Mathematical simulation planning verifications of measuring instruments used in the field of construction, housing and communal services

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Abstract. The scientific and methodological approach to the formation and implementation of optimal plans of technical (metrological) verification of standards and measuring instruments used to equip the technical systems in the field of construction, housing and communal services was developed. The approach consist of three component items: the mathematical model of metrological verification of measuring instruments during operation, the algorithm for constructing optimal verification plans, the method for constructing the sequence of verifications of measuring instruments. The results of mathematical simulation are presented.

1. Introduction

The problem of mathematical simulation of planning verifications of measuring instruments (MI) is an actual practical task in many activities including the spheres of housing services and communal services and construction too [1, 2].

Ensuring the correctness of the transfer of the size of physical quantities units in all parts of the metrological chain is carried out by means of verification schemes: normative documents establishing the subordination of measuring instruments participating in the transfer of the unit size from state standards to working standards (WS) and complexes of measuring instruments (CMI) and MI, indicating methods and error [3, 4].

The transmission of physical quantity unit from standard to other MI means bringing the size of stored in standard to the size of the unit of quantity reproduced by the WS and MI. This procedure is carried out during the verification of MI [3, 4].

A wide variety of methods are used for constructing plans of technical (metrological) verification of MI and CMI. The basic method for constructing optimal or rational plans is the method of mathematical programming, including the method of integer mathematical programming, linear programming problem (LPP), integer linear programming problem (ILPP) [5, 6].

2. Verification scheme

The verification scheme [3, 4] can be represented as a pyramid (Figure 1). At the base of pyramid is the whole set of MI of the physical quantity, the top of the pyramid is the original standard (state standard), and on the intermediate "floors" are WS and primary and secondary measuring standards of various categories in accordance with their accuracy. Various aspects of the use of measuring equipment and methods used to improve the accuracy of measurements are considered in [3, 4]. The

article proposes a scientific and methodological approach to the construction and implementation of optimal plans [5] for the verification of MI by means of WS and secondary MI located in the pyramid between original standard and MI.
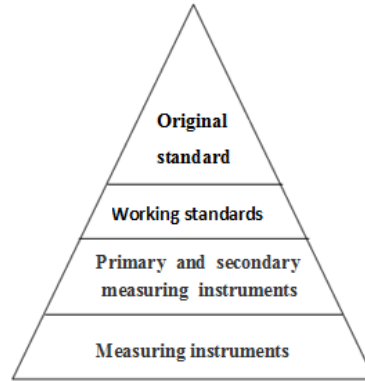

Figure 1. Verification scheme.

3. Formulation of the problem of optimal use of working standards for verifications of measuring instruments

Let consider R units of WS used to transmit M different value units. The characteristic vector for WS with number r, ($r = 1,2,...,R$) of dimension M has the form:

$$V_r^0 = \begin{pmatrix} v_{r1}^* \\ v_{r2}^* \\ ... \\ v_{rM}^* \end{pmatrix} \;,\; r = 1,2,...,R \;.$$

The coordinates of the characteristic vector can be equal zero or one.

Let some set of MI described by means of a set of characteristic vectors be given. We will combine all MI in J CMI (for example, on a territorial principle). If we summarize up all the characteristic vectors included in the CMI, then we obtain the characteristic verification vector for this CMI:

$$S_j^{**} = \begin{pmatrix} s_{j1}^* \\ s_{j2}^* \\ ... \\ s_{jM}^* \end{pmatrix} \;,\; j = 1,2,...,J \;.$$

This vector with integer value nonnegative coordinates shows what physical quantities and in what quantity should be transferred in the process of servicing this CMI.

The characteristic matrix of verification has the form:

$$S_j^* = \begin{pmatrix} s_{j1}^* & s_{21}^* & ... & s_{J1}^* \\ s_{12}^* & s_{22}^* & ... & s_{J2}^* \\ ... & ... & ... & ... \\ s_{1M}^* & s_{2M}^* & ... & s_{JM}^* \end{pmatrix} .$$

It is assumed that during transmission by the WS with the number r of unit of measure with number j there used certain amount of resource. In this paper, the duration of verification is considered as resource t_{rj}. Thus the resource unit cost matrix has the form:

$$T = \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1J} \\ t_{21} & t_{22} & \cdots & t_{2J} \\ \cdots & \cdots & \cdots & \cdots \\ t_{R1} & t_{R2} & \cdots & t_{RJ} \end{pmatrix}.$$

Let us introduce the unknown variables: x_{rj} - the number of units of physical quantities with the sequence number j transmitted by the WS with number r. Then the required service matrix has the form of:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1J} \\ x_{21} & x_{22} & \cdots & x_{2J} \\ \cdots & \cdots & \cdots & \cdots \\ x_{R1} & x_{R2} & \cdots & x_{RJ} \end{pmatrix}.$$

It is required to distribute WS between CMI (or CMI between WS) so that the quality criterion takes the lowest value

$$L(X,T) = \sum_{j=1}^{J} x_{rj} t_{rj} \rightarrow \min , \tag{1}$$

subject to the guaranteed carrying out all necessary verifications of MI:

$$\sum_{r=1}^{R} x_{rm} v_{rj}^{*} \geq s_{jm}^{*}, \quad m = 1,2,...,M , \quad j = 1,2,...,J . \tag{2}$$

After solving (1),(2) and computing the matrix X, the technological matrix TEX is constructed by means of the element-wise multiplication of the matrix T and X: $TEX = T \otimes X$. The technological matrix shows the amount of resources which was spent on each operation (verification by means of WS with number r the MI with number j in the amount of x_{rj} times).

Next, we calculate the estimate of the total resource spent by WS with every number r (the sum of elements of r - row of the matrix). Note that if you select maintenance time as a resource, during this time WS is already engaged in maintenance, and it could not be used to transfer units to other CMI.

And next, we calculate the resource for each WS, which was spent on the verification of the j CMI (the sum of j elements of the relevant matrix column).

It should be noted that in wide variety of practical problems, the constraints (2) can be represented as a simplex form with unit coefficients for unknown. In this case, the optimum solution of the corresponding ILPP will be integer value vector. In this case it possible to use the standard simplex method for solving ILPP, which has significantly less labor input than the general methods for solving ILPP.

Note also that the problem of constructing a sequence of use WS for MI verification is NP - complete (difficult to solve) problem [5-7]. To solve this problem, there are currently sufficiently effective approximate methods and software.

4. The results of mathematical simulation

The real park of MI which used in housing and communal services includes hundreds of thousands of MI samples, several dozen of CMI and tens of thousands of WS. In common case the LPP of such big size dimension and the corresponding problems of construction of sequences of carrying out checks can be solved by means of [6, 7].

It should be noted that for a real park, a significant part of each type of MI can be verified by one single standard. A significantly smaller portion of the MI samples can be verified by several different WS. Therefore, when we simulate planning of verifying, the optimization is applied only to the part of the park of MI for which there is a possibility to verify by means of several WS.

The Figure 2 represents four different types of MI (different types of measured values), a diagram showing the proportions between the number of MI samples that could be verified by only one WS value (left column) and the number of MI samples could be verified by several WS values (right column). Note that the main part of the MI can be verified by only one WS. Therefore, optimization is possible only for a smaller part of park of MI.
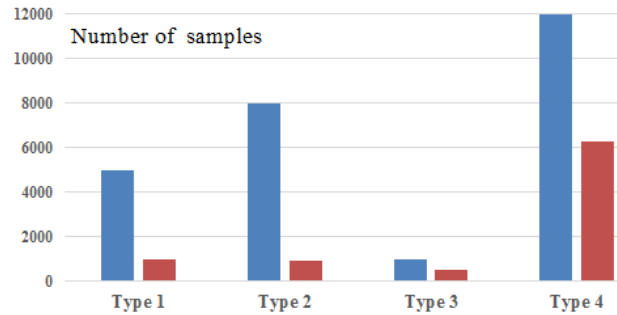

Figure 2. Proportions of the number of MI samples that could be verified by only one WS and the number of MI that could be verified by several different WS.

To understand regularities and peculiarities of solving such a large dimension problem, the article presents the results of solving a simple model problem: $R = 4$, $M = 4$, $J = 3$. The characteristic vectors for WS and the matrix of unit costs are given:

$$V_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad V_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad V_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad V_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad T = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 2 \\ 3 & 2 & 1 \\ 4 & 3 & 4 \end{pmatrix}.$$

The characteristic vectors of verifiable quantities, for MI, entering the first and second CMI:

$$S_{11} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad S_{12} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad S_{13} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad S_{14} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad S_{21} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \quad S_{22} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad S_{23} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

and the characteristic vectors for MI entering the third CMI:

$$S_{31} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad S_{32} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad S_{33} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad S_{34} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \quad S_{35} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad S_{36} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}.$$

Then the vectors of verification values for the three variables CMI:

$$S_1 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 3 \end{pmatrix}, \quad S_3 = \begin{pmatrix} 3 \\ 1 \\ 4 \\ 2 \end{pmatrix}.$$

Then matrix S and matrix $Z = T \otimes S$ - costs of resources (time) on verification are:

$$S = \begin{pmatrix} 3 & 0 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 4 \\ 0 & 3 & 2 \end{pmatrix}, \quad Z = T \otimes S = \begin{pmatrix} 6 & 0 & 3 \\ 6 & 2 & 2 \\ 3 & 2 & 4 \\ 0 & 9 & 8 \end{pmatrix}.$$

Let us introduce the unknown variables: x_{rj} - the amount of physical quantities with a sequence number j transmitted by WS with sequence number r. Then the matrix of service will have a dimension of 4 x 3.

The solution of the problem about optimal use of WS for verification of MI consists of three stages:

1. The solution of the problem of the minimum total time of all CMI verification is sought, provided that WS can serve only one CMI at a time (simultaneous maintenance of several CMI is impossible).

2. The estimate for the service time of all MI is constructed from below.

3. The sequence of verification for each WS is sought. The service time of three CMI in a total is calculated (the minimum possible time during which at least one CMI is in service).

It is necessary to find the minimum total service time of all CMI:

$$L_1 = 6x_{11} + 6x_{21} + 3x_{31} + 2x_{22} + 2x_{32} + 9x_{42} + 3x_{13} + 2x_{23} + 4x_{33} + 8x_{43} \rightarrow \min,$$

and at the same time to provide guarantee verification of three CMI:

$$\begin{cases} x_{11} + x_{31} + x_{41} \geq 3 \\ x_{11} + x_{21} \geq 2 \\ x_{31} \geq 1 \\ x_{21} + x_{41} \geq 0 \end{cases}, \quad \begin{cases} x_{12} + x_{32} + x_{42} \geq 0 \\ x_{12} + x_{22} \geq 1 \\ x_{32} \geq 1 \\ x_{22} + x_{42} \geq 3 \end{cases}, \quad \begin{cases} x_{13} + x_{33} + x_{43} \geq 3 \\ x_{13} + x_{23} \geq 1 \\ x_{33} \geq 4 \\ x_{23} + x_{43} \geq 2 \end{cases}.$$

Since the LPP matrix has a block structure, then, using the decomposition principle, we will solve three independent problems about the minimum of the verification time for each CMI.

Let's formulate each of these three LPP.

It is required to organize the verification of all MI so that the verification time of the 1-st CMI was minimal:

$$L_1 = 6x_{11} + 6x_{21} + 3x_{31} \rightarrow \min, \ x_{11} + x_{31} + x_{41} \geq 3, \ x_{11} + x_{21} \geq 2, \ x_{31} \geq 1, \ x_{21} + x_{41} \geq 0.$$

It is required to organize the verification of all MI so that the verification time of the 2-nd CMI was minimal:

$$L_2 = 2x_{22} + 2x_{32} + 9x_{42} \rightarrow \min, \ x_{12} + x_{32} + x_{42} \geq 0, \ x_{12} + x_{22} \geq 1, \ x_{32} \geq 1, \ x_{22} + x_{42} \geq 3.$$

It is required to organize the verification of all MI so that the verification time of the 3-d CMI was minimal:

$$L_3 = 3x_{13} + 2x_{23} + 4x_{33} + 8x_{43} \rightarrow \min, \ x_{13} + x_{33} + x_{43} \geq 3, \ x_{13} + x_{23} \geq 1, \ x_{33} \geq 4, \ x_{23} + x_{43} \geq 2.$$

The analysis showed that since the third coordinate for the WS characteristic vectors is equal to one only for the third WS so the transfer of the size of the third physical value is possible only with the use of the third WS. Thus the third WS will be used for transferring sizes for the first and second CMI just once, and for the third CMI – exactly for 4 times. So we require $x_{31} = 1$, $x_{32} = 1$, $x_{33} = 4$ and the third inequality in all three of LPP we will not consider. As a result of the solution of the formulated ILPP taking into account the conditions noted above we will receive a matrix of solution and technological matrix:

$$X = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 4 \\ 2 & 2 & 1 \end{pmatrix}, \quad TEX = \begin{pmatrix} 0 & 0 & 0 \\ 12 & 2 & 2 \\ 3 & 2 & 16 \\ 0 & 18 & 8 \end{pmatrix}.$$

Thus, the service time of each CMI are: $L_1 = 15$, $L_2 = 22$, $L_3 = 26$ respectively. At the same time operating time of each WS are: $P_1 = 0$, $P_2 = 16$, $P_3 = 21$, $P_4 = 26$. It should be noted that the first WS is not used at all, and a third WS is advisable to use only for the maintenance of the second CMI and the third CMI.

Lower bound for total service time of all CMI: $L^- = \max\{L_1, L_2, L_3\} = 26$.

It should be noted that the sequence of operation for each WS can be constructed in several different ways. As a rule, the problem has an infinite number of solutions. One of the variants of the sequence is shown in Figure 3.
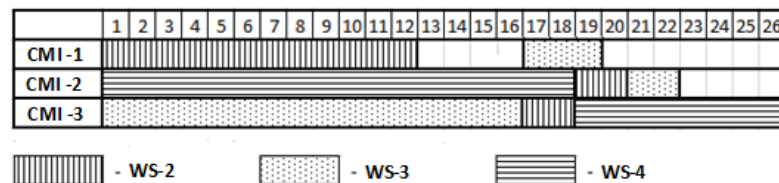

Figure 3. The results of construction of the sequence of use of working standards verification CMI.

It should be noted that the total time spent by a technical system on the metrological service is equal to 26. In the example considered it is the same time as the estimate from below L^-. Note that in solving practical problems for real park of MI and WS, the total time which spent on the system maintenance can significantly exceed the value L^- [6, 7].

5. Conclusion

The main results of the work are as follows:

1. A scientific and methodological approach to the optimization of the process of verification of measuring instruments by means of using working standards, based on the decision of the linear programming problem series was developed. The results of decision of linear programming problem allows developing the optimal plans for verification and to build sequences of verifications, that implementing optimal plans.

2. It is established that if the system of restrictions of linear programming problem can be represented as a simplex a form with unit coefficients at unknowns, then the solution of the corresponding linear programming problem will also be integer. When these conditions are met, it is possible to use the standard simplex method for solving integer linear programming problem, which is significantly less laborious than the general methods for solving integer linear programming problem.

3. The method developed has already shown its effectiveness in the tasks of constructing optimal verification plans in the field of metrology of special equipment used to improve the defense capacity, as well as for special equipment used in emergency situations.

4. The results of research and solution of the model problem presented in this article allow us to understand the basic laws, meaning and structure of optimal plans.

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