

# Monte Carlo Simulation for Modified Parametric Of Sample Selection Models Through Fuzzy Approach

**Y S Triana**

Faculty of Computer Science, Universitas Mercu Buana, Jakarta, Indonesia

E-mail: [yaya.sudarya@mercubuana.ac.id](mailto:yaya.sudarya@mercubuana.ac.id)

**Abstract.** The sample selection model is a combination of the regression and probit models. The models are usually estimated by Heckman's two-step estimator. However, Heckman's two-step estimator often performs poorly. In the context of the parametric method, Monte Carlo simulations are studied. The goal is to simulate and test as early as possible so that we can anticipate the problem of the accuracy of a model. The best approach is to take advantage of the tools provided by the theory of fuzzy sets. It appears very suitable for modeling vague concepts. It is difficult to determine some of the criteria and arrive at a quantitative value. Fuzzy sets theory and its properties through the concept of fuzzy number. The fuzzy function used for solving uncertain of a parametric sample selection model. Estimates from the fuzzy are used to calculate some of equation of the sample selection model. Finally, estimates of the Mean, Root Mean Square Error (RMSE) and the other estimators can be obtained by Heckman two-step estimator through iteration from some parameters and some of values.

**Keywords:** Fuzzy, Heckman's, Monte Carlo, Sample selection model, Simulation

## 1. Introduction

The sample selection was developed by Heckman. Sample selection is widely used in various fields of economics. An early discussion of the problem of self-selectivity was that of [1], who discussed the problem of individuals selecting between hunting and fishing, based on their comparative advantage. The observed distribution of incomes of hunters and fishermen was defined by these choices [3].

The self-selectivity model that focused on selectivity bias was discussed by [4]. [5] introduced a two-step selection model, known as the Heckman two-step sample selection model. The sample selection bias problem in the context of decision by women to participate in the labor force is discussed by [1], [4], [6], [7], [8], [9] and [10].

The disadvantage of this sample selection model is the dependence on the distribution assumption. If the error is heteroskedastic or an abnormal estimate, then it will cause inconsistency [1]. While this may be flexible, through the use of different distribution assumptions, it is interesting to consider alternatives that have limited dependence on parametric assumptions. An alternative to overcome the lack of parametric problems is through the use of semi-parametric methods. This approach will reduce the effective dimension of the estimation problem. This proposal will be present on how the sample selection model works in the context of the parametric model for non-participants.

## 2. Parametric Sample Selection Model

[11] has proposed the parametric sample selection model as follows :

$$\begin{aligned}
 y_i^* &= x_i' \beta + u_i \\
 d_i^* &= w_i' \alpha + v_i \\
 y_i &= y_i^* d_i^* \\
 d_i &= \begin{cases} 1 & \text{if } d_i^* > 0 \\ 0 & \text{otherwise} \end{cases}
 \end{aligned} \tag{1}$$



The variables  $y_i^*$  and  $d_i^*$  are unobserved, whereas  $y_i$  is observed.  $y_i^*$  and  $d_i^*$  are dependent variables,  $x_i$  and  $w_i$  are vectors of independent variables,  $\alpha$  and  $\beta$  are unknown parameter vectors,  $u_i$  and  $v_i$  are error terms.

In Equation (1), there are error terms ( $u, v$ ) which are usually correlated, so that the regression of  $y$  on  $x$  will not give consistent estimates of  $\beta_0$  and  $\beta_1$ . The approach of the error terms ( $u_i, v_i$ ) are assumed to follow a bivariate normal distribution. It is commonly assumed that  $u_i$  and  $v_i$  have a bivariate normal distribution:

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} \sim \text{BN} \left[ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix} \right] \quad (2)$$

According to [12], [13] and [15], there are two parts in Equation (1). The first part is participation equation (a binary decision equation). The second part is the wage equation (outcome equation or selection part). Independent variable  $x_i$  usually contain at least one variable which does not appear in variable  $w_i$ . The outcome equation describes the relationship between the dependent variable  $y_i$  and independent variable  $x_i$ , whereas in the selection equation describes the relationship between the dependent variable  $d_i$  and the independent variable  $w_i$ .

Alpha cuts are simply threshold levels that convert a fuzzy set into a crisp set. The process of converting a fuzzy set to a crisp one is called defuzzification. An alpha-cut  $A$  of a fuzzy number  $A$  is defined as the set  $\{x \in \mathbb{R} \mid A(x) \geq \alpha\}$ .  $A$  is completely determined by the collection  $(A_\alpha) \alpha \in [0,1]$ . An alpha cut is the behaviour sensitivity of the system to the behavior under observation. At some point, as the information value diminishes, one no longer wants to be "bothered" by the data. In many systems, due to the inherent limitations of the mechanisms of observation, the information becomes suspect below a certain level of reliability. The fuzzy model will be written as follows:

$$\begin{aligned} \tilde{y}_i^* &= \tilde{x}_i' \beta + \tilde{u}_i \\ d_i &= 1 \quad \text{if} \quad d_i^* = \tilde{w}_i' \alpha + \tilde{v}_i \leq 0 \\ d_i &= 0 \quad \text{otherwise} \\ \tilde{y}_i &= \tilde{y}_{ic}^* d_i \quad , i = 1, \dots, N \end{aligned} \quad (3)$$

The terms  $\tilde{w}_i$ ,  $\tilde{x}_i$ ,  $\tilde{y}_i^*$ ,  $\tilde{u}_i$  and  $\tilde{v}_i$  are fuzzy numbers with the membership functions  $\mu_{\tilde{w}_i}$ ,  $\mu_{\tilde{x}_i}$ ,  $\mu_{\tilde{y}_i^*}$ ,  $\mu_{\tilde{u}_i}$  and  $\mu_{\tilde{v}_i}$ , respectively. In MPSSM error term assumed to follow the bivariate normal distribution, then the error term for FMPSSM also follows the bivariate normal distribution, namely:

$$(u_{ic}, v_{ic}) \sim N \left( 0, \begin{pmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & 1 \end{pmatrix} \right).$$

The first step is to estimate the coefficient values of  $\alpha$  and  $\beta$ . Then the values of these parameters are applied to the parametric model to obtain the value of the Heckman coefficient estimates of  $\alpha$ ,  $\beta$  and  $\sigma_{u,v}$ . There are two steps in the estimation of the parameter according the Heckman model, namely the first step (probit model) to estimate  $\beta$ :

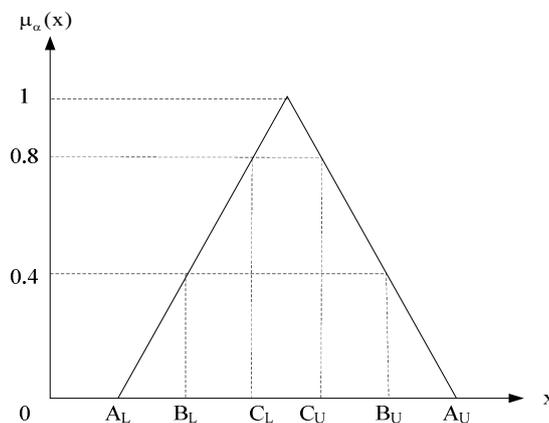
$$P(d_i > 0 \mid x) = 1 = E[d \mid x] = \Phi(w' \alpha) \quad (4)$$

The second step (OLS) is to estimate the regression function by using only observations for  $d_i^* > 0 | x = 1$

$$E(y_i | x_i) = w_i' \alpha + \rho_{uv} \sigma_u \hat{\lambda} \quad (5)$$

Where  $\hat{\lambda} = (-w_i' \hat{\alpha}) / \Phi(w_i' \hat{\alpha})$  is inverse mills ratio. Observed  $y_i$  on  $x_i$  and  $\hat{\lambda}$ , where  $\hat{\lambda}$  is inserted into the decision equations the additional regressor. In this step, an estimation of the parameters of an outcome equation, i.e. selected data is the significant parties of interest. The error terms of the decision and outcome equations should be strongly correlated. Since the real data is generated by a process that satisfies the assumption of the MPSSM, then the coefficient estimates of data generated are quite close to the true coefficients. The following are the procedures of data implemented from crisp data to fuzzy data. First, the real data which involved uncertainties are fuzzified using fuzzy  $\alpha$ -cut. The arithmetic operation on fuzzy  $\alpha$ -cut, for instance the  $\alpha$ -cut method is applied to the data through the fuzzy environment process. This process converts the real data to fuzzy observations  $\tilde{w}_i$ ,  $\tilde{x}_i$  and  $\tilde{y}_i$  with lower and upper membership functions. The defuzzification method is used to convert this fuzzy observations into crisp values  $w_{ic}$ ,  $x_{ic}$  and  $y_{ic}$ . To estimate the parameters fuzzy parametric sample selection model, these values are applied using the Heckman Two Step procedures.

The membership function is cut horizontally at a finite number of  $\alpha$ -levels between 0 and 1. For each  $\alpha$ -level of the parameter, the model is run to determine the minimum and maximum possible values of the output. This information is then directly used to construct the corresponding fuzziness (membership function) of the output which is used as a measure of uncertainty. If the output is monotonic with respect to the dependent fuzzy variable, the process is rather simple since only two simulations will be enough for each  $\alpha$ -level (one for each boundary). Otherwise, optimization routines have to be carried out to determine the minimum and maximum values of the output for each  $\alpha$ -level [2]. Figure 1 shows an illustration of the alpha-cut of triangular fuzzy number.



**Figure 1.** The alpha cut of triangular fuzzy number

From Figure 1, the confidence fuzzy interval defined by different value of alpha cut. For example,  $\alpha(0.4)$  and  $(0.8)$ , then their confidence fuzzy interval are  $[B_L, B_U]$  and  $[C_L, C_U]$ . This relationship denoted by  $(\alpha(0.4), [B_L, B_U])$ , and  $(\alpha(0.8), [C_L, C_U])$  with  $[B_L, B_U] \geq [C_L, C_U]$ .

### 3. The Model

In this section the model of participant, non-participant, and combination of participant and non-participant are discussed. The model is written as follows:

$$\begin{aligned}
 y_i &= \begin{cases} y_0, & \text{if } d_i = 0 \\ y_1, & \text{if } d_i = 1 \end{cases} \\
 d_i &= \begin{cases} 1, & \text{if } w_i \alpha + v_i > 0 \\ 0, & \text{otherwise} \end{cases}
 \end{aligned} \tag{6}$$

Where  $d_i$  is a selection equation of the first stage. The values of  $d_i$  are 0 and 1 which is  $d_i = 1$  for participant and  $d_i = 0$  for non-participant.  $y_i$  is dependent variable of the outcome equation. In the Equation (2) is derived into two, i.e.  $y_0$  and  $y_1$ .  $y_0$  is referred to the outcome of the Equation (2) is non-participant, whereas  $y_1$  for participant.  $x_i$  is independent variable. The details about the equation of  $y_1$  and  $y_0$  are as follows:

$$y_{0i}^* = x_{0i}'\beta_0 + u_{0i} \quad \text{if } d_i = 0 \tag{7}$$

$$y_{1i}^* = x_{1i}'\beta_1 + u_{1i} \quad \text{if } d_i = 1 \tag{8}$$

Where  $u_{0i}$  and  $u_{1i}$  are error terms. The outcome equation for non-participant and participant in Equation (2), can be summarized as follows:

$$y_{i0} = y_{0i}^*(1 - d_i) \quad \text{for non-participant} \tag{9}$$

$$y_{i1} = y_{1i}^*d_i \quad \text{for participant} \tag{10}$$

Hence, the combination of both non-participant and participant, will generate the equation, as follows:

$$y_i = y_{0i}^*(1 - d_i) + y_{1i}^*d_i \tag{11}$$

#### 4. Monte Carlo Simulation Of Parametric Sample Selection Model

The purpose of the Monte Carlo simulation is used to calculate the values of the sample selection model [14]. The fuzzy model for Monte Carlo simulation is as follows:

$$\tilde{y}_i = \beta_0 + \beta_1 \tilde{x}_i + \tilde{u}_i \tag{12}$$

$$d_i = 1(\alpha_0 + \alpha_1 \tilde{w}_i + \tilde{v}_i \leq 0) \tag{13}$$

The following items are considered in the Monte Carlo study:

- The effect of the correlation of  $\tilde{x}_i$  and  $\tilde{w}_i$
- The effect of the correlation of  $\tilde{u}_i$  and  $\tilde{v}_i$

$\tilde{x}_i$  and  $\tilde{w}_i$  are the exogenous or independent variables and the values are as follows:

$$\tilde{w}_i = \tilde{s}_{1i} \tag{14}$$

$$\tilde{x}_i = \frac{[\pi \tilde{s}_{1i} + (1 - \pi) \tilde{s}_{2i}]}{\sqrt{\pi^2 + (1 - \pi)^2}} \tag{15}$$

$\tilde{s}_{1i}$  and  $\tilde{s}_{2i}$  are independent and identically distributed (i.i.d.) random variables distributed uniformly on (0,20).

The values of the exogenous variables,  $\tilde{x}_i$  and  $\tilde{w}_i$  are as follows:

$\tilde{s}_{1i}$  and  $\tilde{s}_{2i}$  are random uniform variables with mean = 0 and variance = 20.  $\pi$  is the correlation coefficient of  $\tilde{x}_i$  and  $\tilde{w}_i$  with  $\pi = 0.0, 0.5,$  and  $0.9$  are considered. The fuzzy error terms  $\{\tilde{u}_i, \tilde{v}_i\}$  are jointly normal and determined as follows:

$$\tilde{v}_i = \tilde{\varepsilon}_{1i}, \quad (16)$$

$$\tilde{u}_i = \frac{\rho_0 \tilde{\varepsilon}_{1i} + (1 - \rho_0) \tilde{\varepsilon}_{2i}}{\sqrt{\rho_0^2 + (1 - \rho_0)^2}} \quad (17)$$

The  $\{\tilde{\varepsilon}_{1i}\}$  are normal random variables with mean = 0 and variance = 1. The  $\{\tilde{\varepsilon}_{2i}\}$  are i.i.d. normal random variables with mean = 0 and variance = 100. The  $\{\tilde{\varepsilon}_{1i}\}$  and  $\{\tilde{\varepsilon}_{2i}\}$  are independently distributed.  $\rho_0$  is the correlation coefficient of  $\tilde{u}_i$  and  $\tilde{v}_i$  with values of  $\rho_0 = 0.0, 0.5,$  and  $0.9$  considered. The error terms are calculated twice, which is for the classical error terms as well as the fuzzy error terms.  $\rho$  from  $[-0.99, 0.99]$  with interval 0.01. The true values of the parameters are  $\beta_0 = -10.0,$   $\beta_1 = 1.0,$   $\alpha_0 = -1.0,$  and  $\alpha_1 = 0.1,$

Our hypothesis is  $H_0: \beta_1 = 0$  against  $H_1: \beta_1 \neq 0$ . If the hypothesis testing fails to reject  $H_0$ , meaning that the model does not reflect our data. The sample sizes  $n=100, 200,$  and  $400$  are considered with 1000 replications on each sample size.

True value is the actual variation that would be measured. In this case, so that the expected results are similar to the entered value, the true value for  $\beta_0$  and  $\beta_1$  are  $-10.0$  and  $1.0$  is expected to result from these estimates and values. The chosen sample size,  $n = 100, 200$  and  $400$  are only example values, this value is expected to meet the minimum value that should be taken, should we take another value, e.g. 101, 213, 379, etc. The larger the sample size and replication, the better. So as the normal random value is generated, this value is exemplified by a normal distribution (0.1) which means normal standards of raw materials, normal distribution (0.100) and the uniform normal distribution (0.20). This simulation will be measured in the range and value, so hopefully the results will be in the range of the accepted values.

## 5. Results

The calculation of Monte Carlo simulation from Table 1 to Table 4 using fuzzy  $\alpha$ -cut 0.2, 0.4, 0.5, 0.6, 0.8, and 1.0. The effect of correlation can be viewed from the fuzzy exogenous variables between  $\tilde{w}$  and  $\tilde{x}$  and the effect of correlation of fuzzy error terms between  $\tilde{u}$  and  $\tilde{v}$ , and the effects of comparison among several different sample sizes are  $n = 100, n = 200,$  and  $n = 400,$  shown in the table on the columns 1, 2 and 3, while columns 4, 5, 6 and 7, 8, 9 show Mean, SD and RMSE of parameter  $\beta_0$  and  $\beta_1$ , where SD is standard deviation and RMSE is root mean square errors.

Result is shown from Table 1 to Table 4 are the Mean, SD, RMSE with  $n=100, 200$  and  $400$  for parameters of  $\beta_0, \beta_1$ . From this table, represented the  $\alpha$ -cuts of triangular fuzzy number with a value of 0.2, 0.4, 0.5, 0.6, and 0.8. The first column shows the parameter of phi with values 0.0, 0.5, 0.9, while the second column shows the parameter of rho with a value of 0.0, 0.5, 0.9.

Table 1 shows that the mean of parameters of  $\beta_0$  and  $\beta_1$  showed about the consistency information of the parameter estimator that approaches the true values, while the SD and RMSE provide information about the level of efficiency for the parameters in the estimation. Sample size  $n = 100, n = 200$  and  $n = 400$  provides the information that the larger the sample size, the smaller the values of SD and RMSE, it means that more efficient and accurate the estimator, if there is no relationship between fuzzy error term. For example in Table 1, for the sample size  $n = 100,$  the values of mean of parameters  $\beta_0 = -10.041$  and  $\beta_1 = 1.004,$  the values of SD = 4.325, RMSE = 4.325 of parameter of  $\beta_0$  and the values of SD = 0.260,

RMSE = 0.260 of parameter of  $\beta_1$ , for the sample size  $n = 200$ , the values of mean of parameters  $\beta_0 = -9.831$  and  $\beta_1 = 0.991$ , the values of SD = 2.964, RMSE = 2.969 of parameter of  $\beta_0$  and the values of SD = 0.171, RMSE = 0.171 of parameter of  $\beta_1$ , while for the sample size  $n = 400$ , the values of mean of parameters  $\beta_0 = -9.969$  and  $\beta_1 = 1.001$ , the values of SD = 2.116, RMSE = 2.116 of parameter of  $\beta_0$  and the values of SD = 0.121, RMSE = 0.121 of parameter of  $\beta_1$ .

Result of Mean, SD, RMSE of FMPSSM with  $n=400$  for  $\beta_0$ ,  $\beta_1$  and  $\alpha$ -cut = 0.2 From Table 1, it can be shown that for the  $\alpha$ -cut = 0.2, with sample size  $n = 100$ , where values of  $\rho_0$  and  $\pi$  are 0 (meaning that there is no relationship between  $\rho$  and  $\pi$ ), so the value becomes small of SD and RMSE for parameter of  $\beta_0$ , the values of SD = 4.325, RMSE = 4.325 and for parameter of  $\beta_1$ , the values of SD = 0.260, RMSE = 0.260. When the sample size increases and become  $n = 200$ , SD and RMSE values for the parameters  $\beta_0$  and  $\beta_1$  decreases, i.e. for value of SD = 2.964, RMSE = 2.969 of parameter  $\beta_0$ , and the value of SD = 0.171, RMSE = 0.171 of the parameter  $\beta_1$ . When the sample size increases again for  $n = 400$ , SD and RMSE values for the parameters of  $\beta_0$  and  $\beta_1$  is smaller again, that is the values of SD = 2.116, RMSE = 2.116 for parameter of  $\beta_0$  and the values of SD = 0.121, RMSE = 0.121 for parameter of  $\beta_1$ . While  $\rho_0$  increases to moderate values of 0.5 (moderate correlation between fuzzy error term), then the values of SD and RMSE becomes larger than when the value of rho = 0 (no error relationship between fuzzy terms).

Increase in value of  $\rho_0$  as shown in Table 1 indicates that the greater correlation of fuzzy error terms between  $\tilde{u}$  and  $\tilde{v}$ , the smaller the value of SD and RMSE. From the above table also illustrates, that when the value of  $\rho_0$  and  $\pi$  strong ( $\rho_0$  and  $\pi = 0.9$ ), then the values of SD and RMSE increased, compared with the value of which no correlation of  $\rho_0$  and  $\pi$  are moderate. This shows that the occurrence of multicollinearity of fuzzy exogenous variables between  $\tilde{w}_i$  and  $\tilde{x}_i$ , but the value of this multicollinearity will be corrected by sample size that continues to expand.

**Table 1.** Mean, SD, RMSE of FMPSSM under normality assumption with  $n=100, 200, 400$  for  $\beta_0, \beta_1$  and  $\alpha$ -cut = 0.2

$\pi$	$\rho_0$	n	$\beta_0$			$\beta_1$		
			Mean	SD	RMSE	Mean	SD	RMSE
0.0	0.0	100	-10.041	4.325	4.325	1.0039	0.2598	0.260
		200	-9.831	2.964	2.969	0.991	0.171	0.171
		400	-9.969	2.116	2.116	1.001	0.121	0.121
	0.5	100	-9.759	6.628	6.633	0.999	0.358	0.358
		200	-9.966	4.594	4.594	1.011	0.265	0.265
		400	-9.967	3.133	3.133	0.999	0.178	0.178
	0.9	100	-9.773	6.244	6.249	1.009	0.323	0.324
		200	-9.861	4.074	4.077	1.001	0.224	0.224
		400	-9.967	2.835	2.835	1.007	0.159	0.159
0.5	0.0	100	-9.967	4.151	4.151	1.017	0.364	0.364
		200	-9.959	2.716	2.716	1.001	0.245	0.245
		400	-10.079	1.876	1.877	1.015	0.171	0.172
	0.5	100	-9.604	6.036	6.049	1.002	0.532	0.532
		200	-9.876	4.023	4.025	1.006	0.371	0.371
		400	-9.960	2.911	2.911	1.005	0.253	0.253
	0.9	100	-9.690	5.555	5.563	1.006	0.437	0.437
		200	-9.785	3.760	3.766	1.011	0.327	0.327
		400	-10.009	2.553	2.553	0.997	0.227	0.227
0.9	0.0	100	-10.038	11.772	11.772	1.012	1.799	1.799
		200	-10.149	5.665	5.667	0.996	1.239	1.239
		400	-9.999	3.345	3.345	1.010	0.880	0.880
	0.5	100	-9.648	14.229	14.233	1.024	2.522	2.522

		200	-9.555	7.524	7.537	1.057	1.816	1.817
		400	-9.542	4.800	4.821	1.101	1.292	1.296
	0.9	100	-10.032	13.081	13.081	0.900	2.364	2.366
		200	-10.046	6.554	6.554	0.957	1.505	1.506
		400	-9.705	4.165	4.175	0.975	1.093	1.093

Table 1 shows that the mean of parameters of  $\beta_0$  and  $\beta_1$  showed about the consistency information of the parameter estimator that approaches the true values, while the SD and RMSE provide information about the level of efficiency for the parameters in the estimation. Sample size  $n = 100$ ,  $n = 200$  and  $n = 400$  provides the information that the larger the sample size, the smaller the values of SD and RMSE, it means that more efficient and accurate the estimator, if there is no relationship between fuzzy error term. For example in Table 1, for the sample size  $n = 100$ , the values of mean of parameters  $\beta_0 = -10.0969$  and  $\beta_1 = 0.9967$ , the values of SD = 4.6709, RMSE = 4.6719 of parameter of  $\beta_0$  and the values of SD = 0.2530, RMSE = 0.2530 of parameter of  $\beta_1$ , for the sample size  $n = 200$ , the values of mean of parameters  $\beta_0 = -9.9669$  and  $\beta_1 = 0.9984$ , the values of SD = 3.0880, RMSE = 3.0882 of parameter of  $\beta_0$  and the values of SD = 0.1737, RMSE = 0.1737 of parameter of  $\beta_1$ , while for the sample size  $n = 400$ , the values of mean of parameters  $\beta_0 = -9.9701$  and  $\beta_1 = 1.0020$ , the values of SD = 2.1179, RMSE = 2.1182 of parameter of  $\beta_0$  and the values of SD = 0.1249, RMSE = 0.1249 of parameter of  $\beta_1$ .

From Table 1, it can be shown that for the  $\alpha$ -cut = 0.2, with sample size  $n = 100$ , where values of  $\rho_0$  and  $\pi$  are 0 (meaning that there is no relationship between  $\rho$  and  $\pi$ ), so the value becomes small of SD and RMSE for parameter of  $\beta_0$ , the values of SD = 4.6709, RMSE = 4.6719 and for parameter of  $\beta_1$ , the values of SD = 0.2530, RMSE = 0.2530. When the sample size increases and become  $n = 200$ , SD and RMSE values for the parameters  $\beta_0$  and  $\beta_1$  decreases, i.e. for value of SD = 3.0880, RMSE = 3.0882 of parameter  $\beta_0$ , and the value of SD = 0.1737, RMSE = 0.1737 of the parameter  $\beta_1$ . When the sample size increases again for  $n = 400$ , SD and RMSE values for the parameters of  $\beta_0$  and  $\beta_1$  is smaller again, that is the values of SD = 2.1179, RMSE = 2.1182 for parameter of  $\beta_0$  and the values of SD = 0.1249, RMSE = 0.1249 for parameter of  $\beta_1$ . While  $\rho_0$  increases to moderate values of 0.5 (moderate correlation between fuzzy error term), then the values of SD and RMSE becomes larger than when the value of  $\rho = 0$  (no error relationship between fuzzy terms).

**Table 2.** Mean, SD, RMSE of FMPSSM under normality assumption with  $n=100, 200, 400$  for  $\beta_0, \beta_1$  and  $\alpha$ -cut = 0.4

$\pi$	$\rho_0$	n	$\beta_0$			$\beta_1$		
			Mean	SD	RMSE	Mean	SD	RMSE
0.0	0.0	100	-9.775	4.539	4.545	0.988	0.249	0.249
		200	-9.911	3.191	3.193	0.999	0.180	0.180
		400	-10.085	2.071	2.073	1.003	0.121	0.121
	0.5	100	-9.515	6.681	6.698	0.982	0.386	0.386
		200	-10.077	4.530	4.530	1.006	0.259	0.259
		400	-10.076	3.225	3.226	0.997	0.180	0.180
	0.9	100	-9.978	5.893	5.893	1.005	0.322	0.322
		200	-9.630	4.182	4.199	0.991	0.230	0.231
		400	-9.972	2.865	2.865	1.003	0.157	0.157
0.5	0.0	100	-10.108	4.383	4.385	1.000	0.363	0.363
		200	-10.065	2.685	2.686	1.006	0.256	0.256
		400	-9.924	1.787	1.789	0.995	0.174	0.174
	0.5	100	-10.071	6.140	6.140	1.001	0.539	0.539
		200	-9.872	4.042	4.044	0.998	0.365	0.365
		400	-10.014	2.850	2.851	0.997	0.252	0.252
	0.9	100	-9.730	5.379	5.386	0.995	0.449	0.449

		200	-10.037	3.785	3.785	0.990	0.311	0.311
		400	-10.065	2.513	2.514	1.008	0.222	0.222
0.9	0.0	100	-10.267	10.585	10.589	1.011	1.855	1.855
		200	-9.653	5.499	5.509	1.066	1.217	1.219
		400	-10.024	3.381	3.381	0.978	0.884	0.884
	0.5	100	-9.198	13.551	13.575	0.982	2.595	2.595
		200	-9.931	9.042	9.043	0.945	1.868	1.869
		400	-10.145	4.712	4.714	0.946	1.278	1.279
	0.9	100	-9.866	11.909	11.910	1.036	2.252	2.252
		200	-9.313	7.034	7.067	1.121	1.593	1.598
		400	-10.096	4.100	4.101	0.946	1.109	1.110

**Table 3.** Mean, SD, RMSE of FMPSSM under normality assumption with n=100, 200, 400 for  $\beta_0$ ,  $\beta_1$  and  $\alpha$ -cut = 0.6

$\pi$	$\rho_0$	n	$\beta_0$			$\beta_1$		
			Mean	SD	RMSE	Mean	SD	RMSE
0.0	0.0	100	-9.961	4.339	4.340	1.000	0.253	0.253
		200	-9.960	3.148	3.148	1.010	0.180	0.181
		400	-9.922	2.042	2.043	1.000	0.123	0.123
	0.5	100	-9.944	6.614	6.614	1.003	0.369	0.369
		200	-10.224	4.475	4.481	1.012	0.254	0.255
		400	-1.000	3.051	3.051	0.997	0.175	0.175
	0.9	100	-9.782	6.230	6.233	1.001	0.328	0.328
		200	-9.998	3.980	3.980	0.996	0.222	0.222
		400	-9.915	2.735	2.736	0.994	0.161	0.161
0.5	0.0	100	-10.112	3.875	3.876	1.005	0.362	0.362
		200	-10.066	2.675	2.676	0.985	0.251	0.251
		400	-9.901	1.881	1.884	0.993	0.178	0.178
	0.5	100	-9.858	5.986	5.987	0.996	0.556	0.556
		200	-9.816	4.018	4.022	1.000	0.358	0.358
		400	-10.060	2.735	2.736	0.998	0.260	0.260
	0.9	100	-9.599	5.601	5.615	1.011	0.440	0.440
		200	-9.848	3.804	3.807	0.996	0.298	0.298
		400	-9.888	2.564	2.566	0.989	0.217	0.217
0.9	0.0	100	-10.226	11.238	11.241	0.959	1.761	1.761
		200	-10.105	5.645	5.646	0.967	1.293	1.293
		400	-10.091	3.203	3.205	1.023	0.851	0.851
	0.5	100	-10.206	13.940	13.942	0.923	2.572	2.573
		200	-9.901	8.333	8.333	1.026	1.829	1.829
		400	-10.052	5.030	5.030	0.902	1.331	1.335
	0.9	100	-10.008	13.863	13.863	0.946	2.307	2.307
		200	-10.068	6.653	6.653	0.929	1.581	1.583
		400	-9.852	4.309	4.311	1.030	1.142	1.142

**Table 4.** Mean, SD, RMSE of FMPSSM under normality assumption with n=100, 200, 400 for  $\beta_0$ ,  $\beta_1$  and  $\alpha$ -cut = 0.8

$\pi$	$\rho_0$	n	$\beta_0$			$\beta_1$		
			Mean	SD	RMSE	Mean	SD	RMSE
0.0	0.0	100	-10.005	4.430	4.430	0.995	0.254	0.254
		200	-10.063	3.007	3.008	0.998	0.180	0.180
		400	-9.995	2.099	2.099	1.004	0.130	0.130

	0.5	100	-9.662	6.554	6.563	0.987	0.366	0.366	
		200	-9.887	4.490	4.491	0.999	0.266	0.266	
		400	-9.815	3.134	3.139	0.999	0.181	0.181	
	0.9	100	-9.869	5.954	5.956	1.018	0.326	0.327	
		200	-9.884	4.202	4.204	1.003	0.222	0.222	
		400	-10.172	2.719	2.725	1.006	0.153	0.153	
0.5	0.0	100	-9.886	3.999	4.000	0.996	0.356	0.356	
		200	-10.121	2.758	2.761	0.991	0.245	0.246	
		400	-9.969	1.883	1.884	1.007	0.176	0.176	
	0.5	100	-9.803	6.135	6.138	0.996	0.523	0.523	
		200	-10.068	4.029	4.030	0.996	0.369	0.369	
		400	-10.062	2.766	2.766	0.988	0.251	0.251	
	0.9	100	-9.766	5.583	5.588	1.022	0.451	0.452	
		200	-9.795	3.692	3.698	0.985	0.316	0.316	
		400	-9.902	2.486	2.488	0.998	0.214	0.214	
	0.9	0.0	100	-10.102	10.428	10.429	0.995	1.805	1.805
			200	-9.960	5.367	5.367	1.039	1.280	1.281
			400	-10.048	3.378	3.378	0.984	0.916	0.916
0.5		100	-9.112	14.978	15.005	1.017	2.707	2.708	
		200	-9.376	7.830	7.855	1.076	1.823	1.825	
		400	-10.149	5.046	5.048	0.926	1.301	1.303	
0.9		100	-9.177	14.470	14.493	1.058	2.217	2.217	
		200	-9.660	6.942	6.950	1.012	1.631	1.631	
		400	-9.832	4.101	4.104	1.015	1.093	1.093	

The consistency for FMPSSM and FMSPSSM has been discussed. The effect of the correlation of variables  $\tilde{x}_i$  and  $\tilde{w}_i$ , then the effect of the correlation of error terms  $\tilde{u}_i$  and  $\tilde{v}_i$  are then observed. The FMSPSSM has used the bandwidth by the Powell estimator. The effects of bandwidth changes are studied and researched, and later observed whether it is consistent or not. Table 1 to Table 4 are calculation of FMPSSM for Mean, SD, RMSE under normality assumption, The values of SD and RMSE decreased with increased sample size, and also decreases with increasing values of  $\alpha$ -cut.

## 6. Conclusion

To reduce the problem of uncertainty that exists in the parametric sample selection models, then created a fuzzy approach. fuzzy concept that used for modified of the parametric sample selection models provides an alternative for the handle to the problem when the model involves the characteristic vagueness, uncertainty and ambiguity. Result of the Parametric Sample Selection Models using a fuzzy approach shows that the larger the sample size, the smaller the values of SD and RMSE, it means that the more efficient and accurate the estimator, then the greater the value of  $\alpha$ -cut, the smaller the values of SD and RMSE, and the more efficient and accurate.

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