($\delta_{po} \geq 0.01$ m and $b_2[i] \leq 0.14$ m) is 10.44%.

Uniform stability in both principal inertia planes is achieved in all optimization solutions (both with constructive constraints and without them). Thus, a set of optimization solutions can serve as a guideline in the process of the constraints' system choosing that meet the specific design conditions. In this case, it becomes possible to compare the optimized versions for the material consumption both among themselves and on the basis of proposed criterion with a material-intensive minimum [1,2]. The criterion presented in this paper, as well as those obtained earlier (presented, for instance, in [3-6]), can also be used for problems' solution of structures' optimal reinforcement and generally for solution of structural analyses' corresponding problems [7-12].

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