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Calculation of two-size particles filtration in a porous medium

L I Kuzmina[1] and Y V Osipov[2]

[1] National Research University Higher School of Economics, 20, Myasnitskaya st., Moscow, 101000, Russia
[2] Moscow State University of Civil Engineering (National Research University), 26, Yaroslavskoe shosse, Moscow, 129337, Russia

yuri-osipov@mail.ru

Abstract. Filtration describes a variety of the construction complex problems: strengthening loose soil to create a solid foundation, the movement of groundwater with solid impurities near underground structures, and many others. A model of two-sized deep bed filtration particles moving with different velocities in a porous medium with three-size pores is considered. The competition of pores and various size particles for deposit formation is modeled. Solutions are constructed at the porous medium inlet and on the concentrations front of the fast particles. For constant filtration coefficients, a global exact solution is obtained. Numerical calculation illustrates the evolution of the filtration process.

1. Introduction

The transportation and retention of solid particles by the fluid flow describe strengthening loose soil to create a solid foundation, the construction of underground and hydraulic structures, and so on. The filtration problems have wide applications in the construction complex [1, 2]. The migration of groundwater solid impurities, the displacement of the grout, pumped into lose ground is described by various mathematical filtration models of a suspension in a porous medium [3, 4]. Exact and asymptotic solutions for some models are obtained [5–7]. In the general case, analytical solutions are unknown, and the problem is solved numerically [8–10].

The paper considers a filtration model for two-size particles d_1, d_2 $(d_1 > d_2)$ moving in a fluid flow with different velocities. If the sizes of particles and pores are of the same order, the main cause of deposit formation is the geometric mechanism of particle capture: the particles freely pass through large pores and get stuck in the pore throats, smaller than the particle diameter [11]. Assume that the porous medium has pores of three various diameters D_1, D_2, D_3, and $D_1 > d_1 > D_2 > d_2 > D_3$. All particles pass freely through the pores D_1 and get stuck at the inlet of the pores D_3. Large particles d_1 get stuck in the pores D_2, and small particles d_2 pass through them unhindered (Figure 1).

The flow rate increases with the cross section of the pore. If large particles pass only through large pores D_1, their velocity α_1 is greater than the average speed α_2 of small particles d_2 passing through the big pores D_1 and medium pores D_2. The filtration of two-size particles moving with different velocities was studied in [12]. However, there are no medium pores in this model, which substantially simplify the structure of the porous medium and the particles-pores interaction.

A mathematical model of the filtration problem for two-size particles moving with different velocities in a homogeneous porous medium is constructed in Section 2. In Section 3, local solutions are obtained on the concentrations front of the fast particles and at the porous medium inlet. In Section 4, a global solution is constructed for constant filtration coefficients. The results of numerical calculations are given in Section 5. Conclusions in Section 6 finalize the paper.

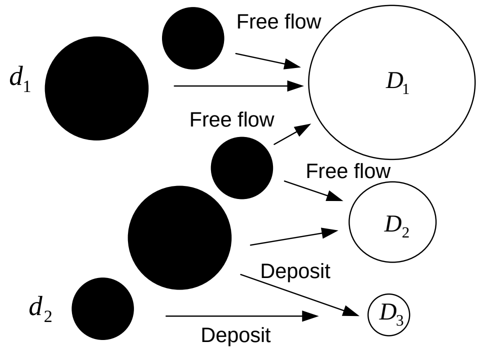


Figure 1. Transport and retention of 2-size particles.

2. Mathematical model

The one-dimensional filtration problem of a two-size particles suspension in a homogeneous porous medium is considered in the domain

$$\Omega = \{(x,t) : 0 < x < 1,\ t > 0\}.$$

For each size of the particle d_i, $i = 1, 2$, the suspended and retained particles concentrations C_i and S_i satisfy the mass balance equation

$$\frac{\partial C_i}{\partial t} + \alpha_i \frac{\partial C_i}{\partial x} + \frac{\partial S_i}{\partial t} = 0, \tag{1}$$

and the kinetic equation of deposit growth rate

$$\frac{\partial S_i}{\partial t} = \Lambda_i C_i, \quad i = 1, 2. \tag{2}$$

Here α_i are the particle velocities, $0 < \alpha_2 < \alpha_1$; $\Lambda_i(S)$ are the filtration coefficients, depending on the deposit concentration.

A suspension of constant concentration is injected into the porous medium inlet; at the initial time, the porous medium does not contain suspended and retained particles. The corresponding boundary and initial conditions have the form

$$C_i\big|_{x=0} = p_i,\ p_i > 0; \tag{3}$$

$$C_i\big|_{t=0} = 0;\ S_i\big|_{t=0} = 0;\ i = 1, 2. \tag{4}$$

Consider the transportation and retention of particles in the pores in detail, the deposit S_2 of small particles d_2 is formed only in small pores D_3 (Figure 1). The deposit S_1 of large particles d_1 is distributed between the pores D_3 and D_2:

$$S_1 = S_1^3 + S_1^2. \tag{5}$$

The deposit in medium pores D_2

$$S^2 = S_1^2;$$

the total deposit in small pores of diameter D_3

$$S^3 = S_1^3 + S_2. \tag{6}$$

When all small and medium pores are clogged, the pore deposits S^2, S^3 reach its maximum values

$$S_{\max}^i = \lim_{t\to\infty} S^i;\ i = 2,3. \tag{7}$$

If the porous medium is homogeneous, the limit deposits $S_{\max}^i$ do will not depend on the coordinate x. The maximum limit deposit

$$S_{\max} = S_{\max}^2 + S_{\max}^3. \tag{8}$$

The maximum deposits (7), (8) are assumed to be known, since they are determined by the concentrations of 3-size pores in a porous medium.

The deposit growth rate depends on the number of free pores that can be blocked by particles. We assume that the filtration coefficients are linear. It follows from (5) - (8) that for a porous medium with 3-size pores equations (2) take the form

$$\frac{\partial S_1^2}{\partial t} = \lambda_1^2 (S_{\max}^2 - S_1^2) C_1;\quad \frac{\partial S_1^3}{\partial t} = \lambda_1^3 (S_{\max}^3 - S_1^3 - S_2) C_1;\quad \frac{\partial S_2}{\partial t} = \lambda_2^3 (S_{\max}^3 - S_1^3 - S_2) C_2. \tag{9}$$

The concentration fronts of fast and slow particles $t = x/\alpha_i$, $i = 1,2$ divide Ω into 3 subdomains

$$\Omega_0 = \{0 < x < 1,\ t < x/\alpha_1\};\quad \Omega_1 = \{0 < x < 1,\ x/\alpha_1 < t < x/\alpha_2\};\quad \Omega_2 = \{0 < x < 1,\ t > x/\alpha_2\}.$$

In the domain Ω_0, the porous medium is empty; in Ω_1 only suspended and retained fast particles d_1 are present; there are all types of particles d_1, d_2 in the domain Ω_2 (Figure 2).

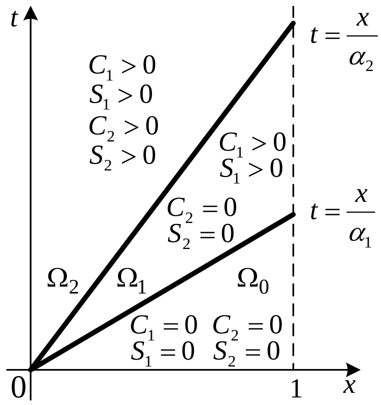


Figure 2. 2-size particles in a porous medium.

The solutions $C_i(x,t)$ are discontinuous on the concentrations front $t = x/\alpha_i$, $i = 1,2$; the solutions S_1^2, S_1^3, S_2 are continuous in the domain Ω.

3. Local solutions

3.1. The solution on the concentrations front of the fast particles

The solution on the concentrations front $t = x/\alpha_1$: $C_2 = 0;\ S_1 = S_2 = 0$ (see Figure 2).

Substitution of (5), (9) into equation (1) for $i = 1$ gives

$$\frac{\partial C_1}{\partial t} + \alpha_1 \frac{\partial C_1}{\partial x} + \Lambda C_1 = 0,\quad \Lambda = \lambda_1^2 (S_{\max}^2) + \lambda_1^3 (S_{\max}^3). \tag{10}$$

The solution of equation (10) with the condition (3)

$$C_1(x,t)\big|_{t=x/\alpha_1} = p_1 \exp(-\Lambda x/\alpha_1)\,. \tag{11}$$

3.2. The solution at the porous medium inlet

According to condition (3), at the porous medium inlet $x=0$ equations (9) have the form

$$\frac{\partial S_1^2}{\partial t} = \lambda_1^2 (S_{\max}^2 - S_1^2) p_1\,;\quad \frac{\partial S_1^3}{\partial t} = \lambda_1^3 (S_{\max}^3 - S_1^3 - S_2) p_1\,;\quad \frac{\partial S_2}{\partial t} = \lambda_2^3 (S_{\max}^3 - S_1^3 - S_2) p_2\,. \tag{12}$$

The solution of the first equation (12) with the condition (4)

$$\int_0^{S_1^2(0,t)} \frac{dS}{\lambda_1^2 (S_{\max}^2 - S)} = p_1 t\,. \tag{13}$$

Addition of the second and third equations (12) and the use of the notation (6) yields

$$\frac{\partial S^3}{\partial t} = \lambda (S_{\max}^3 - S^3),\quad \lambda = \lambda_1^3 p_1 + \lambda_2^3 p_2\,. \tag{14}$$

The solution of equation (14) with the condition (4)

$$\int_0^{S^3(0,t)} \frac{dS}{\lambda\left(S_{\max}^3 - S\right)} = t\,. \tag{15}$$

Using (15), the solutions of the second and third equations (12) are obtained

$$S_1^3(0,t) = p_1 \int_0^t \lambda_1^3 \left(S_{\max}^3 - S^3(0,t)\right) dt;\quad S_2(0,t) = p_2 \int_0^t \lambda_2^3 \left(S_{\max}^3 - S^3(0,t)\right) dt\,. \tag{16}$$

4. Exact solution for constant coefficients

For constant filtration coefficients the system (1), (2) takes the form

- in the domain Ω_1:

$$\frac{\partial C_1}{\partial t} + \alpha_1 \frac{\partial C_1}{\partial x} + \frac{\partial S_1}{\partial t} = 0;\quad \frac{\partial S_1^2}{\partial t} = \lambda_1^2 C_1;\quad \frac{\partial S_1^3}{\partial t} = \lambda_1^3 C_1;\quad S_1 = S_1^3 + S_1^2\,; \tag{17}$$

- in the domain Ω_2:

$$\frac{\partial C_i}{\partial t} + \alpha_i \frac{\partial C_i}{\partial x} + \frac{\partial S_i}{\partial t} = 0;\ i=1,2;\quad \frac{\partial S_1^2}{\partial t} = \lambda_1^2 C_1;\quad \frac{\partial S_1^3}{\partial t} = \lambda_1^3 C_1;\quad S_1 = S_1^3 + S_1^2;\quad \frac{\partial S_2}{\partial t} = \lambda_2^3 C_2\,; \tag{18}$$

with the conditions (3) and conditions on the concentrations fronts

$$C_i\big|_{x=0} = p_i;\quad S_i\big|_{t=x/\alpha_i} = 0;\ i=1,2\,. \tag{19}$$

The solution of the system in the domain Ω is given below (Figure 2).

In the domain Ω_0:

$$C_i(x,t) = 0;\quad S_i(x,t) = 0;\ i=1,2\,. \tag{20}$$

In the domain Ω_1:

$$C_1(x,t) = p_1 e^{-\Lambda x/\alpha_1}; \quad S_1^3(x,t) = \lambda_1^3 p_1 e^{-\Lambda x/\alpha_1}(t - x/\alpha_1); \quad S_1^2(x,t) = \lambda_1^2 p_1 e^{-\Lambda x/\alpha_1}(t - x/\alpha_1);$$
$$C_2(x,t) = 0; \quad S_2(x,t) = 0. \tag{21}$$

In the domain Ω_2:

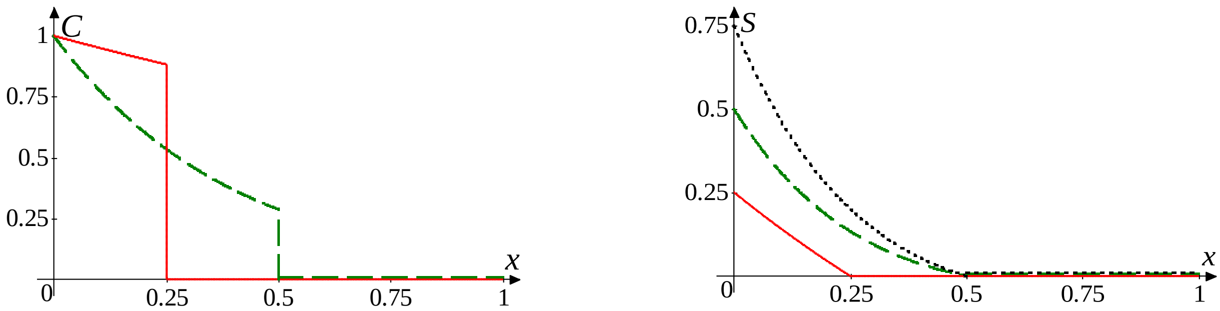
$$C_1(x,t) = p_1 e^{-\Lambda x/\alpha_1}; \quad C_2(x,t) = p_2 e^{-\lambda_2^3 x/\alpha_2}; \quad S_2(x,t) = \lambda_2^3 p_2 e^{-\lambda_2^3 x/\alpha_2}(t - x/\alpha_2);$$
$$S_1^3(x,t) = \lambda_1^3 p_1 e^{-\Lambda x/\alpha_1}(t - x/\alpha_1); \quad S_1^2(x,t) = \lambda_1^2 p_1 e^{-\Lambda x/\alpha_1}(t - x/\alpha_1); \quad \Lambda = \lambda_1^2 + \lambda_1^3. \tag{22}$$

5. Numerical modeling

To calculate the solution (20) - (22), the following parameters are selected:

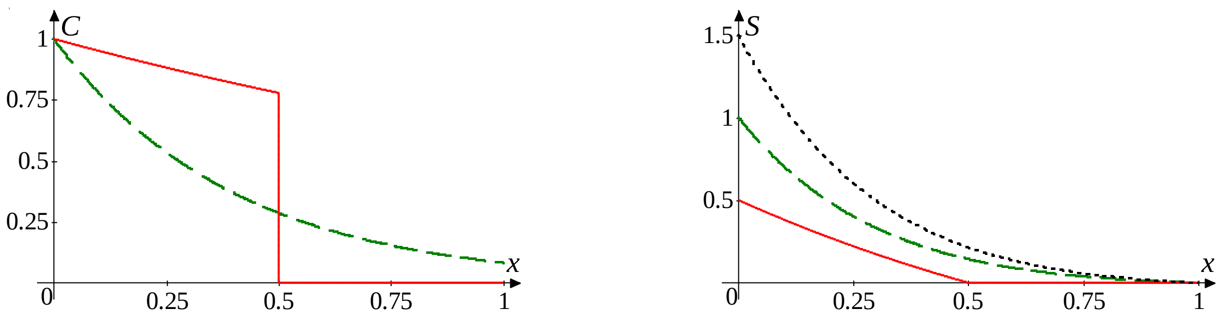
$$\alpha_1 = 1; \ \alpha_2 = 0.5; \ p_1 = p_2 = 1; \ \lambda_2^3 = 0.5; \ \lambda_1^3 = 1; \ \lambda_1^2 = 1.5$$

In Figures 3-5 graphs of suspended and retained particles concentrations are presented for $t = 0.5; 1; 5$. The graphs of C_1, S_1^3 are drawn by a dashed line, C_2, S_2 - by a solid line, and S_1^2 - by a dotted line.



(a) (b)

Figure 3. Concentrations Suspended $C_1|_{t=0.5}$; $C_2|_{t=0.5}$ (a); Retained $S_1^3|_{t=0.5}$; $S_1^2|_{t=0.5}$; $S_2|_{t=0.5}$ (b).



(a) (b)

Figure 4. Concentrations Suspended $C_1|_{t=1}$; $C_2|_{t=1}$ (a); Retained $S_1^3|_{t=1}$; $S_1^2|_{t=1}$; $S_2|_{t=1}$ (b).

In Figures 3-5, the suspended and retained particles concentrations are zero before its front and are positive behind the front. The large particles concentrations front moves faster than the small particles front. Graphs 3-5 b) show that the formation of large and small particles deposit occurs most intensively at the porous medium inlet; the suspended and retained concentrations of both particles types decrease with increasing distance x to the inlet.

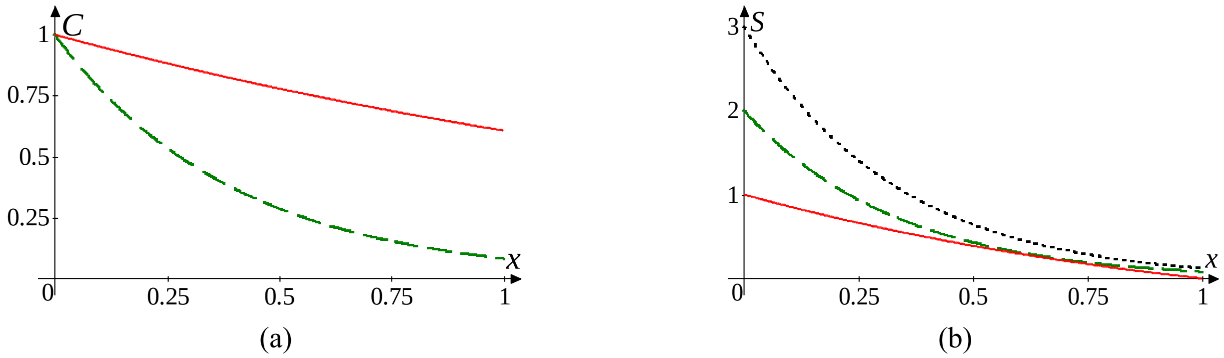


Figure 5. Concentrations Suspended $C_1\big|_{t=2}$; $C_2\big|_{t=2}$ (a); Retained $S_1^3\big|_{t=2}$; $S_1^2\big|_{t=2}$; $S_2\big|_{t=2}$ (b).

6. Conclusion

The filtration problem of a suspension with two-size particles in a porous medium with three-size pores is considered. A mathematical model describing the competitions of large and small particles for blocking small pores and of small and medium pores for large particles retention is constructed.

It is shown if large and small particles move with different velocities, two concentrations fronts propagate in a porous medium. Before the fast particles front, the porous medium is empty. Only large suspended and retained particles are present between the fast and slow particles fronts. There are particles of all types behind the slow particles front.

Local exact solutions are constructed on the characteristics of the system, at the porous medium inlet and on the concentrations front of the fast particles. For constant filtration coefficients, a global solution is obtained.

Exact solutions give way to construct the asymptotics, which depends explicitly on the parameters of the problem, allowing fine-tuning of the experiments and production processes [13]. This is the subject of another study.

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