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Probabilistic analysis and reliability evaluation of the harmonically loaded rod systems with dynamic vibration dampers, based on the use of modern software complexes

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Abstract. The problems of determining the probabilistic properties of the stress-strain state parameters of dynamically loaded systems with the application of modern software are considered. The advantages and possible risks of using the multifunctional ANSYS software in probabilistic analysis and reliability calculations of dynamically loaded systems are analyzed. Peculiarities of software application have been revealed taking into account the character of the changes in the probability density distribution of the stress-strain state parameters of dynamic system in the frequency intervals in the vicinity of the resonance zone. Qualitative and quantitative comparison of the results of probabilistic characteristics calculated in ANSYS software with analytical solutions has been carried out.

1. Introduction

One of effective ways to reduce structures and constructions vibrations is to use dynamic vibration dampers (DVD) [1]. In the deterministic formulation full damping of vibration can be realized as the result of optimum selection of damper's parameters. But because of all design parameters of system with DVD have the stochastic nature, even their small accidental deviations from those values on which setup of DVD is made, lead to sharp increasing of the dynamic system stress-strain state (SSS) parameter's amplitudes, that is the peculiarity caused by effect of natural frequencies crowding together in the area of the frequency damper setup. In this aspect probabilistic calculation is necessary [4] for the correct evaluation of reliability, that is actual according to requirements of the Russian [2] and European Standards [3].

Calculation of reliability needs that the probabilistic properties of system's output parameters have to be known (such as dynamic and/or full displacements, forces, stresses, etc.). Definition of their probabilistic characteristics (density distribution functions, the means, standard deviations (SD)/variance and so forth) is one of the most difficult stages of reliability analysis and demands taking in account the design features (structure, the sizes, arrangement of masses, etc.) and quantities of system's elements, and also types and frequency properties of excitations, etc. For the considered systems the obtaining of accurate analytical expressions for the distribution density

$$p_f(f)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty} p_X(X)\cdot\left|\frac{d\,x_1(f,x_2,\ldots,x_n)}{d\,f}\right|dx_2\ldots dx_n\,, \tag{1}$$

of mean and variance

$$\overline{f} = \int_{-\infty}^{\infty} f \cdot p_f(f)\,df; \quad \widehat{f} = \int_{-\infty}^{\infty} \left(f - \overline{f}\right)^2 p_f(f)\,df \tag{2}$$

for some output parameter $f(X)$ which is the function of n entrance random variables $X = \{x_1 ... x_n\}$ is difficult or may be mathematically impossible even if the analytical mathematical model defining relations between input and output parameters of system is known.

Because of mentioned difficulties, some approximate methods can be considered as the rational alternative to creation of the exact decision on determination of probabilistic characteristics of the dynamic system SSS parameters. Namely there are statistical tests method (STM) and the statistical linearization method (SLM) [5]. Both of them are based on the repeated recalculations with varying values of system's input parameters. Large number of needed recalculations is the main problem for SLM and STM application in practical designs since the general time of the calculation can be unacceptably large, especially in comparison with the deterministic dynamic analysis. In spite of SLM is more favorable on the volume of calculations, it does not allow to reveal type of output parameters distribution. Also it can give big deviations from the exact decision in values of variance in the problems with strongly nonlinear functions $f(X)$ [6]. As for STM, it is capable to provide the results with small errors, but the large volume of calculations may be demanded.

Modern program complexes allow solving STM implementation problem even for complex systems – there are procedures of statistical processing and the stochastical data analysis in some programs (Matlab, MathCAD, STATISTICA, EXEL, VaP, etc.). Using the STM algorithms, histograms, the form of the density distribution, and the numerical probabilistic characteristics of the output parameters [7] can be obtained. In this case, both the known analytical dependences of the output parameters on the input parameters and the finite-element description of the problem can be used for the design model creation. The objective of the work is studying the peculiarities of probabilistic analysis and reliability evaluation of harmonically loaded systems with DVD taking into account effects of nonlinear influence input parameters on the SSS of system using modern program complexes; evaluation of computer calculations results' accuracy.

2. The main part

2.1. Theoretical bases, methods and algorithms of the solution of problems

In the majority of the program complexes (SCAD, Lira, MicroFe, STARK ES, etc.) used in Russian Federation's design practice there are no modules allowing determining probabilistic characteristics of the system SSS parameters. Their use requires the creation of additional program modules. Possibility of probabilistic analysis performance with use of finite-elements method (FEM), and also calculation for known analytical equations is implemented in the ANSYS software (Workbench environment, Design Exploration module – Six Sigma Analysis application). The solution of problem consists of three main steps:

1) computing experiment with use analytical or finite-elements models, including its planning and implementation;

2) approximation of relation output parameters from the input ones (creation of so-called "metamodel" – response surface), that leads to reduction of calculations volume in theory;

3) determination of functional and numerical probabilistic characteristics.

The vastest stage of probabilistic calculations in ANSYS software is Design of Experiments (DOE). On this step description for probabilistic properties of system's input parameters are set. Then the set of input and output parameters for used model (so-called "Design Points") is calculated for further creation of metamodel based on this data. Quality of metamodel depends on number of design points which is dictated by the method of determination and type of the solved problem. Thus, if a certain parameter is highly nonlinearly dependent on input parameters (for example, in problems of dynamics, stability, etc.), then substantially more design points are required, as well as the application of appropriate methods for their statistical modeling.

At the stage of Response Surface creation according to the data obtained on the previous step, different metamodels are formed by different ways and quality of approximations is estimated. It is important to note that quality of metamodels influences to correctness of probabilistic calculation.

Final step is direct determination of probabilistic characteristics (Six Sigma Analysis procedure). Based on the formed metamodels multiple recalculations (> 10^4) with variation of input parameters are executed, then the generation of histograms is carried out and finally the probabilistic characteristics of output parameters are calculated.

The results of researches have shown, that even for simple models in problems of some types the application of the ANSYS software implementing FEM with using metamodels for probabilistic calculation demands large time expenditure. The examples allowing to evaluate possibilities of the ANSYS software in the field of probabilistic calculations and quality of obtained results in comparison with the available analytical decisions are presented below.

2.2. Examples

In order to obtain testing estimates of computer calculations results the models of systems with the existing analytical solutions for dynamic problems are considered.

Example A. Beam of constant rigidity with one degree of freedom, loaded by the point harmonic load (Figure 1).

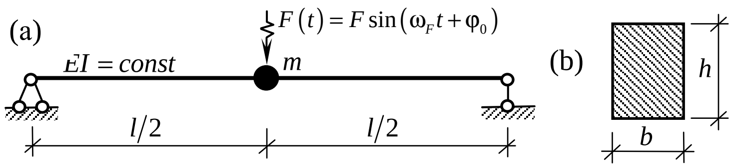

Figure 1. Design model (a); cross-section (b).

It is required to obtain probabilistic characteristics of the mass's dynamic displacement amplitude $\tilde{y}_{dyn}$ determined as $\tilde{y}_{dyn} = \tilde{y}_{st}\tilde{\mu}$ ($\tilde{y}_{st}$ – static displacement of loading point; $\tilde{\mu}$ – dynamic magnification factor on displacement).

The following software products were used to define the characteristics of $\tilde{y}_{st}$, $\tilde{\mu}$ and $\tilde{y}_{dyn}$ stochastic properties: ANSYS software (modules Static Structural, Harmonic Response, Mechanical APDL, Design Exploration); VaP (non-commercial license).

1. Probabilistic properties of static displacement $\tilde{y}_{st} = \tilde{F}\tilde{l}^3 \big/ \left(48\tilde{E}\tilde{I}\right)$ were specified at the following characteristics of random variables: $\bar{F} = 20$ kN; $\hat{F} = 0{,}2$ kN; $\bar{E} = 2 \cdot 10^8$ kPa; $\hat{E} = 2 \cdot 10^6$ kPa, other parameters were accepted as deterministic ($l = 3$ m; $b = 0{,}05$ m; $h = 0{,}2$ m) by the reason of small stochastic variability [4]. The results of calculations with the use of different programs/modules, presented in Table 1, slightly differ.

Table 1. Results of calculations.

Parameter	VaP	ANSYS Mechanical APDL	ANSYS Static Structural
Mean (10^{-3} m)	1.68779	1.68784	1.71084
SD (10^{-3} m)	$2.39068 \cdot 10^{-2}$	$2.39658 \cdot 10^{-2}$	$2.42449 \cdot 10^{-2}$

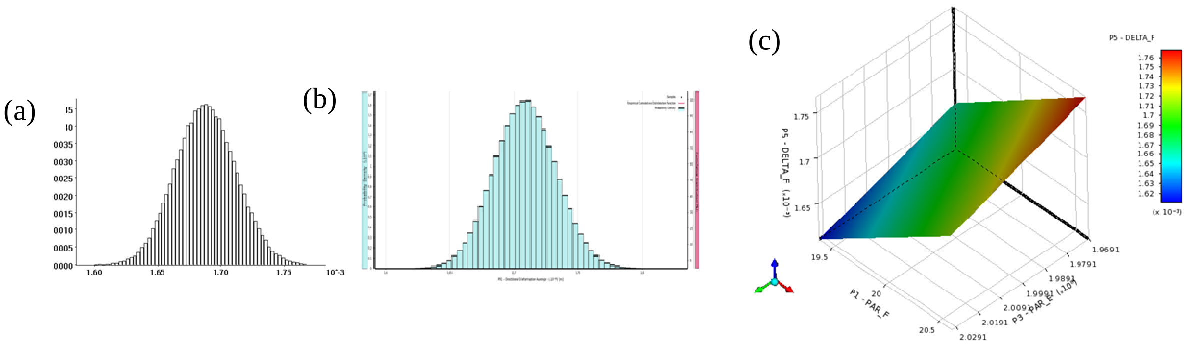

Figure 2. The histograms of static displacement received on VaP (a); on ANSYS Mechanical APDL (b); metamodel of displacement dependence on *F* and *E* (c).

Type of the density distribution function graph (Figure 2 (a), (b)) corresponds to the normal distribution law. It should be noted that time spent on problem solving by the formula for $\tilde{y}_{st}$ using ANSYS software is by an order of magnitude more than with program VaP when the Mechanical APDL module is used as a solver, and when solving with the FEM (Static Structural module) is used it is several orders of magnitude greater. This is connected with specific feature of the probabilistic ANSYS module implementation in option with approximations. It’s necessary to note that the metamodel (Figure 1(c)) is weakly nonlinear, that is why it’s well coordinated with the analytical decision.

As a result of computing experiments it is revealed that in case of increase in number of the varied input parameters considerably bigger (many times) number of design points is necessary to be set for creation of qualitative metamodel, if dependence of output parameter from entrance poorly nonlinear, and much more with an everyone new considered parameter at strongly nonlinear dependence. The quantity of design points for each task has to be evaluated by individually consecutive refinements. As the number of the considered varied parameters strongly influences on time expenditure, it is necessary to evaluate the importance of each of them when modeling.

2. Dynamic magnification factor (DMF) on mass displacement for system with one degree of freedom taking into account damping is:

$$\tilde{\mu} = \left[\left(1 - \tilde{k}_{\omega F}^2\right)^2 + \left(\gamma \tilde{k}_{\omega F}\right)^2 \right]^{-1/2}, \tag{3}$$

where $\tilde{k}_{\omega F} = \tilde{\omega}_F / \tilde{\omega}_0$; $\tilde{\omega}_F$ – frequency of external excitation; $\tilde{\omega}_0$ – natural frequency of system; γ – coefficient of inelastic resistance.

The special aspects of changes of the probability density function and probabilistic characteristics of DMF on operating frequency were described in details in work [6]. The data obtained with the use of ANSYS software are well agreed with the analytical decision.

3. Vertical dynamic displacement of mass (Figure 1 (a)) is defined on expression (with damping)

$$\tilde{y}_{dyn} = \tilde{y}_{st}\tilde{\mu} = \tilde{F}\tilde{l}^3 \cdot \left[48\tilde{E}\tilde{I}\sqrt{\left(1 - \tilde{k}_{\omega F}^2\right)^2 + \left(\gamma \cdot \tilde{k}_{\omega F}^2\right)^2} \right]^{-1}. \tag{4}$$

The following varied parameters have been used in calculations of displacement $\tilde{y}_{dyn}$ probabilistic characteristics: $\tilde{F}$, $\tilde{E}$, $\tilde{k}_{\omega F}$ ($\bar{k}_{\omega F} = 0{,}97$; $\hat{k}_{\omega F} = 0{,}01371$).

Results of calculations are presented in table 2 and on Figure 3 at various γ values. Figure 3 (a–c) show metasurfaces $\tilde{y}_{dyn}\left(\gamma, F, k_\omega\right)$. At small γ (0,005; 0,01) bigger quantity of design points are required for receiving high-quality approximations because of strong nonlinearity dependence

$\tilde{y}_{dyn}(k_\omega)$ in confidential interval of values k_ω. In Figure 3 (d – f) the histograms $\tilde{y}_{dyn}$ are shown at various γ (in different scales). The results obtained with using of listed above software slightly differ. It is should be noted that distribution densities according to histograms in Figure 3 (d – f), are significantly Different to the normal ones.

Table 2. Results of calculations.

Program	γ	Mean (10^{-3} m)	SD (10^{-3} m)
VaP / ANSYS Mechanical APDL	0.1	14.7064 / 14.7129	1.46658 / 1.4666
	0.01	36.5868 / 36.6435	26.2439 / 26.3189
	0.005	40.4086 / 40.4580	41.3332 / 41.3733

If expression of displacement without damping

$$\tilde{y}^0_{dyn} = \tilde{y}_{st}\tilde{\mu}^0 = \tilde{F} l^3 \cdot \left[48\tilde{E}\tilde{I}\sqrt{\left(1-\tilde{k}^2_{\omega F}\right)^2}\right]^{-1}, \quad (5)$$

is used as mathematical model then the metamodels obtained in the ANSYS software quantitatively and qualitatively strongly differ from (5). This result is due to the fact that function $\tilde{y}^0_{dyn}\left(\tilde{k}_{\omega F}\right)$ has discontinuity with resonance frequency ($k_{\omega F}=1$), and mathematical tools for construction of metamodels for such functions are absent in used software. But in the areas remote from gap the obtaining of qualitative good results is possible.

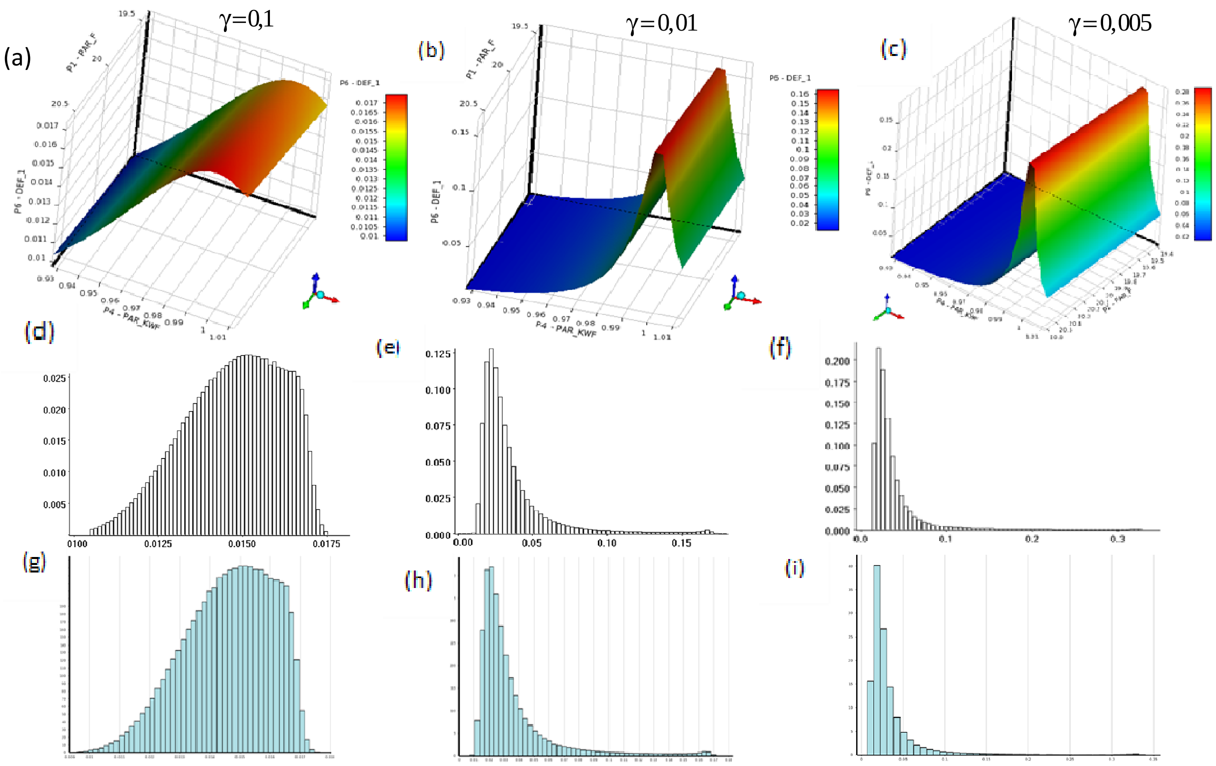

Figure 3. Graph of metasurfaces $\tilde{y}_{dyn}(\gamma, F, k_\omega)$ (a – c); histograms of displacement $\tilde{y}_{dyn}$ on VaP (d – f) and ANSYS Mechanical APDL (g – i) at various γ (in different scales).

Example B. A specific feature of a system with a DVD (Figure 4 (a)) is the presence of a resonant frequency near the tuning frequency (working frequency of the load) that is less than the working frequency (Figure 4 (b)). Depending on design features of system with damper and probabilistic properties of input parameters, the effective work zone of DVD can be placed in area of strong nonlinearity. Previous studies [4] have shown that the greatest impact on the reliability of the system is caused by variations in the parameters of the damper (mass $\tilde{m}_d$ and stiffness $\tilde{c}_d$) and the frequency of external excitation $\tilde{\omega}_F$, so taking into account stochastic properties of these parameters is necessary. The Beam188 (beam element), Mass21 (mass of system and damper) and COMBIN14 (deforming element of damper) were used when modeling system (Figure 4, a). Calculation was executed by means of the Harmonic Response module of the ANSYS software with the following input data: $\bar{m}_d = 0{,}1$ t; $\hat{m}_d = 0{,}0005$ t; $\bar{c}_d = 758{,}47\ \mathrm{kN \cdot m^2}$; $\hat{c}_d = 37{,}92\,\mathrm{kN \cdot m^2}$; $\bar{\omega}_F = 13{,}861$ Hz ($\overline{k}_{\omega F} = 0{,}8$ from mean of the natural frequency of the unprotected system); $\hat{\omega}_F = 0{,}1386$ Hz. Parameters: $F = 20$ kN; $E = 2 \cdot 10^8$ kPa; l = 3 m; b = 0,005 m; h = 0,2 m; m = 1 t; frequency of damper setup $\omega_F^* = 13{,}861$ Hz and parameter of damping in damper's elastic element $C_{V1} = 200\ \mathrm{N \cdot s/m}$ were accepted determined.

Required output parameters are the dynamic displacements of the system's mass and the damper's mass which dependences on the load frequency received by calculation in the Harmonic Response module with the above noted input parameters are shown in Figure 4.

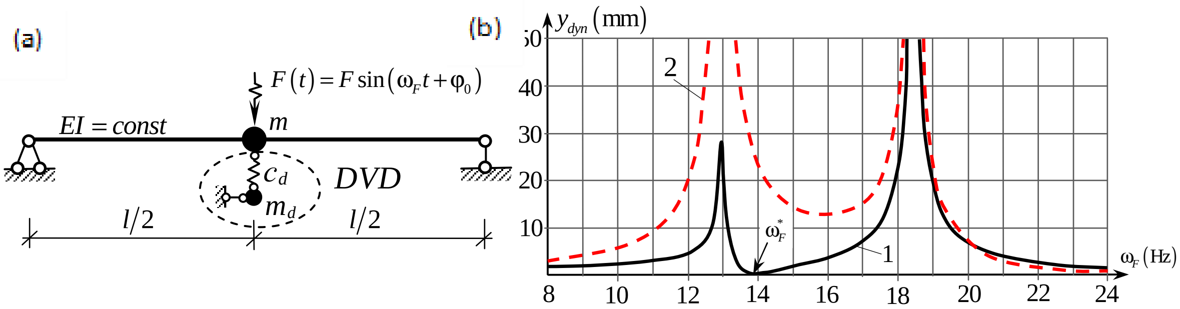

Figure 4. Design model of system with damper (a); the diagrams of mass's displacements (mm) – mass of system (1) and damper (2) in dependence from the frequency ω_F (Hz) (b).

The histograms of the masses displacements (system and damper) which found by using the Design Explorer software module are shown in Figure 5 (a), (b). It's seen that the distribution is different from the normal density function for the displacement of system's mass (Figure 5 (a)) and practically coincides with the normal distribution for the displacement of damper's mass (Figure 5 (b)). Figures 5 (c), (d) shows obtained with using ANSYS software metasurfaces for displacement of system's mass well illustrate effect of vibration damping by means of DVD. Results of calculations are presented in Table 3.

Table 3. Results of calculations.

Parameter	Mean (10^{-3} m)	SD (10^{-3} m)
Y_1	$1.363775 \cdot 10^{-3}$	$7.633240 \cdot 10^{-4}$
Y_2	–26.369	0.13282

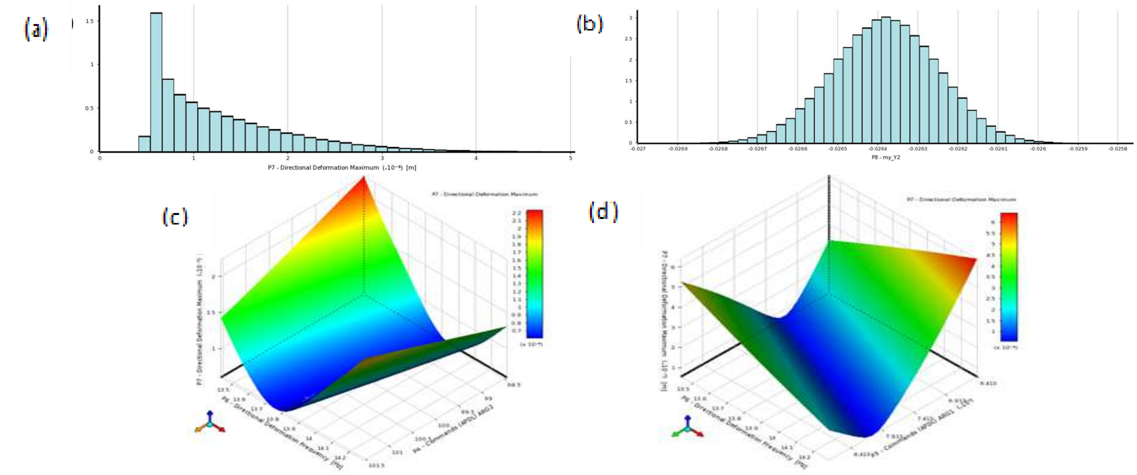

Figure 5. Histograms of displacements – system’s mass (a) and mass of damper (b); metasurfaces of system’s mass displacements dependences – on frequency and the damper’s mass (c); on frequency and damper stiffness (d).

Note that if we include F, E, b, h, C_{v1} in the group of stochastically varied quantities, repeatedly large volumes of calculations are required for the acceptable accuracy of metamodels creation. If attenuation in the system and damper is not considered, a satisfactory solution to the problem of determining the probabilistic characteristics of the output parameters can be obtained in the ANSYS software only at a distance from the resonant frequencies.

2.3. Determination of reliability

The calculation of reliability uses the technique based on the definitions of resistance $\tilde{R}$, load effect $\tilde{Q}$ and reserve of the generalized serviceability $\tilde{S} = \tilde{R} - \tilde{Q}$ [8]. In case of complex system it’s recommended to form the serviceability criteria set according to the approach described in [9].

In the general case, the calculations of the reliability P_S or probability of failure P_f based on specified criteria,-require to integrate the actually obtained function of the reserve capacity density:

$$P_f = \int_{-\infty}^{0} p_s(S)\,dS \,. \tag{6}$$

For example, if limitations of displacements of masses (system and damper) are used as a criteria of non-failure operation in form of $Y_1 < [Y_1]$ and $Y_2 < [Y_2]$ in dynamic analysis of system with DVD (Figure 4, a), then

$$\tilde{S}_1 = [Y_1] - \tilde{Y}_1 \text{ и } \tilde{S}_2 = [Y_2] - \tilde{Y}_2 , \tag{7}$$

where $[Y_1]$ and $[Y_2]$ – targeted allowed values of masses displacements (system and damper respectively).

Calculations on (6) with the use of specified descriptions (see above) for density function of masses displacements have given the following values for probabilities of failure: $P_{f,Y_1} = 0{,}0027834$, $P_{f,Y_2} = 6{,}118 \cdot 10^{-5}$ at $[Y_1] = 4{,}5 \cdot 10^{6}$ m and $[Y_2] = 0{,}0269$ m respectively.

If we use assumption of a normal distribution $\tilde{S}$ while calculating P_f and apply the Laplace function, it turns out that $P_{f,Y_1} = 2{,}0 \cdot 10^{-5}$, $P_{f,Y_2} = 3{,}16 \cdot 10^{-5}$ (the errors of 99.29% and 48.40%, respectively), that is practically unacceptable.

3. Conclusion

1. Determination of probabilistic properties of state's parameters for mechanical systems in the software ANSYS with use of approximating metamodels leads to qualitatively and quantitatively correct results for weakly nonlinear functions.

2. For systems with parameters of state characterized by strongly nonlinear dependence on the input varied parameters, generating of qualitative metamodels demands large number of design points; at relatively small amount of the varied parameters (up to 4 – 6) the number of necessary design points can be comparable to direct calculations for the Monte-Carlo method without using metamodels.

3. To obtain acceptable evaluations of reliability (or failure probability), sufficiently accurate descriptions of the probabilistic characteristics of the calculated parameters should be used, wherefore software with the appropriate capabilities has to be used.

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