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Stress-strain state in constructions corner areas

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Abstract. The topical problem of investigation, development of numerical and analytical methods to research constructions, buildings and structures is stress-strain analysis of building structures with intricate shape of the boundary. Geometrically nonlinear shape of boundaries (notches, crosscuts) determines the occurrence of stress concentration zones, deformations with significant enormities and gradients. Theoretical analysis of stress-strain state (SSS) for angled cut-out zones of area boundaries under the action of ruptural forced deformations resolves itself to study the singular solutions of uniform problem of elasticity theory with degree type features. The novelty of the research in the present work is determined by the fact that SSS near the vertex of angled cut-out zones of area boundary is characterized by limit strains, similar to the stress-intensity factors KI, KII, KIII when applying force criteria in mechanics of damage. Two-dimensional Betti formula is used to determine the intensity factors as limit strains, for the area constrained by the contour-circle of small radius $r = \varepsilon$. The independence of the Betti integral from the integration path is taken into account, which allows us to consider the contour integral along the length of arc close to the circle $r = \varepsilon$. The limit strains obtained in angled cut-out zones of area boundary are analyzed depending upon the area cut apex angle and the eigenvalues of elastic problem for the case of boundary conditions homogeneous for stresses.

1. Introduction

Stress-strain state (SSS) of composite conduits in the corner areas of elements interface under the action of forced deformations, ruptural in the line (surface) of elements contact, characterized by the peculiarities of stresses and deformations. The purpose of this work is to determine the SSS in the area of arbitrary opening angled cut-out of area boundary having used stress intensity factors as limit strains.

The elastic problem near non-regular points on area boundary singular line resolves itself into the two plain tasks [1–4]: planar deformation and out-of-plane deformation or out-of-plane shear.

The peculiarity of the SSS due to the geometry of shape of boundaries (notches, crosscuts) is determined by the singularity of the solution, the order of which depends on the eigenvalues of the homogeneous elastic boundary value problem [1, 4–6]. The eigenvalues of the homogeneous boundary value problem depend on the shape of the boundary, the type of boundary conditions, and the mechanical characteristics of the material in the area, and have range space [7, 8]. Let us consider in more detail the solution of the elastic problem of planar deformation in the area of the vertex of planar domain angled cut-out.

2. Materials and Methods

2.1. Setting and Solution to Problem

To obtain asymptotic solution near non-regular point at vertex of angled cut-out, a solution of homogeneous boundary value problem for wedge is considered. Planar wedge area with opening $2\alpha \in [\pi, 2\pi]$ consists of two symmetric areas: Ω_1, $\theta \in [0,\alpha]$ and Ω_2, $\theta \in [-\alpha, 0]$, one of which has forced cooling strains $\varepsilon_{ij} = \alpha T \delta_{ij}$. On the boundary of the contact areas $L = \Omega_1 \cap \Omega_2$ there is a rupture (jump) of deformations $\Delta\varepsilon_{ij} = \alpha T \delta_{ij}$, going out to vertex of angled cut-out at area boundary. The forces concentrated are not considered.

We introduce the polar system with the polar pole O (0.0) in vertex of angled cut-out at boundary and with the polar axis along wedge symmetry axis. Solution of a homogeneous elastic problem in movements for wedge infinite domain near corner at boundary is vertex of arbitrary opening angled cut-out is sought in the form of a product of two functions [1, 4, 6]:

$$u_r = r^{\lambda} g(\theta)\,,\ u_\theta = r^{\lambda} f(\theta)\,, \tag{1}$$

where λ is unknown parameter, $f(\theta)$, $g(\theta)$ is unknown functions of angle θ, to be determined. Applying (1) to the Lame equations, we obtain SSS depending on four arbitrary constants A, B, C, D, to be determined:

$$\begin{gathered}
\mathrm{r}^{-\lambda}\,\mathrm{u}_r = \mathrm{A}\cos\left[(1+\lambda)\theta\right] + B\sin\left[(1+\lambda)\theta\right] + C\cos\left[(1-\lambda)\theta\right] + D\sin\left[(1-\lambda)\theta\right], \\
\mathrm{r}^{-\lambda}\,\mathrm{u}_\theta = B\cos\left[(1+\lambda)\theta\right] - A\sin\left[(1+\lambda)\theta\right] + \nu_2\, D\cos\left[(1-\lambda)\theta\right] - \nu_2\, C\sin\left[(1-\lambda)\theta\right], \\
\mu^{-1}\mathrm{r}^{1-\lambda}\,\sigma_\theta = -2\lambda A\cos\left[(1+\lambda)\theta\right] - 2\lambda B\sin\left[(1+\lambda)\theta\right] - \\
-(1+\lambda)(1-\nu_2)\,C\cos\left[(1-\lambda)\theta\right] - (1+\lambda)(1-\nu_2)\,D\sin\left[(1-\lambda)\theta\right], \\
\mu^{-1}\mathrm{r}^{1-\lambda}\,\tau_{r\theta} = -2\lambda A\sin\left[(1+\lambda)\theta\right] + 2\lambda B\cos\left[(1+\lambda)\theta\right] - \\
-(1-\lambda)(1-\nu_2)\,C\sin\left[(1-\lambda)\theta\right] + (1-\lambda)(1-\nu_2)\,D\cos\left[(1-\lambda)\theta\right], \\
\mu^{-1}\mathrm{r}^{1-\lambda}\sigma_r = 2\lambda\{(A\cos\left[(1+\lambda)\theta\right] + B\sin\left[(1+\lambda)\theta\right] + \frac{3-\lambda}{k-\lambda}C\cos\left[(1-\lambda)\theta\right] + \frac{3-\lambda}{k-\lambda}\mathrm{D}\sin\left[(1-\lambda)\theta\right]\},
\end{gathered} \tag{2}$$

where $\mu = \dfrac{\mathrm{E}}{2(1+\nu)}$, E, ν – modulus of volume elasticity, the Poisson ratio of area material respectively, $\nu_2 = \dfrac{3+\lambda-4\nu}{3-\lambda-4\nu}$, $1-\nu_2 = \dfrac{-2\lambda}{k-\lambda}$, $k = 3-4\nu$, λ are the eigenvalues of the homogeneous boundary value problem.

Satisfying homogeneous boundary conditions: $\sigma_\theta = \tau_{r\theta} = 0$ at $\theta = \pm\alpha$, two homogeneous systems of equations are obtained:

$$\begin{cases} 2\lambda \mathrm{A}\cos[(1+\lambda)\alpha] + (1+\lambda)(1-\nu_2)C\cos[(1-\lambda)\alpha] = 0, \\ 2\lambda \mathrm{A}\sin[(1+\lambda)\alpha] + (1-\lambda)(1-\nu_2)C\sin[(1-\lambda)\alpha] = 0, \end{cases} \tag{3a}$$

$$\begin{cases} 2\lambda B\sin[(1+\lambda)\alpha] + (1+\lambda)(1-\nu_2)D\sin[(1-\lambda)\alpha] = 0, \\ 2\lambda \mathrm{B}\cos[(1+\lambda)\alpha] + (1-\lambda)(1-\nu_2)D\cos[(1-\lambda)\alpha] = 0, \end{cases} \tag{3b}$$

In order that homogeneous boundary value problem has non-zero solutions; it is necessary and sufficient that the determinants of each of the systems (3a) and (3b) are zero. The determinant of (3a):

$$\lambda\sin 2\alpha + \sin 2\lambda\alpha = 0 \ \text{ or } \ \sin 2\lambda\alpha = -\lambda\sin 2\alpha\,. \tag{4}$$

For λ, which is the root of this equation, for SSS (2), we must keep the summands with coefficients A and C (B = D = 0). Let us indicate λ_i^- root system of equation (4).The second determinant of system of linear equations (3b):

$$-\lambda \sin 2\alpha + \sin 2\lambda\alpha = 0 \text{ or } \sin 2\lambda\alpha = \lambda \sin 2\alpha . \tag{5}$$

For λ, which is the root of this equation (5), for SSS (2), we must keep the summands with coefficients B and D (A = C = 0). Let us indicate λ_i^+ root system of equation (5).

Characteristic equations (4), (5) have an infinite aggregate of eigenvalues λ_i^+, λ_i^-. The values $\lambda_i > 1$ result in unlimited stresses at infinity. The values $\lambda < 0$ result in unlimited displacements at vertex of angled cut-out in the absence of loads on the boundary of area cut. According to the physical meaning of value task $\lambda_i \in [0,1]$. Among many roots of (4), (5) to build asymptotic solutions to (2) the following is selected $\lambda^+ = \min \mathrm{Re}\, \lambda_i^+$, $\lambda^- = \min \mathrm{Re}\, \lambda_i^-$, because subsequent large values λ_i result in unlimited increase in deformation energy.

Taking into account the boundary conditions (3a), (3b), the relations for the coefficients *A* and *C*, *B* and *D* are obtained:

$$A = \frac{(1-\lambda^-)\sin\left[(1-\lambda^-)\alpha\right]}{\left(k-\lambda^-\right)\sin\left[(1+\lambda^-)\alpha\right]} C \cdot \tag{6}$$

$$B = \frac{(1+\lambda^+)\sin\left[(1-\lambda^+)\alpha\right]}{\left(k-\lambda^+\right)\sin\left[(1+\lambda^+)\alpha\right]} D \cdot \tag{7}$$

We indicate limit strains near non-irregular point of the area boundary as:

$$\mathrm{K}_I = \lim_{r\to 0} r^{1-\lambda^-} \sigma_{\theta,\,\theta=0} \cdot \tag{8}$$

$$\mathrm{K}_{II} = \lim_{r\to 0} r^{1-\lambda^+} \tau_{r\theta,\,\theta=0} \cdot \tag{9}$$

Taking into account designations (8), (9) and the relations (6), (7) the coefficients A and B, C and D are expressed through limit strains in the form of:

$$A = \frac{\left(1-\lambda^-\right)\sin\left[\left(1-\lambda^-\right)\alpha\right]}{2\lambda^-\mu\left[\left(\lambda^- - 1\right)\sin\left[\left(1-\lambda^-\right)\alpha\right] + \left(\lambda^- + 1\right)\sin\left[\left(1+\lambda\ \right)\alpha\right]\right]} K_I , \tag{10a}$$

$$C = \frac{(\text{k-}\lambda^-)\sin\left[\left(1+\lambda^-\right)\alpha\right]}{2\lambda^-\mu\left[\left(\lambda^- - 1\right)\sin\left[\left(1-\lambda^-\right)\alpha\right] + \left(\lambda^- + 1\right)\sin\left[\left(1+\lambda^-\right)\alpha\right]\right]} K_I , \tag{10b}$$

$$\mathrm{B} = \frac{\left(1+\lambda^+\right)\sin\left[\left(1-\lambda^+\right)\alpha\right]}{2\lambda^+\mu\left[\left(\lambda^+ + 1\right)\sin\left[\left(1-\lambda^+\right)\alpha\right] - \left(1-\lambda^+\right)\sin\left[\left(1+\lambda^+\right)\alpha\right]\right]} K_{II} , \tag{10c}$$

$$D = \frac{(\text{k-}\lambda^+)\sin\left[\left(1+\lambda^+\right)\alpha\right]}{2\lambda^+\mu\left[\left(\lambda^+ + 1\right)\sin\left[\left(1-\lambda^+\right)\alpha\right] - \left(1-\lambda^+\right)\sin\left[\left(1+\lambda^+\right)\alpha\right]\right]} K_{II} . \tag{10d}$$

2.2. Stress-strain state near vertex of area boundary angled cut-out

Taking into account the coefficients (10), the solution of the homogeneous boundary value problem (2) near vertex of area boundary angled cut-out is written as:

$$\mathrm{u}_r = \mathrm{r}^{\lambda^-}\{\mathrm{A}\cos\left[\left(1+\lambda^-\right)\theta\right] + C\cos\left[\left(1-\lambda^-\right)\theta\right]\} + \mathrm{r}^{\lambda^+}\{B\sin\left[\left(1+\lambda^+\right)\theta\right] + D\sin\left[\left(1-\lambda^+\right)\theta\right],$$

$$\mathrm{u}_\theta = \mathrm{r}^{\lambda^-}\{-A\sin\left[\left(1+\lambda^-\right)\theta\right] - \nu_2^- \, C\sin\left[\left(1-\lambda^-\right)\theta\right]\} + \mathrm{r}^{\lambda^+}\{B\cos\left[\left(1+\lambda^+\right)\theta\right] + \nu_2^+ \, D\cos\left[\left(1-\lambda^+\right)\theta\right]\},$$

$$\begin{aligned}\sigma_\theta &= \mu r^{\lambda^- -1}\{-2\lambda^- A\cos\left[\left(1+\lambda^-\right)\theta\right]-\left(1+\lambda^-\right)\left(1-\nu_2^-\right)C\cos\left[\left(1-\lambda^-\right)\theta\right]\}+\\ &+\mu r^{\lambda^+ -1}\{-2\lambda^+ B\sin\left[\left(1+\lambda^+\right)\theta\right]-\left(1+\lambda^+\right)\left(1-\nu_2^+\right)D\sin\left[\left(1-\lambda^+\right)\theta\right]\},\\ \tau_{r\theta} &= \mu r^{\lambda^- -1}\{-2\lambda^- A\sin\left[\left(1+\lambda^-\right)\theta\right]-\left(1-\lambda^-\right)\left(1-\nu_2^-\right)C\sin\left[\left(1-\lambda^-\right)\theta\right]+\\ &+\mu r^{\lambda^+ -1}\{2\lambda^+ B\cos\left[\left(1+\lambda^+\right)\theta\right]+\left(1-\lambda^+\right)\left(1-\nu_2^+\right)D\cos\left[\left(1-\lambda^+\right)\theta\right],\\ \sigma_r &= 2\mu\lambda r^{\lambda^- -1}\{(A\cos\left[\left(1+\lambda^-\right)\theta\right]+\frac{3-\lambda^-}{k-\lambda^-}C\cos\left[\left(1-\lambda^-\right)\theta\right]\}+\\ &+2\mu\lambda r^{\lambda^+ -1}\{B\sin\left[\left(1+\lambda^+\right)\theta\right]+\frac{3-\lambda^+}{k-\lambda^+}D\sin\left[\left(1-\lambda^+\right)\theta\right]\},\end{aligned} \tag{11}$$

where the coefficients *A, B, C, D* satisfy relations (10).

Taking into account the aggregate of eigenvalues λ_i of solutions to equations (4), (5) the solution of the elastic problem in angled cut-out zones of planar domain boundaries has the form:

$$\begin{aligned}u_r &= \sum_{i,\lambda_i^-} r^{\lambda_i^-}\{A_i\cos\left[\left(1+\lambda_i^-\right)\theta\right]+C_i\cos\left[\left(1-\lambda_i^-\right)\theta\right]\}+\\ &+\sum_{i,\lambda_i^+} r^{\lambda_i^+}\{B_i\sin\left[\left(1+\lambda_i^+\right)\theta\right]+D_i\sin\left[\left(1-\lambda_i^+\right)\theta\right]+u_r^s,\\ u_\theta &= \sum_{i,\lambda_i^-} r^{\lambda_i^-}\{-A_i\sin\left[\left(1+\lambda_i^-\right)\theta\right]-\nu_2^- C_i\sin\left[\left(1-\lambda_i^-\right)\theta\right]\}+\\ &+\sum_{i,\lambda_i^+} r^{\lambda_i^+}\{B_i\cos\left[\left(1+\lambda_i^+\right)\theta\right]+\nu_2^+ D_i\cos\left[\left(1-\lambda_i^+\right)\theta\right]\}+u_\theta^s,\\ \sigma_\theta &= \mu\sum_{i,\lambda_i^-} r^{\lambda^- -1}\{-2\lambda_i^- A_i\cos\left[\left(1+\lambda_i^-\right)\theta\right]-\left(1+\lambda_i^-\right)\left(1-\nu_2^-\right)C_i\cos\left[\left(1-\lambda_i^-\right)\theta\right]\}+\\ &+\mu\sum_{i,\lambda_i^+} r^{\lambda^+ -1}\{-2\lambda_i^+ B_i\sin\left[\left(1+\lambda_i^+\right)\theta\right]-\left(1+\lambda_i^+\right)\left(1-\nu_2^+\right)D\sin\left[\left(1-\lambda_i^+\right)\theta\right]\}+\sigma_\theta^s\\ \tau_{r\theta} &= \mu\sum_{i,\lambda_i^-} r^{\lambda^- -1}\{-2\lambda_i^- A_i\sin\left[\left(1+\lambda_i^-\right)\theta\right]-\left(1-\lambda_i^-\right)\left(1-\nu_2^-\right)C_i\sin\left[\left(1-\lambda_i^-\right)\theta\right]\\ &+\mu\sum_{i,\lambda_i^+} r^{\lambda_i^+ -1}\{2\lambda_i^+ B_i\cos\left[\left(1+\lambda_i^+\right)\theta\right]+\left(1-\lambda_i^+\right)\left(1-\nu_2^+\right)D_i\cos\left[\left(1-\lambda_i^+\right)\theta\right]+\tau_{r\theta}^s,\\ \sigma_r &= 2\mu\sum_{i,\lambda_i^-}\lambda_i^- r^{\lambda_i^- -1}\{(A_i\cos\left[\left(1+\lambda_i^-\right)\theta\right]+\frac{3-\lambda_i^-}{k-\lambda_i^-}C_i\cos\left[\left(1-\lambda_i^-\right)\theta\right]\}+\\ &+2\mu\sum_{i,\lambda_i^+}\lambda_i^+ r^{\lambda_i^+ -1}\{B_i\sin\left[\left(1+\lambda_i^+\right)\theta\right]+\frac{3-\lambda_i^+}{k-\lambda_i^+}D_i\sin\left[\left(1-\lambda_i^+\right)\theta\right]\}+\sigma_r^s,\end{aligned} \tag{12}$$

where $\boldsymbol{u}_i^s$, σ_{ij}^s - displacements and stresses due to the action of the required loads or the general field of displacements and stresses - the set forced cooling strains having discontinuity along the axis of wedge symmetry $\theta=0$ of the type: $\Delta\varepsilon_{ij}=\alpha T\delta_{ij}$.

Coefficients *A, B, C, D* according to (10) are expressed in terms of limit strains K_I, K_{II}; therefore, the stress-strain state near non-regular point at area boundary depends on the parameters K_I, K_{II}, and is written as:

$$u_i = \mu^{-1}K_I\, r^{\lambda^-} f_{i\lambda^-}(\theta)+\mu^{-1}K_{II}\, r^{\lambda^+} f_{i,\lambda^+}(\theta)+\mu^{-1}\sum_i[r^{\lambda_i^-}C_i f_{i,\lambda_i^-}(\theta)+r^{\lambda_i^+}C_i f_{i,\lambda_i^+}(\theta)]+u_i^s, \tag{13a}$$

$$\sigma_{ij} = \mathrm{K}_I\, r^{\lambda^-} f_{ij,\lambda^-}(\theta) + \mathrm{K}_{II}\, r^{\lambda^+} f_{ij,\lambda^+}(\theta) + \mu \sum_{i,j} [r^{\lambda_i^-} C_i f_{ij,\lambda_i^-}(\theta) + r^{\lambda_i^+} C_i f_{ij,\lambda_i^+}(\theta)] + \sigma_{ij}^s , \qquad (13b)$$

where $\lambda^- = \min \mathrm{Re}\, \lambda_i^-$, $\lambda^+ = \min \mathrm{Re}\, \lambda_i^+$ are the solutions of characteristic equations (4), (5), respectively; $f_{i,\lambda^\pm}(\theta), f_{ij,\lambda^\pm}(\theta)$ are the angle functions $\theta \in [-\alpha, \alpha]$, $u_i = (u_r, u_\theta)$ are the displacements, $\sigma_{ij} = (\sigma_\theta, \tau_{r\theta}, \sigma_r)$ are the stresses in which the main expansion term is allocated recorded by the intensity factors $\mathrm{K_I}$, $\mathrm{K_{II}}$; u_i^s, σ_{ij}^s are the displacements and stresses due to action of set forced cooling strains.

2.3. The definition of limit strains $\mathrm{K_I}$, $\mathrm{K_{II}}$ *planar area boundary angled cut-out*

Taking into account SSS of the form (13), the principal member of the asymptotic solution of the problem near non-regular point at the area boundary takes the form:

$$u_i = \mu^{-1} \mathrm{K}_I\, r^{\lambda^-} f_{i\lambda^-}(\theta) + \mu^{-1} \mathrm{K}_{II}\, r^{\lambda^+} f_{i,\lambda^+}(\theta) + o(r^{\lambda}) , \qquad (14a)$$

$$\sigma_{ij} = \mathrm{K}_I\, r^{-1+\lambda^-} f_{ij,\lambda^-}(\theta) + \mathrm{K}_{II}\, r^{-1+\lambda^+} f_{ij,\lambda^+}(\theta) + o(r^{-1+\lambda}) , \qquad (14b)$$

Where $\lambda = \min(\lambda^-, \lambda^+)$. Near non-regular point O (0.0) of area boundary the order of singular behavior for the stresses is $(\lambda - 1)$, $\lambda \in [0.5; 1]$, $\lim_{r\to 0} r^{\lambda-1} = +\infty$, in addition average stress (14b) in a small neighborhood of this point is finite. This follows from the convergence of the improper integral:

$$\iint_{\Omega_\varepsilon} \mathrm{K}_I\, r^{-1+\lambda} f_{ij,\lambda}(\theta) = K_I \lim_{n\to\infty} \iint_{C_\varepsilon \setminus U_n} f_{ij,\lambda}(\theta) \frac{r dr d\theta}{r^{1-\lambda}} = K_I \lim_{n\to\infty} \int_{\rho_n}^{1} \int_{-\alpha}^{\alpha} f_{ij,\lambda}(\theta) r^{\lambda} dr d\theta ,$$

where $\{\rho_n\}$ is a sequence of neighborhoods centered at the beginning of the polar system (not including the vertex of boundary angled cut-out) in the form of central sectors which radii are ρ_n, $\lim_{n\to\infty} \rho_n = 0$. For angle function $f_{ij,\lambda}(\theta) = 1$ the value of improper integral:

$$\frac{2\alpha}{1+\lambda} \lim_{n\to\infty} \rho^{\lambda+1} \Big|_{\rho_n}^{1} = \frac{2\alpha}{1+\lambda} K_I (K_{II}) .$$

Vertex area of boundary angled cut-out can include multiple subdomains: plastic deformation area where the finite deformations, the area of elastic (linear and nonlinear) deformations, for which, in the framework of the linear elastic problem, a singular SSS with a power singularity is written with the aid of limit strains in the form (14). If the area of material non-linear properties is sufficiently small, then there exists a neighborhood near non-regular point in which the expressions (14) define the stress distribution reasonably accurate confirmed by investigational studies of solutions in such an area [9-16]. For such an area of non-regular point of area boundary, the coefficients $\mathrm{K_I}$, $\mathrm{K_{II}}$ of the type (8), (9) of solution (14) are defined.

We consider the solution of the problem in a planar domain D^+ near non-regular point O (0.0) limited by contour C, which consists of arc of circle C_ε with radius ε, of straight portion MN and QP at boundary $\theta = \pm\alpha$, of arc C_L of other circle of arbitrary small radius $C_L > \varepsilon$, with the positive direction of area boundary girdle. Functions u_i, σ_{ij} are continuous and have continuous derivatives up to the second order inclusive in the area D^+. Let us write the second Green's formula for the area D^+:

$$\iint_{D^+} \left(\xi \Delta^* \eta - \eta \Delta^* \xi \right) d\Omega = \int_C \left(\xi T_N \eta - \eta T_N \xi \right) dl = \int_{C_\varepsilon} \left(\xi T_N \eta - \eta T_N \xi \right) dl + \int_{C_L} \left(\xi T_N \eta - \eta T_N \xi \right) dl . \qquad (15)$$

The contour integral over the segments MN and QP is equal to zero. Functions η, ξ are harmonic, they satisfy Lame equations, hence the integral (15) as a two-dimensional integral Betty is rewritten in the form of:

$$\int_{C_\varepsilon} (\xi T_N \eta - \eta T_N \xi)\, dl = -\int_{C_L} (\xi T_N \eta - \eta T_N \xi)\, dl\,, \tag{16}$$

where $\xi = (u_r, u_\theta)$, $T_N \eta = (\sigma_\theta, \tau_{r\theta})$ are the components of balanced stress-strain state in the area D^+ near the point O (0.0), satisfying homogeneous boundary conditions (3) and Lame homogeneous equations.

According to (16) for calculations K_I, K_{II} two balanced stress-strain states of a homogeneous boundary value problem of the form (14) are chosen, satisfying (3).

The first SSS has the form (14), the second SSS can be taken as a solution (14) for the parameter (-λ). The contour integral (16) for the arc C_ε, $r = \varepsilon$ will be written in a form of:

$$\int_{C_\varepsilon} (\tilde{\sigma}_r u_r - \tilde{\tau}_{r\theta} u_\theta - \sigma_r \tilde{u}_r + \tau_{r\theta} \tilde{u}_\theta) d\theta = \int_{-\alpha}^{\alpha} (\tilde{\sigma}_r u_r - \tilde{\tau}_{r\theta} u_\theta - \sigma_r \tilde{u}_r + \tau_{r\theta} \tilde{u}_\theta) r d\theta, ... \tag{17}$$

where $u_r, u_\theta, \sigma_r, \tau_{r\theta}$ is SSS of a type (14), $\tilde{u}_r, \tilde{u}_\theta, \tilde{\sigma}_r, \tilde{\tau}_{r\theta}$ is SSS obtained according to SSS of a type (14) by substitution λ for ($-\lambda$). Subintegral function at quadrature around $C_\varepsilon : r = \varepsilon$ depends on angle $\theta \in [-\alpha, \alpha]$ and computed explicitly $I_\varepsilon = C_I K_I + C_{II} K_{II}$, where C_I and C_{II} are unknown constants, $C_i \in R$.

Taking into account the independence of contour integral from paths of integration (16), for definition of coefficients K_I, K_{II} another paths of integration is considered C_L close to $C_\varepsilon : r = \varepsilon$. The contour integral along the arc, C_L will be written in a form of:

$$\int_{L_\varepsilon} (\tilde{\sigma}_r u_r - \tilde{\tau}_{r\theta} u_\theta - \sigma_r \tilde{u}_r + \tau_{r\theta} \tilde{u}_\theta) d\theta, \tag{18}$$

where u_r, u_θ, σ_r, $\tau_{r\theta}$ are the SSS of a type (11), written taking into account the homogeneous boundary conditions, i.e. relations (6), (7), $\tilde{u}_r, \tilde{u}_\theta, \tilde{\sigma}_r, \tilde{\tau}_{r\theta}$ are the SSS obtained according to SSS of a type (11) by substitution λ for ($-\lambda$). The integral of a type (18) depends on C_1, C_2. As a result of the integration on the second curve C_L expression is obtained: $C_1 I_1 + C_2 I_2$, where values I_1, I_2 are obtained by integrating (18). Equating the expression (17), (18), the coefficients are obtained K_I, K_{II}.

Examples of integrals (17), (18) calculations, values K_I, K_{II} for different angles of wedge opening and eigenvalues of homogeneous boundary value problem are given in Tables 1 - 3. Table 3 shows limit strains $K_I = \lim_{r\to 0} r^{1-\lambda} \sigma_{\theta,\theta=0}$, $K_{II} = \lim_{r\to 0} r^{1-\lambda^+} \tau_{r\theta,\theta=0}$ (intensity coefficients) in the vertex area of boundary angled cut-out, that are proportional to some constants, which can be selected the regulatory limit strains for designs or materials. According to the results obtained, the coefficients K_I, K_{II} as the angle of the wedge opening decreases, which corresponds to a decrease in the influence of the "singularity" of the form $r^{\lambda-1}$, while the change in coefficients occurs in different ways.

Table 1. Example of integral calculation (17) for different angles of wedge opening of boundary.

Eigenvalues (3a) $\min \mathrm{Re}\,\lambda^-$	Eigenvalues (3b) $\min \mathrm{Re}\,\lambda^+$	Angle of wedge opening, 2α	Integral (17)
0.5	0.5	360°	$2.35\, C_1\, K_1 + 3.93\, C_2\, K_2$
0.512	0.73	300°	$2.41\, C_1\, K_1 + 2.3\, C_2\, K_2$
0.563	0.98	260°	$2.81\, C_1\, K_1 + 1.48\, C_2\, K_2$

Table 2. Example of integral calculation (18) for different angles of wedge opening of boundary.

Eigenvalues (3a) $\min \mathrm{Re}\,\lambda^-$	Eigenvalues (3b) $\min \mathrm{Re}\,\lambda^+$	Corner of wedge opening, 2α	Integral (18)
0.5	0.5	360°	$2.36\, C_1 + 5.25\, C_2$
0.512	0.73	300°	$2.29\, C_1 + 2.96\, C_2$

Table 3. Intensity coefficients K_I,K_{II} in the vertex area of boundary angled cut-out.

Eigenvalues (3a) $\min \operatorname{Re} \lambda^-$	Eigenvalues (3b) $\min \operatorname{Re} \lambda^+$	Corner of wedge opening, 2α	The values of the coefficients K_I, K_{II}
0.5	0.5	360^o	$K_I = 1.004\,K_I^s,\ K_{II} = 1.34\,K_{II}^s$
0.512	0.73	300^o	$K_I = 0.95\,K_I^s,\ K_{II} = 1.28\,K_{II}^s$
0.563	0.98	260^o	$K_I = 0.79\,K_I^s,\ K_{II} = 0.45\,K_{II}^s$

3. Conclusions

In this paper, stress strain state near vertex of angled cut-out of planar domain is obtained. The principal member of the asymptotic of elastic problem solution near non-regular point at area boundary is given. Using the 2D Betti formula for a planar domain with angled cut-out at the boundary, limit strains are obtained that allow analyzing the influence of the change in the opening angle of angled cut-out at planar domain, the eigenvalues of the homogeneous boundary value elastic problem, on the limit strains at the vertex of angled cut-out of domain (the "singularity" of the solution). The proposed approach to the SSS study in the area of angled cut-out at planar domain boundary makes possible to determine and analyze the SSS in the area of arbitrary opening angled cut-out of area boundary by means of the stress intensity factors, as the limit strains.

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