The calculation of a culvert in a high mound of a road as a contact problem with friction

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Abstract. The long large span culvert in a high mound is reviewed in the article. Based on the calculations series results, if we consider the separation and slip of a soil on the culvert, the distribution of internal forces can differ a lot from the linear formulation.

1. Introduction

Researching stress-strain state (SSS) of culverts and building a model of the system "culvert – mound" must take into account the scheme of interaction of pipes and soil. A simple scheme of interaction assumes the replacement of soil action by forces [1]. The specified scheme assumes the account of unilateral interaction of pipe and soil. In [2, 3] for this purpose, contact elements with characteristics whose values are difficult to justify are used. In this paper we consider the option of frictional one-way interaction, which characterizes this model only by the friction coefficient.

The main goal of the research is to found the influence of the friction considering between culvert and mound on the distribution of internal forces in the pipe.

Series of calculations were carried out for the large spanned culvert with different profiles (three-radial, parabolic, sinusoidal, elliptic) and gentleness (camber and span ratio) under the mound of homogeneous soil (clay, coarse sand, disintegrated rock), which is the only load on the pipe. The load on a day surface was not considered by virtue of big soil layer thickness above the pipe.

2. Estimated scheme

Culvert length is enough to consider the plane deformation state. The culvert is modeled by an arch because it has open contour. By virtue of symmetric object and load, it is considered half of the arch and adjacent soil to it (Figure 1). For the calculations FEM is used.

The arch is modeled by plane frame elements and the mound by 4-node elements of the plane problem of elasticity theory

3. Model and algorithm

Nonlinear unilateral contact model of arch and soil with Coulomb friction is considered in the article.

Interaction model of arch and soil with frictionless unilateral constraints in discredited problem, if the displacement method (DM) is chosen, is describing by the linear complementary (LCP) [4, 5] problem:

$$\mathbf{x_n} = \mathbf{R_{nn}} \cdot \mathbf{z_n} + \mathbf{R_{Fn}};\ \mathbf{z_n} \geq 0;\quad \mathbf{x_n} \geq 0;\quad \mathbf{z_n}^T \cdot \mathbf{x_n} = 0\,. \tag{1}$$

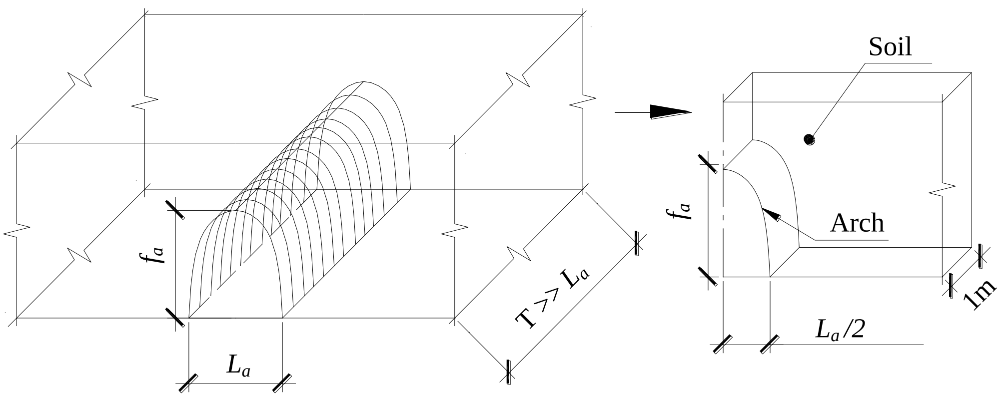

Figure 1. Conversion to the plane state.

For the model with friction and at known limiting forces $\mathbf{r_f}$ LCP form:

$$\begin{bmatrix}\mathbf{x}_\tau^+\\ \mathbf{x}_\tau^-\end{bmatrix}=\begin{bmatrix}\mathbf{R}_{\tau\tau} & -\mathbf{R}_{\tau\tau}\\ -\mathbf{R}_{\tau\tau} & \mathbf{R}_{\tau\tau}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{z}_\tau^+\\ \mathbf{z}_\tau^-\end{bmatrix}+\begin{bmatrix}\mathbf{R}_{\mathbf{F}\tau}\\ -\mathbf{R}_{\mathbf{F}\tau}\end{bmatrix}+\begin{bmatrix}\mathbf{r_f}\\ \mathbf{r_f}\end{bmatrix}; \tag{2}$$

$$\mathbf{x}_\tau^+\geq 0;\quad \mathbf{x}_\tau^-\geq 0;\quad \mathbf{z}_\tau^+\geq 0;\quad \mathbf{z}_\tau^-\geq 0;\quad \mathbf{z}_\tau^{+T}\cdot\mathbf{x}_\tau^+=0;\quad \mathbf{z}_\tau^{-T}\cdot\mathbf{x}_\tau^-=0.$$

Here $\mathbf{z}_\tau=\mathbf{z}_\tau^+-\mathbf{z}_\tau^-$, $\mathbf{z_n}$ are vectors of mutual displacements of the contact pairs points (assumed points of two bodies contact) on the tangent and along the normal to the L respectively (Figure 2); L is the assumed contact zone (in this work it is along the arch outline); $\mathbf{x}_\tau=\left(\mathbf{x}_\tau^+-\mathbf{x}_\tau^-\right)/2$, $\mathbf{x_n}$ are the vectors of contact pair interaction forces on the tangent and along the normal to the L respectively, f is the friction coefficient between bodies; $\mathbf{R_{nn}}$ is the contact stiffness matrix (CSM) with components – the forces in introduced constraints (will be described later) along the normal to L from the unit dislocations of these contact pairs along the normal; $\mathbf{R}_{\tau\tau}$ is CSM with components – the forces in introduced constraints along the tangent from the unit dislocation of these contact pairs along the tangent; $\mathbf{R_{Fn}}$, $\mathbf{R}_{\mathbf{F}\tau}$ are contact load vectors (CLV) with components – the forces in introduced constraints along the normal and along the tangent from external load (q', q'' as an example on Figure 2).

To solve problems (1), (2), effective algorithms are developed [6 - 8]. On this basis “the algorithm of iteration over limit friction forces” has been implemented using methods solving LCP [4, 5].

The algorithm for calculating one-way contact of the arch and soil along the contact boundary normal with considering the friction on the tangent to arch outline is as follows. On the first step, it is supposed that there is no friction. Solving the problem of frictionless unilateral contact as the LCP (1), we get normal contact forces. On the second step, we solve the LCP (2) for the contact with friction at known limit friction forces, which we found on previous step, and we get tangent contact forces. On the third step, we solve again the LCP for the frictionless unilateral contact, adding tangential contact forces to existing load, which we get on the previous step. As the result we have normal contact forces. Iteration process proceeds until the difference of SSS parameters of neighboring steps does not become sufficiently small.

In that way, at each step, it is accepted or invariable contact normal forces (odd steps) or contact tangent forces (even steps).

To solve the LCP, the DM is chosen [4, 5]. DM basic system is gained by changing unilateral constraints to bilateral, joining arch and soil along the tangent and along the normal to arch outline in each contact pair (Figure 3, nodes are separated for more visibility). All nodes within the contact pair have the same coordinates (coincident). Denote the number of contact pairs as m, and the number of bilateral constraints as $k=2m$.

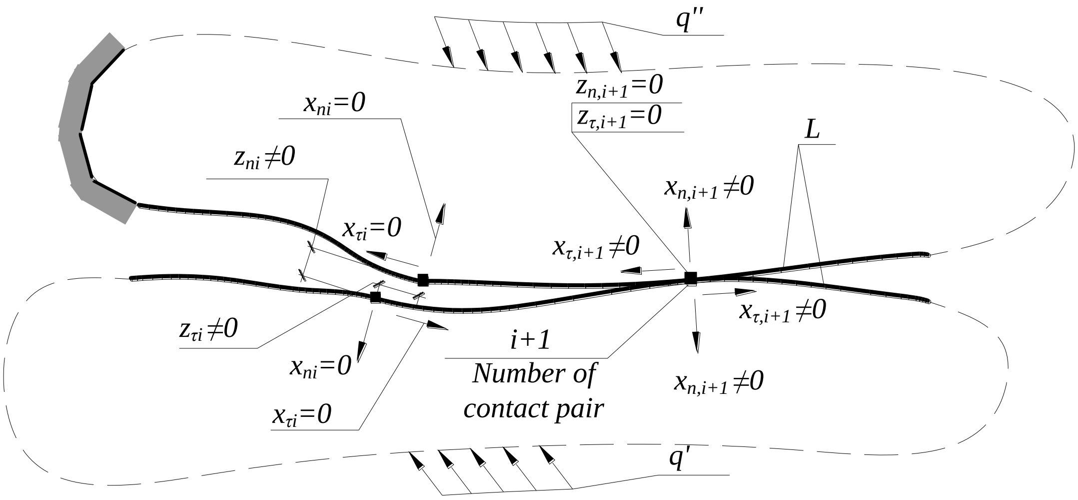

Figure 2. Contact of deformed bodies after deformation. The detachment of the *i* contact pair and the contact of the *i*+1 contact pair.

To solve the problem, it is needed to form CLV $\mathbf{R_F}$. Formation of $\mathbf{R_F}$.is carried out in the DM basic system under a given load (Figure 3, b). Each i element of the $\mathbf{R_F}$.equals to the force in the introduced i bilateral constraint.

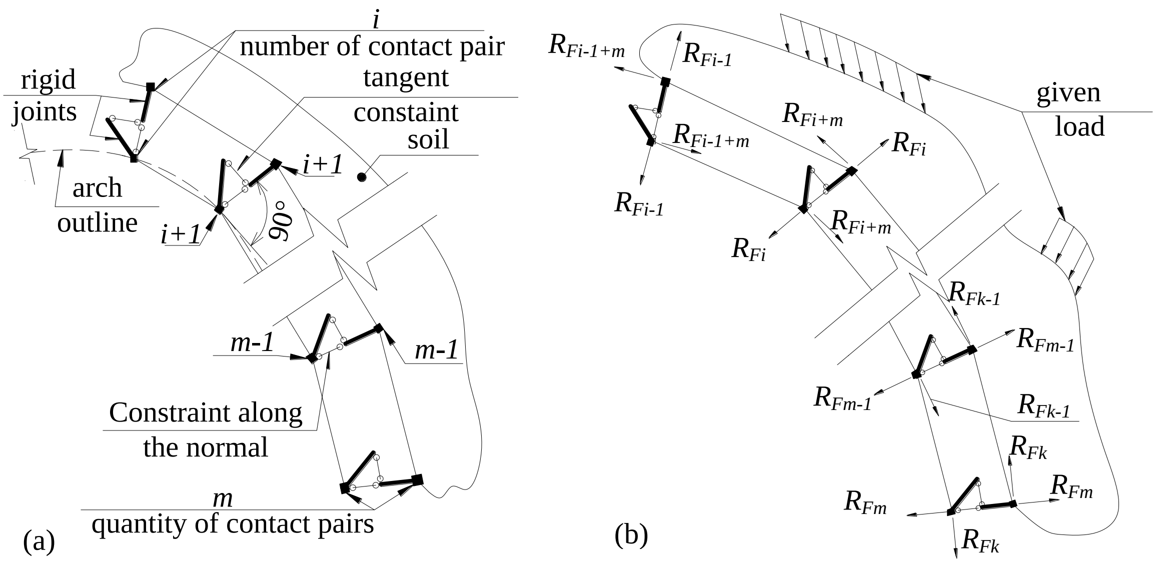

Figure 3. Constraints layout: Numbering (a), Formation of CLV (b).

Also it needs to form CSM. CSM components are forces in the introduced bilateral constraints for a unit dislocation along the introduced constraints direction.

For the problem, the numbering of constraints is first taken along the normal, and then along the tangent to the L – contact zone (arch outline).

In order to form a column j CSM $\mathbf{R}$ (Figure 4), it needs to provide a unit dislocation of j contact pair along the normal to the arch outline and find forces in all i -ths constraints, ie, to find all the components $R_{i,j}$, where $i = 1, 2, ..., k$.

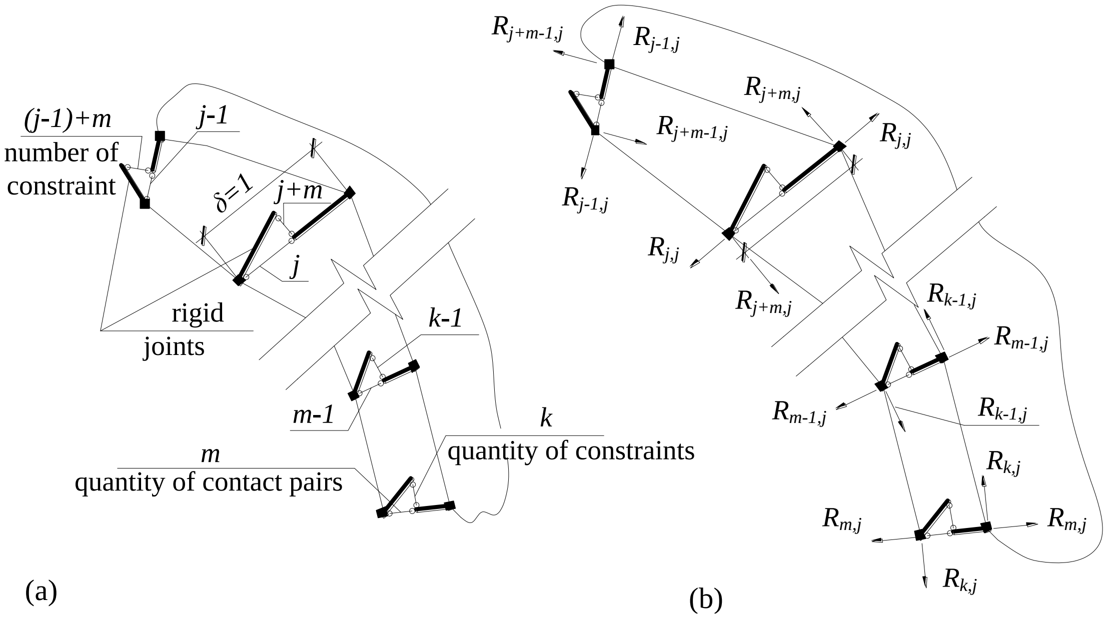

Figure 4. Formation of CSM: Numbering of introduced constraints (a), Formation of *j* CSM column (*j*-th constraint introduced along the normal to arch outline) (b).

For the CSM formation, it is convenient to use null-elements [9], which makes it possible to transform the kinematic effect (unit dislocation) to the force and to ensure a unit dislocation of the contact pair in any direction, even for coincident nodes [10]. Finding the forces of contact pairs interaction is also simplified and reduced to finding the force in the null-element connecting the contact pair.

4. Example and results

An estimated scheme example of the three-radius arch, considered in the article, for the changeover contact is shown in Figure 6 (the arch nodes are corresponding to the numbers of the contact pairs). Three-radius arch was chosen as one of the popular on practice [11] variants of arch outlines.

Input for calculation: arch camber $f_a = 10.047\ m$, arch span $L_a = 13.622\ m$, mound height $h = 26.662\ m$, soil elastic modulus $E_s = 4.5 \cdot 10^7\ Pa$, soil Poisson's ratio $\mu = 0.27$, width of cut strip (scheme depth) $t = 1.2\ m$,soil specific gravity $\gamma = 2.266 \cdot 10^4\ N/m^3$, arch elastic modulus $E_a = 2 \cdot 10^{11}\ Pa$, moment of inertia of arch $I_{xa} = 2.110 \cdot 10^{-4}\ м^4$, cross-sectional area of the arch $A_a = 2.366 \cdot 10^{-2}\ м^2$, the friction coefficient between arch and soil $f = 0.6$. Further results will be presented using these parameters except for the described changes. A solution without accounting friction (frictionless unilateral contact) is considered in [12].

The algorithms described above are programmatically realized in C#. Series of calculations were carried out, the results shown below are obtained using this program.

The nodal tangent interaction forces of the arch with the soil and mutual displacement soil points relative to the arch along the tangent as the LCP solution result for the half of the arch is shown in Figure 7 (the first and the last nodes of the arch are not shown, because there are supports). It is clear from the figure that forces are equal to the limiting ($x_{\tau i} = x_{ni} \cdot f$), in the zone with friction, there is a slippage of the soil along the arch, and where the friction forces are less than the limiting, the mutual displacement along the tangent is zero.

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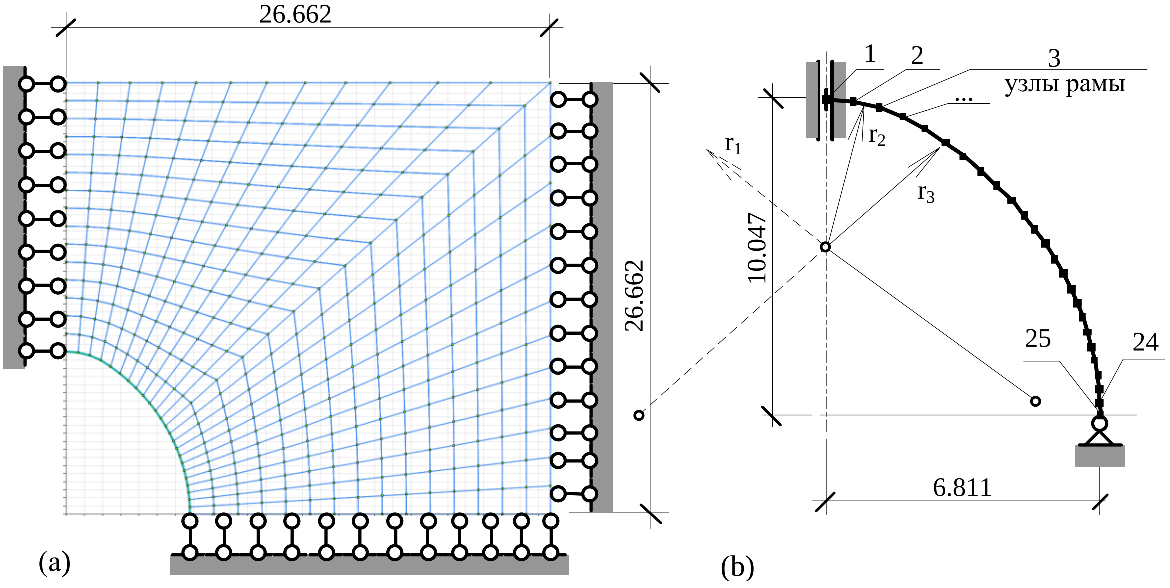

Figure 5. Estimated scheme. Boundary conditions for soil (a) and three-radius arch (b).

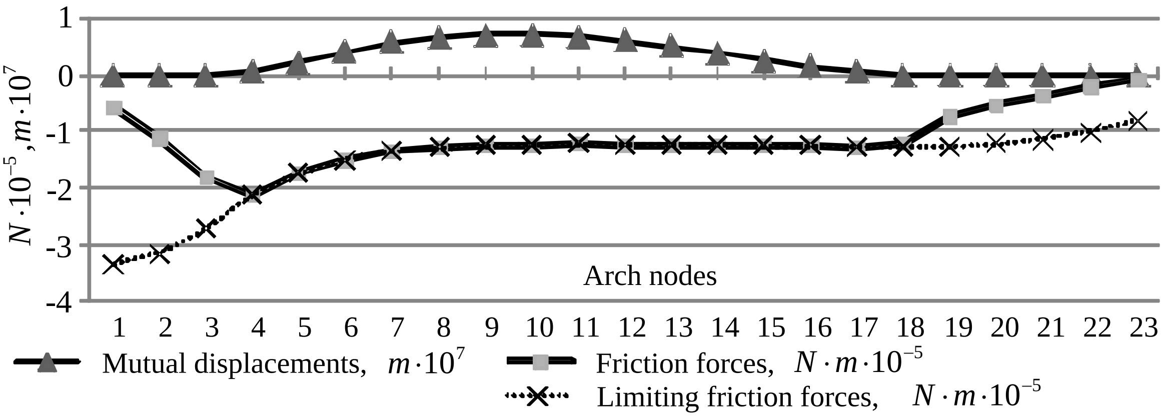

Figure 6. The result of solving LCP for tangent mutual forces and displacements.

Figure 8 shows the bending moment diagrams for half of the three-radius arch for various interaction models with the soil. When different models of the soil and the arch interaction are compared, a solutions convergence of the frictionless contact and contact with friction is observed with friction coefficient $f \to 0$. With f increase, a convergence of the bilateral contact solution (linear formulation) and the contact with the friction is observed. Figure 9 shows bending moment diagrams for half of the arch with different outlines types with other matching parameters.

Further, an arch of an elliptical outline was considered, because the smallest maximum bending moment emerges in it (see Figure 9). Figure 10 shows bending moment diagram for half of the elliptical arch with different soil characteristics: disintegrated rock: $E = 4.5 \cdot 10^7\, Pa$, $\gamma = 2.266 \cdot 10^4\, N/m^3$, $\mu = 0.27$; coarse sand: $E = 4.903 \cdot 10^7\, Pa$, $\gamma = 2.858 \cdot 10^4\, N/m^3$, $\mu = 0.35$; clay: $E = 2.059 \cdot 10^7\, Pa$, $\gamma = 2.923 \cdot 10^4\, N/m^3$, $\mu = 0.3$ taking into account the detachment and friction of the soil along the arch.

Figure 11 shows the maximum modulo values for the bending moment diagrams for half of the elliptical arch with different gentleness f_a / L_a (see Figure 1) with taking into account the detachment and friction of the soil along the arch. The figure shows a clearly defined minimum, which allows us to talk about the selection of optimal geometric parameters of culvert.

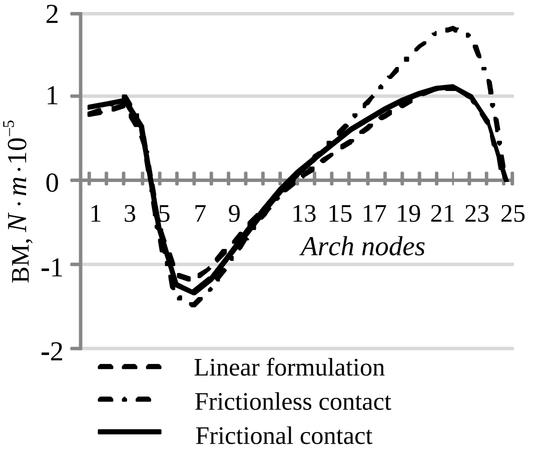

Figure 7. Bending moment diagram of half of the arch. Comparison of models.

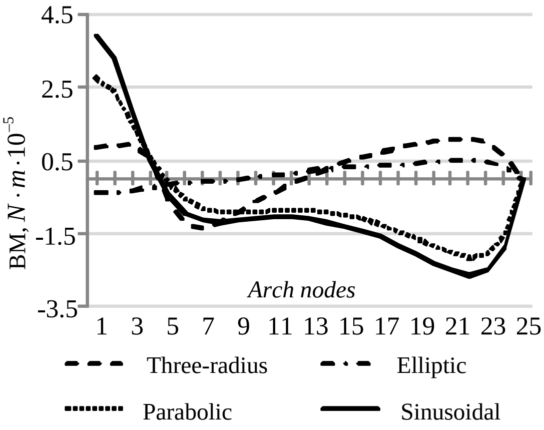

Figure 8. Bending moment diagram of half of the arch. Various arch outlines.

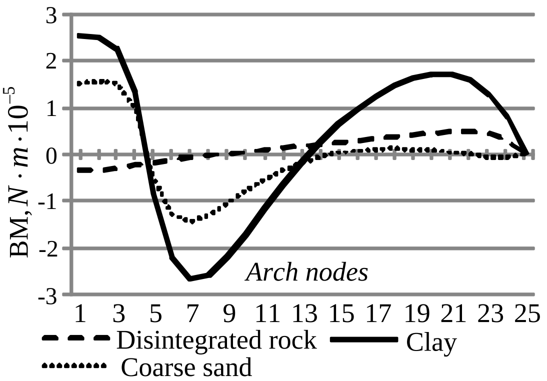

Figure 9. Bending moment diagram of half of the arch. Various soil types**.**

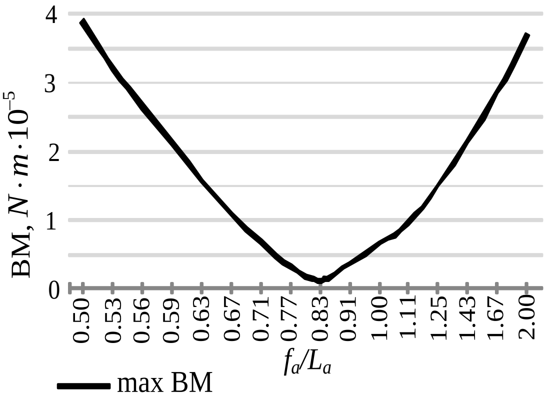

Figure 10. Bending moment diagram of half of the arch. Different variants of gentleness of the arch.

5. Conclusions

From the comparison of the calculation results for different models of the interaction of the culvert and the soil, it is clear that the solution for contact with friction is in the interval between the solutions for ideal and bilateral contact. At low friction coefficients, the bending moment can be increased up to 1.5 times compared to the bilateral contact. In the process of constant height of the mound maintaining and the soil characteristics and the types of the arch outlines changing, these dependences are preserved.

For the case of the arch gentleness change, a clearly defined minimum is observed for the maximum value of the bending moment, which allows you to choose the most optimal culvert dimensions.

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