

Non-Linear calculation and optimization of mast construction, strengthened by stretches

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Abstract. The analytical calculation technique of high-altitude mast structures, reinforced with cable stretches, was developed in conditions of longitudinal-transverse bending. A numerical algorithm for deformed scheme calculating based on the stepwise loading of the finite element mast model was proposed. Comparative calculations of the masts selected versions with the use of developed software modules and SCAD and ANSYS software complexes are carried out. Parametric optimization problems of constant mast structures and two-step stiffness, with fixed and variable position of stretch marks were formed and solved. On the basis of the developed two-level algorithm, an effective design of a two-stage mast with an optimal arrangement of stretch marks was obtained.

1. Introduction

The design scheme of mast structures deformation is adopted in the form of a rod model, it occurs, as a rule, under conditions of longitudinal-transverse bending. In this case, the main transverse load is wind load, and longitudinal loads include the equipment process weight and maintenance personnel, as well as the weight of the structure. If the through variant of the mast in a spatial rod model form is unacceptable for any reasons (first of all technological). Wind influence on the outer side surface of the mast is maximum one and measures are needed to enhance the mast resistance to loads. Such measures include a fixed jamming device in the lower section of the mast and cable stretch marks installation.

The most rational from the perception of the wind impact position is a round circular section of the mast. However, it is inconvenient from the placing technological equipment position, and a square circular mast cross-section is adopted in this study. The presence of cable stretch marks, which are unilateral ties, complicates the calculation of the mast in longitudinal-transverse bending, because it leads to the appearance of additional concentrated loads that depend not only on the transverse wind load, but also on the non-linear calculation results of the mast.

In connection with the foregoing, the development and software implementation task of the algorithm for non-linear calculation of a high-altitude mast with stretching, which would be convenient later in the formulation and solution of the corresponding optimization problems, seems topical.



2. Mast calculation scheme. Determination of dangerous wind direction and rational arrangement of stretch marks

The calculation scheme of the system under consideration is shown in Figure 1. The wind load quasistatic, including static and dynamic components [1], is conveniently represented as a trapezoid (Figure 1). The intensity of the wind pressure in the upper section of the mast is determined according to the relation:

$$q_v = q_0 \cdot k(z_e) \cdot c \quad (1)$$

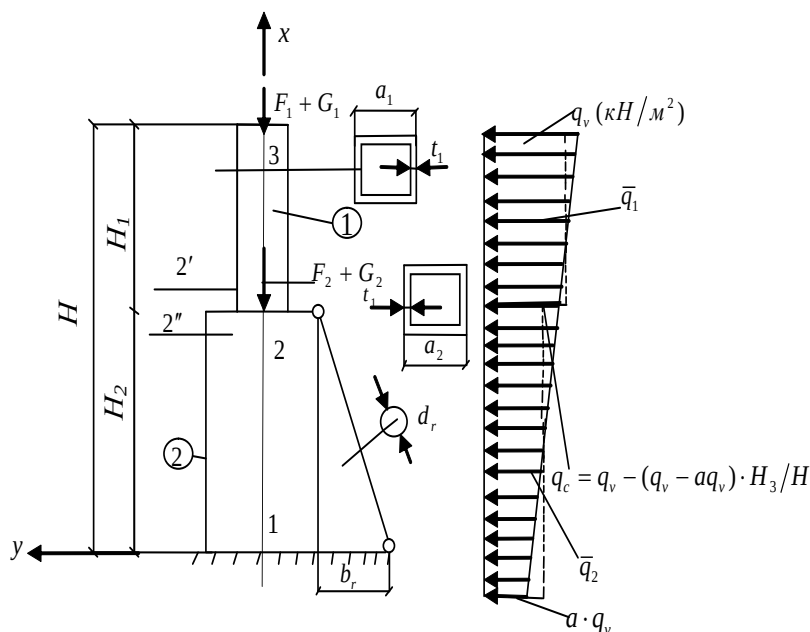


Figure 1. The calculation scheme of the system under consideration.

where q_0 – normative wind pressure, appointed by BCR 2.01.07-85* «Loads and impacts»; $c(z_e)$ – mast height factor; c – aerodynamic coefficient. Value $a < 1$ depends on the area of the wind pressure diagram, determined according to BCR.

Analysis of the mast stressed state under the action of only transverse loads shows that the most dangerous wind direction for a given square circular section of the mast is the direction along the diagonal. It is not difficult to show that the maximum normal stresses in c directions, d are respectively:

$$\sigma_{\max}^c = \frac{M_{c \max} \cdot a_c}{2 \cdot I}; \quad \sigma_{\max}^d = \frac{\sqrt{2} M_{c \max} \cdot \sqrt{2} a_c}{2 \cdot I} = 2 \sigma_{\max}^c. \quad (2)$$

Consequently, the wind direction along the diagonal is twice dangerous than the direction along the side of the section. Analysis of the four stretches work showed that the option of fixing them in the middle of the square annular section sides and the direction along the sides is more advantageous, because two stretches work in the dangerous direction of the wind. However, in general, it is more preferable to use the direction of stretch marks along for design diagonals, taking into account the convenience of stretch marks fastening, as well as the absence of local bending of mast walls.

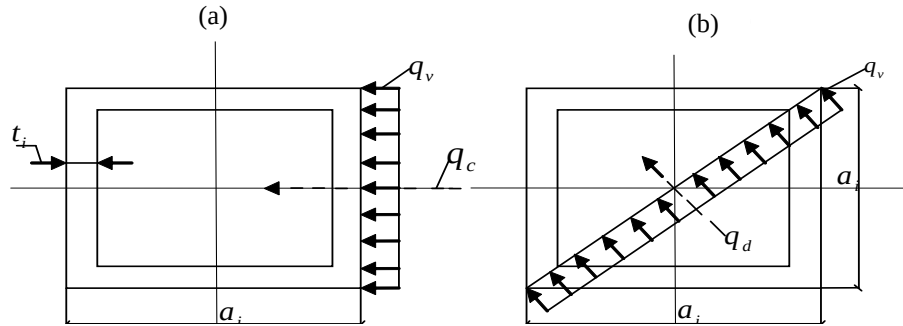


Figure 2. Directions of the wind.

3. Algorithms of non-linear mast calculation

Non-linearity of the dependencies between load parameters and of stress-strain state parameters is due, in this case, to the so-called "constructive non-linearity", or change in the design scheme during loading process. The calculation according to the deformed calculation scheme for longitudinal-transverse bending is widely represented in educational [1, 2, 3], normative [4, 5] and scientific [6, 7] sources. The algorithms proposed in this paper are presented below, and comparative analysis of the calculation results using various algorithms and software is performed.

3.1. Using the exact solution

The exact solution will be found on the basis of differential equation solution of the mast longitudinal-transverse bending in displacements [4]:

$$V^{IV}(x) = q(x)/EI_z(x) + N(x)/EI_z \cdot V''(x), \quad (3)$$

where $V(x)$ – function of sections transverse displacements in the integration region; $q(x)$, $N(x)$ – intensity functions of the transverse load and longitudinal forces in the integration region. It was more convenient to change the direction of the x , y axes (Figure 3). In addition, the following simplifications are introduced.

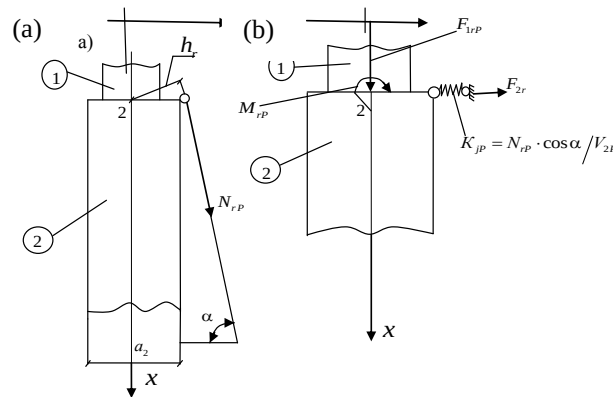


Figure 3. Effects of the tensile force on the mast.

1. Flexural stiffness of the sections (Figure 1) and wind load on the mast (Figure 1) are considered constant within the plots. The wind pressure intensity per unit length of the mast in sections 1 and 2 is equal to:

$$\begin{aligned} q_1 &= \bar{q}_1 \cdot \sqrt{2a_1} = (q_v + q_c)/2 \cdot \sqrt{2} \cdot a_1; \quad q_2 = \bar{q}_2 \cdot \sqrt{2a_2} = \\ &= (q_c + a \cdot q_v)/2 \cdot \sqrt{2} \cdot a_2 \end{aligned} \quad (4)$$

2. Effort effects N_{rp} in the mast banner is transmitted in the node 2 (Figure 3 (b)) in the form of an external bending moment $M_{rp} = N_{rp} \cdot h_r$, longitudinal compressive force $F_{lrp} = N_{rp} \cdot \sin \alpha$ and «girder» lateral force $F_{2r} = K_{jp} \cdot V_2$, in this case N_{rp} is the tensile force from lateral load. Stiffness K_{jp} of conditional elastic horizontal connection in node 2 (Figure 3 (b)) is based on the transverse wind load calculation.

3. Distributed longitudinal loads of self-weight are reduced to equivalent nodal loads G_1, G_2 (Figure 1).

The simplifications introduced made it possible to find solutions of equation (3) on sections 1, 2 by the initial parameters method.

3.2. Calculation according to the deformed calculation scheme by the finite element method

The prerequisites for linear calculation of the system using the finite element method assume the undeformed design scheme use. In this case, there are errors associated with the change in the position of the system nodes during deformation. To refine the linear calculation results of the mast constructions type under consideration, a procedure for stepwise loading with the change in the nodes coordinates of the finite element circuit at each step is proposed:

At each step of the calculation, the system's stiffness matrix is constructed $K^{(j)}$ current values of the nodes coordinates with a linear calculation using the specified load F is performed:

$$K^{(j)} = \sum_{i=1}^{kel} \bar{K}_i^{(j)}; \delta^{(j)} = K^{(j)-1} \cdot F \quad (5)$$

where j is the step number; kel – number of system elements; $\bar{K}_i^{(j)}$ – stiffness matrix i CE for j – calculation step. The coordinates of the nodes and the values of the system state parameters on j , the loading step changes according to the relations:

$$\begin{aligned} x^{(j)} &= x^{(j-1)} + 1/n \cdot \delta^{(j)} \cdot (\Delta_x, \Delta_y, \Delta_z); \\ S_{ik}^{(j)} &= S_{ik}^{(j-1)} + 1/n \cdot S_{ik}^{(j)}(\delta^{(j)}, F), \end{aligned} \quad (6)$$

where $\delta^{(j)}(\Delta_x, \Delta_y, \Delta_z)$ – linear displacement vector of system nodes; $S_{ik}^{(j)}$ – state parameter i element of the system. As the calculations show, the proposed algorithm for non-linear calculation converges rather quickly in 30-40 steps, and then is slightly refined with the number of steps increase to 100

3.3. Comparative analysis of calculation results using various algorithms and software modules

As examples, the systems were chosen with the following options for the mast parameters and loads values: 1) $H=60$ m, $b_r=18$ m, $H_1=20$ m, $a_1=a_2=0.5$ m, $t_1=t_2=0.006$ m, $d_r=0.01$ m, $q_v=0.56$ kH/m², $aq_v=0.46$ kN / m², $F_1=10$ kN, $F_2=20$ kN, $\gamma=78$ kN/m³ (steel); 2) $H=60$ m, $b_r=18$ m, $H_1=20$ m, $a_1=a_2=1.0$ m, $t_1=t_2=0.01$ m, $d_r=0.01$ m, $q_v=0.56$ kN/m², $aq_v=0.46$ kN/m², $F_1=10$ kN, $F_2=20$ kN, $\gamma=78$ kN/m³ (steel). Some calculations results of the considered mast variants are presented in tables 1,2,3 with the help of the developed author's program modules and SCAD and ANSYS programs. The calculations were carried out under the calculated loads action (Tables 1, 2) and loads increased by the safety factor $c_s = 1.5$ on the loads, (Table 3).

Table 1. Mast calculations results. Option 1) $c_s=1.0$.

Options VAT	Calculation Algorithms		Inaccuracy of linear calculation %
	Linear calculation by SCAD	Nonlinear calculation by deformed scheme	
V_3 (m)	0.769	0.954	24
M_2	108.1	135.97	25.8

Table 2. Results of mast calculations. Option 2), $k_p=1.0$.

Options VAT	Algorithms for mast calculating			
	Analytical calculation	Nonlinear calculation by deformed scheme	Nonlinear calculation by ANSYS	Linear calculation by SCAD
V_3 (m)	0.4799	0.4385	0.4508	0.4305
M_1 (kNm)	731.06	683.5	693.1	677.5

Table 3. Results of the mast calculations. Option 2), $k_p=1.5$.

Options VAT	Algorithms for mast calculating			
	Analytical calculation	Nonlinear calculation by deformed scheme	Nonlinear calculation by ANSYS	Linear calculation by SCAD
V_3 (m)	0.767	0.6653	0.6376	0.6476
M_1 (kNm)	1149.07	1033.0	1032.4	1019

Analyzing the calculations results, we note the following. Linear calculation error by SCAD program in version 1) of a more flexible mast is 25.8% for maximum force and 24.0% for maximum displacement in comparison with the deformed scheme calculation algorithm. If the load was increased by 1.5 times (variant 2)), the forces and displacements in the analytical calculation increased 1.59 times, which indicated an increase in the state parameters as compared to the linear dependence by 6%. For a more flexible mast (option 1)), the nonlinear effect of longitudinal-transverse bending was much higher. In addition, calculations of the flexible mast showed an intensive increase in the state parameters values when the load parameter approached the value corresponding to the loss of the mast stability.

4. Setting and solving the optimizing problem of the mast structure with stretches

At present, there is a great number of different optimization of the constructions problems [8-12]. Mast optimizing issue with stretches in the form of a nonlinear mathematical programming problem is formulated in this paper [5, 11]: $\min f(X, P(X))$ is required to find, $X \in E_n$, subject to the limitations

$$h_j(X, P(X)) = 0, j = 1, \dots, m_1;$$

$$g_j(X, P(X)) \leq 0, j = m_1 + 1, \dots, m, \quad (7)$$

where $f(X, P(X))$ are the objective function; X is the vector of variable parameters; $P(X)$ is the vector of structural state parameters. We will not take into account the equality constraints (of state equation) in (7) solving the formulated problem. In this case, an iterative algorithm for solving the formulated problem is used, with the construction of an approximation of $P^l(X)$ vector of state parameters on iterations.

4.1. Selection of variable parameters. Formation of objective function and constraints

The following parameters are accepted as variable: X_1, X_2 are the external dimensions of 1,2; X_3 , sections, X_4 is the mast walls thickness on the sections; X_5 – diameter of stretch marks section; X_6 – coordinate of the extension attachment to the mast.

The function of the system steel elements volume is adopted as the target. Taking into account the introduced variable parameters, expression for the chosen objective function takes the form:

$$f(X) = (X_1^2 - (X_1 - 2X_2)^2) \cdot X_6 + (X_3^2 - (X_3 - 2 \cdot X_4)^2) \cdot (H - X_6) + \\ + k_r \cdot 0,25 \cdot X_5^2 \cdot l_r \cdot n_r, \quad (8)$$

where H is the mast height; l_r is the length of the rod stretch; k_r is the "weight" stretch coefficient; n_r is number of the stretches. "Weight" coefficient k_r is introduced with the different cost account of materials volume unit of a mast and cable stretch marks.

Limitations on strength and rigidity are formed from the conditions of the load calculation [2]. Test calculations showed that it is possible to limit itself to checking the strength according to normal stresses calculating the mast for strength:

$$g_1(X, P(X)) = ((F_1 + G_1) \cdot k_p / AM_1 + \max MOM(H_1) / W_{z1}) / (k_p \cdot R) - 1 \leq 0, \quad (9)$$

where $AM_1 = X_1^2 - (X_1 - 2X_2)^2$ – mast cross-sectional area 1; W_{z1} – moment of mast section resistance relative to axis Z_1 ; R – rated resistance of the mast material section; k_p – load coefficient; $\max MOM(H_1)$ – maximum bending moment in section sections 1, were found as a result of nonlinear calculation for the maximum load. Similarly, restrictions are formed g_2, g_3 on the strength of the mast in section 2 and on the cable stretch marks strength. Limitation g_4 the mast rigidity was superimposed on the maximum displacement of the mast sections horizontally from the maximum load:

$$g_4(X) = \max |v(x)| / k_p [v] - 1 \leq 0, \quad (10)$$

where $[v]$ is allowable movement. In addition, parametric constraints that indirectly took into account the stability constraint and excluded the possibility of the mast sections degeneration:

$$g_5(X) = X_1 / (200 \cdot X_2) - 1 \leq 0; \quad g_6(X) = X_3 / (200 \cdot X_4) - 1 \leq 0. \quad (11)$$

4.2. The algorithm for the optimal solution finding

Recently, in connection with the computer performance increasing, it seems more attractive to organize optimization processes without constructing approximations of the state parameters, within the framework of the search algorithm is being implemented. The method of mobile external penalty as the search algorithm for solving the conditionally-extreme problem of the mast construction optimization is proposed to use in this paper [13]. According to this method, the initial conditional-extremal problem reduces to an unconditionally extremal form: we find the function minimum

$$F^t(X, P(X)) = k_f^t f(X) + \sum_{j=1}^6 k_j^t \max(0, g_j(X) + \Delta Z_j^t)^2, \quad (12)$$

where $k_f^t, k_j, k_j^t, \Delta Z_j^t$ are the regulating parameters of the method; t is the iteration number. In order to reduce the influence of the overall geometric parameter X_6 to choose the optimal parameters of the cross sections, it is suggested to take the parameter X_6 on an external level and use a two-level search algorithm, with the construction of optimal solution interpolation with respect to the parameter X_6 .

4.3. The results of the optimization problem solving

As a "base" project, a mast of constant cross section with a height $H = 60$ m was taken. Four cable ropes were fixed at the corners of the mast square ring section at the level $H_1 = 30$ m in the base of $0.3 H$ distance from mast edges to diagonals. In the case of a "basic" project, the number of variable parameters is reduced to three: X_1, X_2 are the dimensions of a rectangular annular section; X_3 is the diameter of the stretches rod cross-section.

The solving results of the optimization problems of a two-stage project with two variants $[V]$:

- 1) $k_r = 80, [V] = H/500 = 0.12$ m; $f(X^*) = 6.29$ m³, $X_1^* = 1.66$ m; $X_2^* = 0.00836$ m; $X_3^* = 0.0368$ m;
- 2) $k_r = 80, [V] = H/200 = 0.3$ m; $f(X^*) = 2.83$ m³, $X_1^* = 1.18$ m; $X_2^* = 0.00589$ m; $X_3^* = 0.023$ m

The calculations show that the greatest impact on the optimization results has a limitation on the stiffness. Solving the optimization problems of a two-stage project with two variants $[V]$ gives:

- 1) $k_r = 80, [V] = H/500 = 0.12$ m; $f(X^*) = 4.57$ m³, $X_1^* = 1.18$ m; $X_2^* = 0.00836$ m; $X_3^* = 2.37$ m; $X_4^* = 0.119$ m; $X_5^* = 0.004$ m

- 2) $k_r=80$, $[V]=H/200=0.3$ m; $f(X^*)=2.53$ m³, $X_1^*=0.803$ m; $X_2^*=0.00769$ m; $X_3^*=0.0696$ m; $X_4^*=0.0696$ m; $X_5^*=0.017$ m

According to the calculations results, with $k_r=80$, $[V]=H/500=0.12$ m; the optimal two-stage mast design is 38% more efficient than the "base" one. If the horizontal movement tolerance of the mast section increase ($[V]=H/200=0.3$ m), saving material is lowering to 11%.

Optimization of the two-stage mast be done according to the upper level parameter. As it was noted above, it is of interest to identify the optimal level of stretch marks attachment to the mast. For this purpose, the variable parameter X_6 was placed on an external level. According to the developed algorithm for given values of X_6 parameter and variable parameters $X_1 - X_5$ the response curve of the optimal solution for the parameter change is constructed on the minimum value of the objective function $\min f(X)$ and the optimal values of the variable parameters. The results of solving the posed optimization problem for the case were showed $k_r=80$, $[V]=H/200=0.3$ m; $X_6^*=46.7$ m; $f(X^*)=0.287$ m³; $X_1^*=0.3$ m; $X_2^*=0.002$ m; $X_3^*=0.556$ m; $X_4^*=0.0028$ m; $X_5^*=0.011$ m.

5. Conclusion

- The algorithm and the program module of analytical non-linear calculation of the considered type constructions were developed, taking into account the features of cable stretch marks imposed on the masts' longitudinal-transverse bending.
- A step-by-step algorithm for masts non-linear calculation for a deformed scheme based on the finite element method was proposed and programmed. The convergence of the proposed numerical algorithm.
- A two-level algorithm and software modules for optimizing the masts reinforced with cable extensions were developed. The efficiency of optimal two-stage mast structures was shown in comparison with the "basic" variants of constant rigidity.
- It is shown that the optimal attachment point of the stretch marks can deviate from the top of the two-stage mast.

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