Identifying the Nonlinearity of Structures Dynamics by Wavelet Packet Decomposition

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Abstract. In this study, an application of Wavelet Packet Decomposi (WPD) for modal analysis of a circular cylinder supported by two elastic cantilever beams and wires was developed. The Frequency Response Functions (FRFs) analysis was typically calculated by means of Fourier transform method that worked well with linear systems but had limitations when nonlinearities were present mainly due to their inability to examine the non-stationary data. More recently, a WPD-based technique that calculates complex, time-varying FRFs for input/output relationships has been developed. This method represents a unique fundamental advance regarding time-frequency measurement techniques, since, time-varying transfer function is computed while the direct response time-frequency decomposition on wavelets is not, which have been introduced as an alternative method to FRF calculation.

Keywords: *Nonlinearity of structure, damping estimation, natural frequency, wavelet packet decomposition*

1. Introduction

The dynamic response and the nonlinearity vibration of structure to the surroundings are critically determined by the damping mechanisms, and its value is very important in designing and analysing the vibrating structures. When the structure is modelled, the stiffness and mass distributions are normally well determined, but there is usually great uncertainty regarding the energy dissipating mechanism provided by the damping structure because it is the least well-understood aspect. However, to validate these models, the damping must be estimated by applying an experimental modal analysis.

Fast Fourier transform and Hibert transform are effective methods for detecting the nonlinearity of the structures. However, these methods are difficult in detecting the nonlinearity of harmonic response of damping structured system which has the weak and softening nonlinearity. In this study, we used wavelet packet decomposition to identify the modal parameter of a structure system. Many studies have utilized the wavelet analysis in system-identification applications. For instance, frequency localization properties allow the detection and decoupling of individual vibration modes of Multi-degree-of-freedom (MDOF).

The wavelet transform has been promoted as an elegant multi-resolution signal processing tool [1]. Mohallem and his colleagues presented a study about WPD to identify the linear system in frequency subband in the longitudinal flexible model of a simulated aircraft [2]. Wavelet analysis of the free response of a system allows the estimation of the natural frequency and viscous damping ratios [3]. The wavelet analysis to the free response of the system represents a good improvement for the technique based on Hilbert transform. When the identification technique is performed on a data, the estimated natural frequency of wavelet will be agreed well. Analytic wavelet transforms based on the identification of modal parameter system has been able to predict the utility distribution system damping parameters efficiently and precisely [4]. The experimental modal analysis is the analysis of the structural dynamic properties in terms of its modal parameters. Frequency Response Function (FRF) of mechanical structures can be identified by using modal testing techniques. The structure is excited by exerting force, and the response is measured with vibration sensors.

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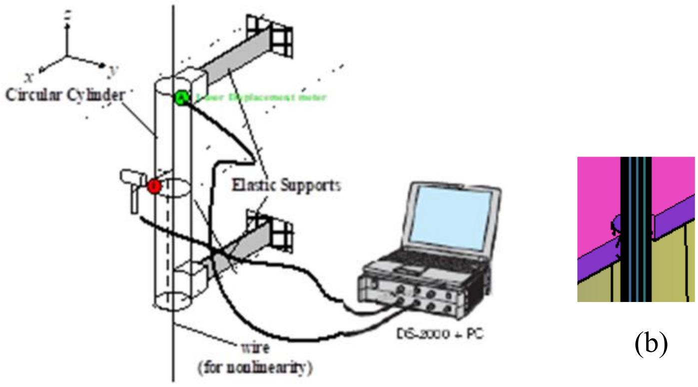
2. Experimental Techniques

In this study, we used a cylinder that is supported by two clamped beams and a wire as shown in Figure 1 (a). A circular plate with a small hole is attached at the centre of the cylinder. There is a clearance between the wire and the circular plate with a small hole (Dia. 0003 m), which is attached at the centre of the cylinder as shown in Figure 1 (b). The test cylinder is a polycarbonate tube with cross-section of outer diameter, D = 0.045 m, length, L = 0.32 m, and mass, m = 0.06642 kg. Beams are made of stainless steel with the length of 0.185 m, the height of 0.03 m, the thickness of 0.006 m and the Young's modulus of 206 GPa. The material of the wire is SWP-A with Young's modulus of 208.1 GPa and tensile strength of 1600 MPa.

The Laser displacement meter (Keyence LB-040/LB-1000) was used for detecting the change in the system displacement. Simultaneously, the impulse force generated by the hammer was recorded; an impact is acted at the point 1 of the cylinder. In our measurement, the sampling frequency was 100 kHz, and the sampling times were 1024. The output signal from the laser displacement meter (point A) was simultaneously acquired using an FFT analysis (Ono-sokki DS200), which allowed a direct vibration amplitude read-out (acceleration, velocity or displacement). The output of the FFT analysis was in turn fed to a computer via an interface card.



(a)

(b)

Figure 1. An elastically supported cylinder with a wire

3. Identification of the structure with wire system

In this section, the implementation of wavelet packet decomposition to obtain the forced vibration response of the nonlinear SDOF system was subjected to the vector of amplitude harmonic excitation (L). This system can be expressed as follow:

$$m\ddot{x} + c\dot{x} + kx + f_{nl}(x,\dot{x}) = L(t) \ , \tag{1}$$

where m, c, k, and $f_{nl}(x,\dot{x})$ respectively are the mass, the damping, the stiffness and the nonlinear terms which depend on the spatial displacements x and their derivatives. The main problem is the recast in the next subspace spanned by Daubechies scaling function. The appropriate expansion [6] for the response $x(t)$ and the excitation $L(t)$ is given by

$$x(t) = \sum_{k=I}^{J} \overline{\alpha}_k \, 2^{j/2} \, \omega\left(2^j t - k\right) \text{ and } L(t) = \sum_{k=I}^{J} \overline{\beta}_k \, 2^{j/2} \, \omega\left(2^j t - k\right) \tag{2}$$

Substituting the analytic signal forms of $x(t)$ with the two derivatives of the wavelet function, i.e.

$$\dot{x}(t) = 2^j \sum_{k=I}^{J} \overline{\alpha}_k \, 2^{j/2} \, \dot{\omega}\left(2^j t - k\right) \text{ and } \ddot{x}(t) = 2^{2j} \sum_{k=I}^{J} \overline{\alpha}_{j,k} \, 2^{j/2} \, \ddot{\omega}\left(2^j t - k\right) \tag{3}$$

into Eq. (1), one can derive the representation of the corresponding modal parameters

$$m\,2^{2j}\sum_{k=I}^{J}\bar{\alpha}_{j,k}\,2^{j/2}\,\ddot{\omega}\left(2^{j}t-k\right)+c\,2^{j}\sum_{k=I}^{J}\bar{\alpha}_{k}\,2^{j/2}\,\dot{\omega}\left(2^{j}t-k\right)+k\,2^{j/2}\,\omega\left(2^{j}t-k\right)+f_{nl}(x,\dot{x})=\sum_{k=I}^{J}\bar{\beta}_{k}\,2^{j/2}\,\omega\left(2^{j}t-k\right)\tag{4}$$

When $m = 0$ is Daubechies scaling function with the number of vanishing moments N, $\varphi_{j,k}(t)$ is a compact support in [2-jk ,2-j (k +2N-1)], the lower (I) and upper (J) bounds component to the Eqs. [3, 4] so that it was obtained $I = k_0$-L +2 and $J = k_1$-1, where L=2N, k0 = $2^j t_0$ and $k_1=2^j t_f$ are integer values and t_0 and t_f denote respectively, the in initial and final time. The function $f_{nl}(x,\dot{x})$ can be assumed as:

$$f_{nl}(x,\dot{x})=d\,x\,\dot{x}\tag{5}$$

The product $(x,\dot{x})$ in the Eq. 3 can be estimated at level j by

$$\sum_{m}\sum_{k}a_{j,m}(x)a_{j,k}(x)\Omega_{j,k,l}^{j,m,m(1,0)}\tag{6}$$

with three terms of the inner product methods in the differential operation that can be defined as

$$\Omega_{j,k,l}^{j,m,m(1,0)}=\int_{0}^{2n-m}\varphi(y-m)\dot{\varphi}(y-k)\varphi(y-l)dy\tag{7}$$

The assumption of the Eq. 8 into Eq. 6 can be shown as follow:

$$m\sum_{k}\alpha_{j,k}(x)\Gamma_{j,k}^{j,l^{(2)}}+c\sum_{k}\alpha_{j,k}(x)\Gamma_{j,k}^{j,l^{(1)}}+k\,\alpha_{j,k}(x)+d\sum_{m}\sum_{k}\alpha_{j,m}(x)\alpha_{j,k}(x)\Omega_{j,k,l}^{j,m,m^{(1,0)}}=\alpha_{j,k}(L)\tag{8}$$

where $\alpha_{j,k}(x)$, $\alpha_{j,l}(x)$, and $\alpha_{j,k}(L)$ respectively refer to the scaling coefficients of the displacement and the excitation at discrete translations k and l at level j. The equation 8 is the inner product of the wavelet function, where the terms $\Gamma_{k}^{(n)}$ in the latter expression denote the so-called 2 whiles $\omega^{(n)}(t)$ is n-the derivatives of wavelet function $\omega(t)$. The wavelet function $\omega(t)$ cannot be defined in the explicit form, but the differentiation method can be defined with some n. In identifying an unknown non-linearity, it is suggested to try several types of non-linearity. The accuracy of the model can be compared with the response generated by experimental excitation. The implements of wavelet to obtain the response of MDOF nonlinear systems and weak nonlinearities as formulated in the equation 9 following these approximations with z DOFs, can be written as follow:

$$m\sum_{k}\alpha_{j,k}(x_f)\Gamma_{j,k}^{j,l^{(2)}}+c\sum_{k}\alpha_{j,k}(x_f)\Gamma_{j,k}^{j,l^{(1)}}+k\,\alpha_{j,k}(x_f)+d\sum_{m}\sum_{k}\alpha_{j,m}(x_i)\alpha_{j,k}(x)\Omega_{j,k,l}^{j,m,m^{(1,0)}}=(L_f)\tag{9}$$

The matrix, $\Gamma_{j,k}^{j,l^{(1)}}$ and the equation can be written as

$$\tilde{\ddot{x}}_{lf}=-\lambda_f^2\tilde{x}_{lf}\qquad \tilde{\dot{x}}_{lf}=-i\lambda_f\tilde{x}_{lf}\ \text{ where }\ \tilde{x}_{lf}=\Phi^{-1}x_{lf}\qquad \tilde{\ddot{x}}_{lf}=\Phi^{-1}\ddot{x}_{lf}\qquad \tilde{\dot{x}}_{lf}=\Phi^{-1}\dot{x}_{lf}\tag{10}$$

After substituting the Eq. 10 into Eq. 9, we could get:

$$\lambda_{j,k}^2 m\sum_{k}\alpha_{j,k}(x_f)-i\lambda_{j,k}c\sum_{k}\alpha_{j,k}(x_f)+k\,\alpha_{j,k}(x_f)+d\sum_{m}\sum_{k}\alpha_{j,m}(x_f)\alpha_{j,k}(x_f)\Omega_{j,k,l}^{j,m,m^{(1,0)}}=(L_f)\tag{11}$$

The governing equation for an MDOF with z-DOFs is given the equation which is mentioned earlier. Here, each displacement is approximated in time using Daubechies scaling function as shown in Eq. 11. Equation 11 can be used to obtain x_{lf} with f = 0, 1, 2, ... z-1 and l = 0, 1, 2, …, z.

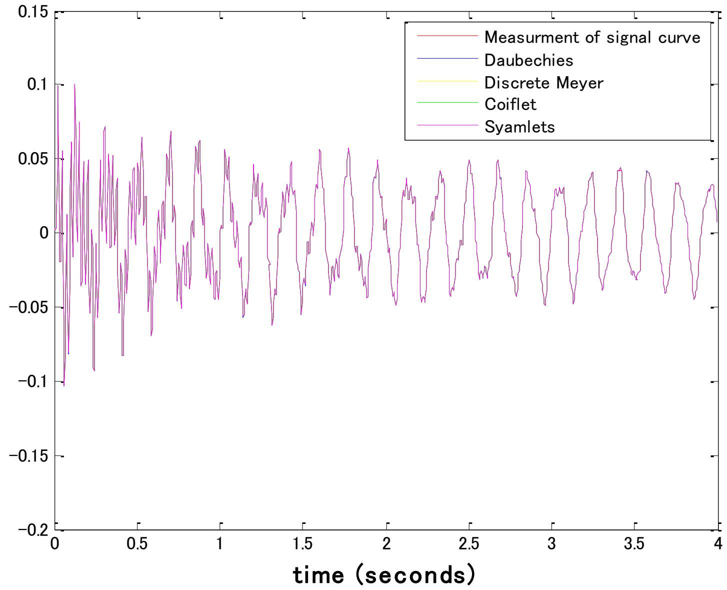
4. Results and Discussion

This section summarizes the result of wavelet packet decomposition for natural frequency and damping estimation of the cylinder that is supported by two clamped beams and a wire. We used SNR which was equal to ∞ dB, 30 dB and 40 dB. The decomposition level was set at 2 therefore $2^2 = 4$-coefficient sets are generated at the wavelet toolbox in the Matlab.

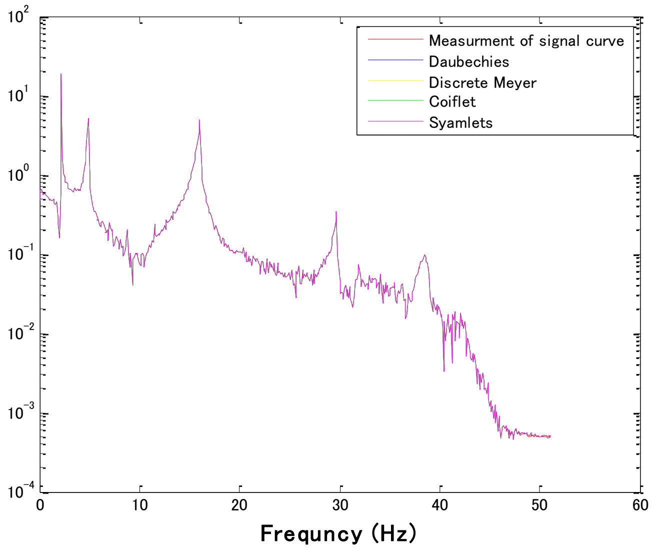
The response was measured in the horizontal direction by means of laser displacement sensor (type LB-040/LB-1000) at the cylinder and beam. Simultaneously, the impulse force generated by the hammer was recorded. The vertical vibration and the curve-fitting resulted from a library of wavelet packet decomposition bases at cylinder (point A) shown in Figure 2 (a). The figure 2 (b) shows the frequency response function of vertical vibration in Figure 2 (a). The analysis of the displacement response and frequency response functions can use wavelet packet family obtained by adding the effects of two selected nodes with the damping ration, which is 0.0202 with no noise. The results showed a better accuracy of estimation even for data measurement with noise. By decreasing the SNR from ∞ dB to 40 dB in the case of the second mode, the percentage error slightly deviated from -11 to 6.654.

The point frequency response functions have also been carried out for single to the three-dof system with the same locations which were measured. The point FRFs corresponding to 1, 2, and 3 DOFs systems are presented in figure 3 (b).

Based on the FRF presented in this figure, there is a large frequency interval between 20 and 30 Hz within the test structure which has no natural frequency. The amplitude of different modal components of the response varies with the different degrees of freedom when the frequency response function is measured.



(a) Displacement response



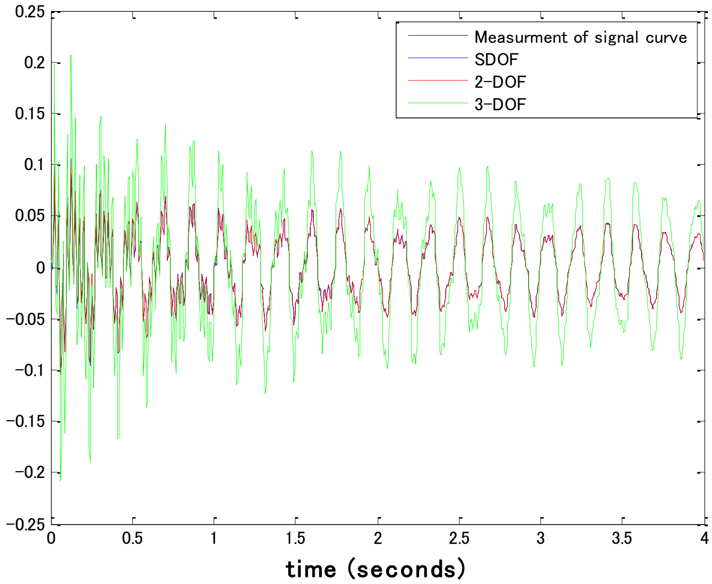
(b) FRF

Figure 2. (a) Displacement response; and (b) FRF with wavelet packet family

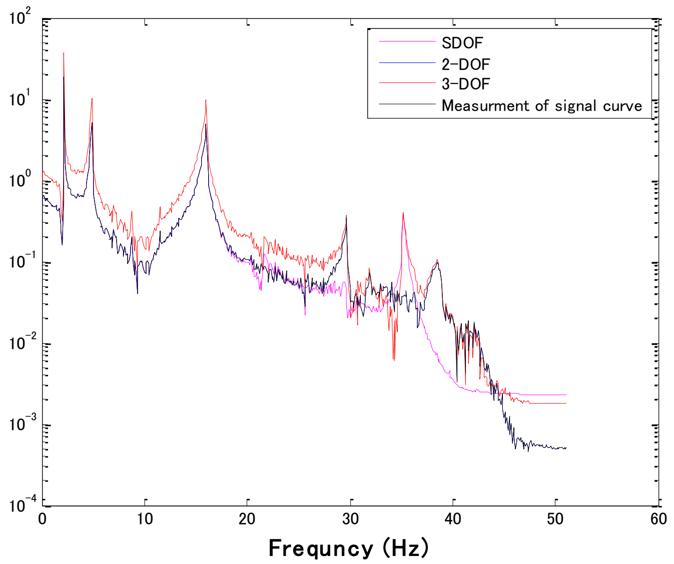
The effect of all modes with a natural frequency of above 20 Hz is relatively small with an interval ranging from 0 to 30 Hz. It was considered that the behaviour of the system could be adequately described in terms of the natural frequency below 20 Hz. Thus, the range of characterization frequency was set from 0 to 20.

The amplitudes of the different modal components of the response vary with the DOF at which the FRF point is measured. The magenta line, corresponding to the point FRF at a single-DOF system, showed that in this location, the fourth mode was not effectively excited, whereas the red line, corresponding to the 3-DOF system, indicates that none of the four modes can be excited effectively from this location.

The blue line corresponds to a point FRF at the 2-DOF system. The entire mode appears to be well excited by this method. However, the peak corresponding from the first to the fifth modes is seeing to correspond to the same frequency as that of the FRF for measurement signal curve. Figure 3 shows the mathematical model of this system which can be used with the 2-dof system.



(a) Details of the time series



(b) Frequency response function

Figure 3. (a) Displacement response; (b) FRF at Daubechies wavelet packet with different DOFs

5. Conclusions

This study attempts to develop a wavelet packet-based method that allows the identification of system parameters directly from data generated through a dynamic test structure, for its application into systems with multi-degree-of-freedom. Here, the use of wavelet packets revealed accurate results of frequency calculation and damping estimation. Moreover, the types of these wavelet packets were suitable for testing the elastic dynamic structure supported with beam and a wire. However, the most fundamental problem was the determination of the appropriate types of wavelet packets and their parameters, as well as the computation which still becomes a challenge for signal analysts. Currently, all basic type wavelet packets seem reasonable and provided adequate results.

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