

# A comprehensive study on the buckling of sandwich plates

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**Abstract.** Advancements in aeromechanical materials is a significant topic of research in recent days. Better understanding of structural behaviour is vital for the manufacturing of aircraft and spacecraft components. Sandwich plates can be employed for several applications, especially for making airplane and rocket structures. These all motivate researchers to investigate on the modelling and analysis of advanced materials like carbon nanotubes, laminated composites, functionally graded materials, sandwich structures, etc. Sandwich plates have thin outer facings containing a core in between. The objective of the present work is to conduct a detailed study of the buckling response of sandwich plates employing first order and higher order shear deformation theories considering various aspects. Numerical formulation has been carried out initially. Programing in Matlab environment has been done. Detailed parametric studies also conducted to provide clear understanding of the behaviour of buckling of sandwich structures.

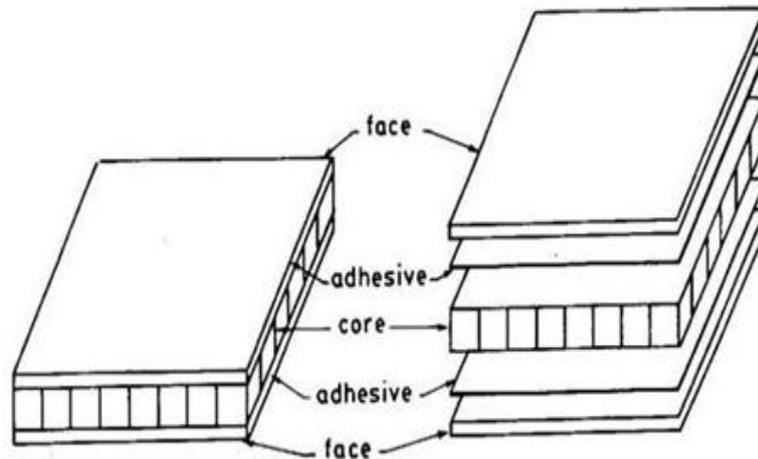
## 1. Introduction

Sandwich plates have thin outer facings containing a core in between. Sandwich plate in the present analysis is made up of composite at the top and bottom and low strength and lightweight materials in the middle thicker portion. The strong composites at the outer layers support axial stresses. Also, the resistance to bending increases because of the thick core in the middle. Sandwich plates can be employed for several applications, especially for making airplane and rocket structures. Numerous plate and shell type structural elements are used in aerospace industry. This necessitates the development of exact simulations to calculate structural response. In the perspective of aircraft wing components made of sandwich square or rectangular plates, it is essential to have a vital understanding of their buckling/structural instability characteristics.

A few significant works which are relevant to the present topic are discussed in this paragraph. Analytical and finite element derivations for laminated composite plates were discussed by Reddy [1] in detail. Zienkiewicz [2] discussed elaborately about von Karman nonlinearity problems. He discussed in detail the geometric stiffness matrix in relation to buckling phenomenon. Recently, a review on various shear deformation theories was conducted by Sreehari et al. [3]. Certain aspects on sandwich plates were discussed in recent literature [4, 5, 6]. They presented detailed results on the bending and vibration behaviour of sandwich plates. The buckling behaviour for various dimensions of the plate is an interesting area to be investigated in detail. So in this work, buckling behavior of



sandwich plates is discussed. The goal of the current investigation is to conduct a comprehensive study of the buckling response of sandwich plates using first order (FSDT) and higher order shear deformation theories (HSDT) considering aspects like various dimensions and loadings.



**Figure 1.** Sketch of a sandwich structure.

## 2. General numerical modelling

The formulation adopted for present problem is described in brief here. Higher order shear deformation theory is considered at first. The components of displacements ( $u$ ,  $v$ ,  $w$ ) in  $x$ ,  $y$ , and  $z$  directions are as shown below:

$$\begin{bmatrix} u(x, y, z, t) \\ v(x, y, z, t) \\ w(x, y, z, t) \end{bmatrix} = \begin{bmatrix} u_o(x, y, z, t) \\ v_o(x, y, z, t) \\ w_o(x, y, z, t) \end{bmatrix} + z \begin{bmatrix} \phi_x(x, y, t) \\ \phi_y(x, y, t) \\ 0 \end{bmatrix} + z^2 \begin{bmatrix} \beta_x(x, y, t) \\ \beta_y(x, y, t) \\ 0 \end{bmatrix} + z^3 \begin{bmatrix} \psi_x(x, y, t) \\ \psi_y(x, y, t) \\ 0 \end{bmatrix} \quad (1)$$

where  $u_o, v_o, w_o$  indicates the displacements of a point on the mid plane. Rotations of the cross-section perpendicular to  $x$  and  $y$  axes are  $\phi_x, \phi_y$  respectively and  $\beta_x, \beta_y, \psi_x, \psi_y$  are the second and third order terms in Taylor's series expansion. Displacement field for FSDT can be simplified as shown below,

$$\begin{bmatrix} u(x, y, z, t) \\ v(x, y, z, t) \\ w(x, y, z, t) \end{bmatrix} = \begin{bmatrix} u_o(x, y, z, t) \\ v_o(x, y, z, t) \\ w_o(x, y, z, t) \end{bmatrix} + z \begin{bmatrix} \phi_x(x, y, t) \\ \phi_y(x, y, t) \\ 0 \end{bmatrix} \quad (2)$$

Introductory concepts and the mathematical formulations for bending and buckling behaviour of laminated composite and smart composites were studied and discussed earlier [7, 8, 9]. So details are not discussing here for the sake of brevity. Finally,  $K^{(e)}$  is computed numerically and can be written as:

$$K_{ij}^{(e)} = \int_{-1}^{-1} \int_{-1}^{-1} B_i^T D B_j \det J d\xi d\eta \quad (3)$$

Where,  $J$  is the Jacobian Matrix. The element stiffness matrix can be written as below after employing numerical integration:

$$K_{ij}^{(e)} = \frac{1}{2} \sum_{p=1}^N \sum_{q=1}^N W_p W_q B_i^T D B_j \det J \quad (4)$$

Where  $W_p$ ,  $W_q$  are the weights of the Gaussian quadrature technique. Consequently an eigenvalue problem can be framed considering buckling aspects as shown below,

$$([K] + \lambda [K_G]) \{\Delta\} = \{0\} \quad (5)$$

where  $[K]$  denotes global stiffness matrix and  $[K_G]$  represents global geometric stiffness matrix. Also,  $\lambda$  denotes in-plane magnification factor whereas  $\{\Delta\}$  is the global displacement field. The eigenvalue problem is solved to get minimum eigenvalues (which are critical buckling loads). Also detailed mathematical formulations and matlab coding are discussed in few existing literature [10, 11].

### 3. Results and discussions

Results for the present analysis are obtained by examining the formulation and performing computer programming in Matlab. A sandwich plate with length  $a$ , width  $b$ , and total thickness  $h$  is considered. Critical buckling loads are presented using below non-dimensionality relation.

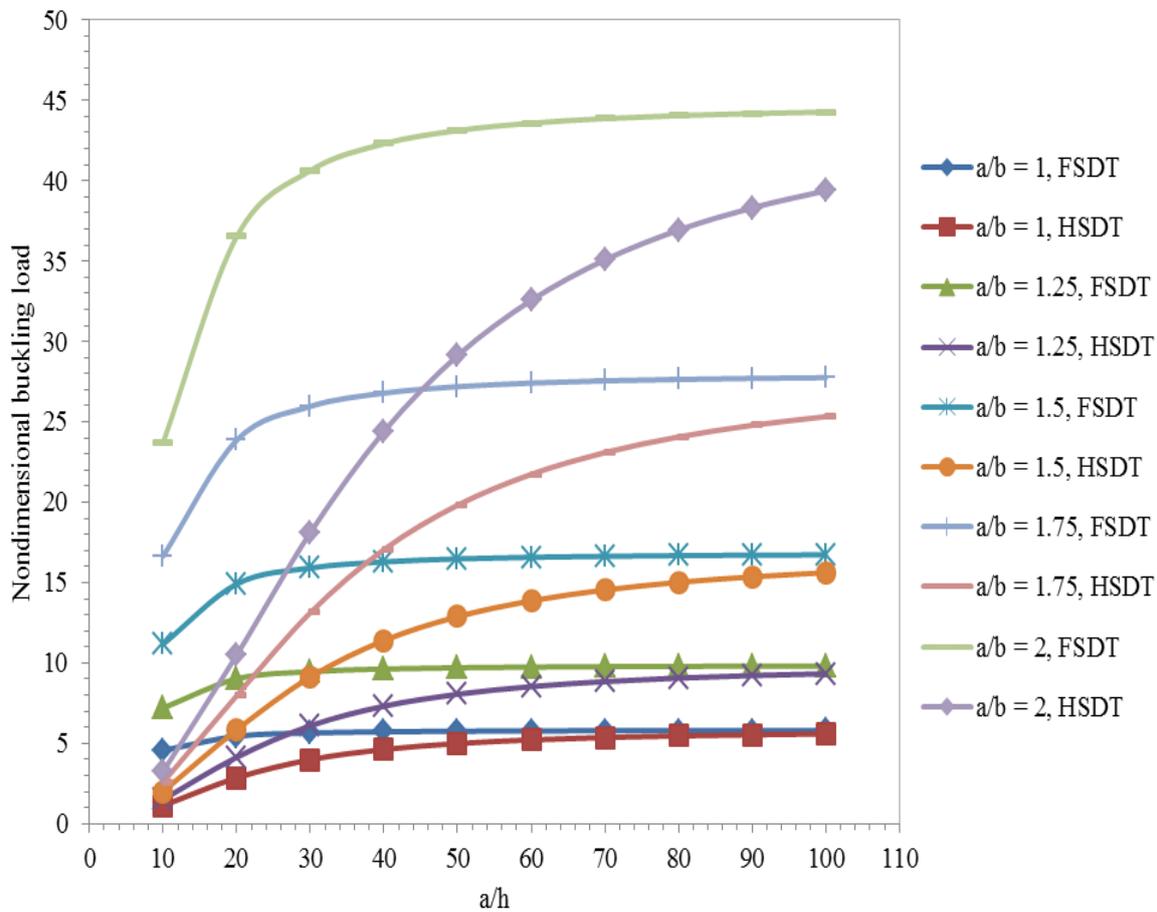
$$\bar{P}_{cr} = P_{cr} \left( \frac{a^2}{E_2 h^3} \right) \quad (6)$$

Navier solutions for sandwich plates are formulated and the formulation and code is validated with existing result for central deflection of a sandwich plate (Table 1). A shear correction factor of 5/6 is used for FSDT. All inputs including material property, geometry, and loadings for this validation study are as in Ref [4]. Core thickness to face sheet thickness ratio is considered as 10 in present problem. The present results are well matching and the minor difference is due to the difference in the shear deformation theory employed in both works.

**Table 1.** Validation study for sandwich plate.

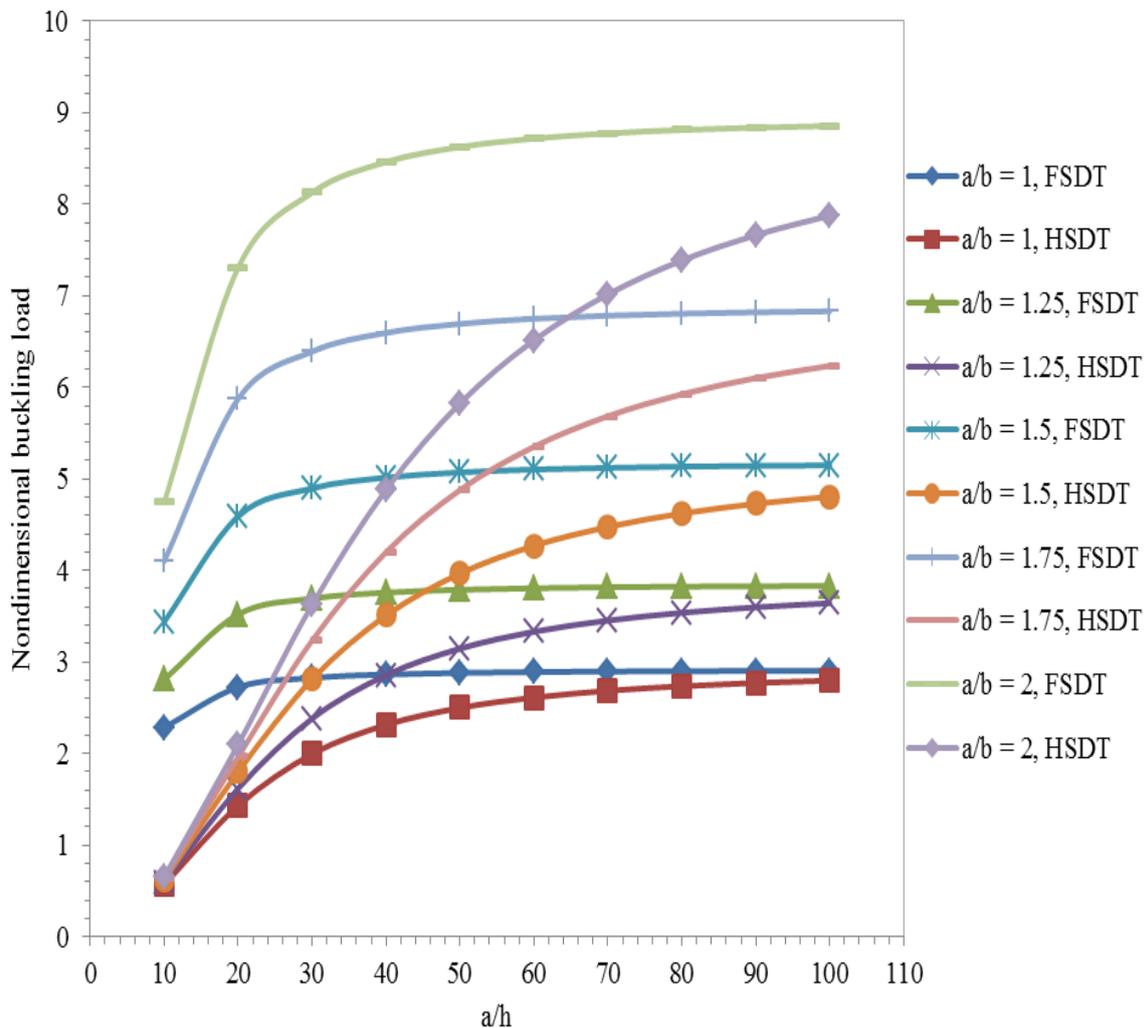
$a/h$	Non-dimensionalised transverse deflection (Present)	Non-dimensionalised transverse deflection (Reference [4])
10	2.0531	2.0190
50	0.9226	0.9212

As discussed in previous section, the main results from current work are presented in Fig 2 and Fig 3. A five layered sandwich plate is considered for present analysis (0/90/core/0/90). The top and bottom facesheets are having material property as:  $E_1 = 131$  GPa,  $E_2 = 10.34$  GPa,  $G_{12} = G_{23} = 6.895$  GPa,  $G_{13} = 6.205$  GPa,  $\nu = 0.22$ ,  $\rho = 1627$  kg/m<sup>3</sup>. The middle isotropic core is having material property as:  $E = 6.9$  MPa,  $G = 3.45$  MPa,  $\rho = 97$  kg/m<sup>3</sup>. All boundaries are simply supported (SS1). The results for uniaxial loading are presented in Fig 2.



**Figure 2.** Non-dimensionalized critical buckling load versus side-to-thickness ratio of a (0/90/core/0/90) sandwich plate under uniaxial inplane load.

The non-dimensionalized critical buckling load has been calculated and presented for various plate dimensions. Initially, a square plate was considered, i.e.,  $a/b = 1$ . Then the study was conducted for  $a/b = 1.25, 1.5, 1.75, \text{ and } 2$ . The results are prepared employing FSDT and HSDT for  $a/h$  ratios 10 to 100 at intervals of 10. Thus the program was run for 100 times and results are plotted in the form of graph. It is noted from the present analysis that the non-dimensionalized critical buckling load increases as side-to-thickness ratio increases. The variation between corresponding FSDT and HSDT results are higher for thick plates and the variation is lesser for square plate compared to plates having  $a/b = 1.25, 1.5, 1.75, \text{ and } 2$ . Then the study for various  $a/b$  ratios of sandwich plate is conducted after applying biaxial inplane loads. Similarly the code was run again for another 100 times and the obtained non-dimensionalized critical buckling loads are noted down. The results obtained are presented in Fig 3.



**Figure 3.** Non-dimensionalized critical buckling load versus side-to-thickness ratio of a (0/90/core/0/90) sandwich plate under biaxial inplane load.

#### 4. Conclusion

Buckling responses of a sandwich plate structure has been calculated employing FSDT and HSDT. Numerical modelling using FSDT as well as HSDT has been made and computer programme for the whole analysis has been prepared. The buckling of sandwich plates for various cases has been evaluated. Results from present analysis were compared with certain available results from existing literature and thus validated the formulation as well as programming. Code is run for more than 200 times and several results and observations are made. As the  $a/h$  ratio increases, the non-dimensionalized critical buckling load increases. Also it is witnessed that for  $a/h$  ratio larger than 40, the non-dimensionalized critical buckling load becomes almost constant when FSDT is employed. The variation amongst FSDT and HSDT results reduces as the thickness of plate reduces. So HSDT is advisable for employing in studies related to buckling of sandwich plates. Thus the current study

discussed the buckling response of sandwich plates using FSDT and HSDT considering many aspects like various dimensions and loadings.

## 5. References

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