

Fixed Transmission Cost Allocation Based on a Nucleolus Solution with Economic Incentive on Power Market

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Abstract. The economic incentive of fixed transmission cost under power market is investigated in this paper. The fixed transmission cost allocation based on cooperate game model gives the economic signals of the difference of loads, which stimulates the customers to choose the reasonable way to get the power. The nucleolus is the only one solution with character of exit and unique in all cooperate game solution. The fixed transmission cost is allocated to each customer according to the nucleolus. The economic attribute of nucleolus also determines the responsibility of customers for the allocation. What is more, the fixed transmission cost allocated to one customer will change if different generator is chosen to supply power to this customer. So customers choose the generators economically and reasonably. The effectiveness of the economic stimulation based on the nucleolus solution for fixed transmission cost allocation is verified by the 5-bus system.

1. Introduction

Under power deregulation, customers can choose an economical way to get electricity. As for the transmission cost, a large part is fixed cost. So when the customer choose, the fixed cost allocated to this customer is an important reference to him or her. The customer's load structure, geographical location and the distribution of surrounding power will cause the difference between the customer's influence on the transmission network and the degree of use. So the indiscriminate charge to customer through some kinds of fixed cost allocation method cannot generate economic incentives for the customers. The customers are enthusiasm of participate in the power market.

There are many ways to allocate fixed costs of transmission grid, each with its own strengths. A class of methods based on cooperative games has a good economic and mathematical background [1, 2]. Only the nucleolus solution always exists in the concept of cooperative games and defines a unique fixed cost allocation vector. As for promoting sustainable development of electricity, alleviating pressure on power supply and implementing demand side management, motivating customers to save electricity is an important aspect. In terms of the transmission grid, the cost-effective signal encourages the customer to choose the appropriate transmission way, and can also reduce the transmission pressure of the grid and avoid the transmission congestion.

This paper analyzes the fixed cost allocation of transmission grids using nucleolus solution show the economic incentives to customers. The nucleus solution clarifies the difference between fixed costs allocated by different customers in different ways of transmission, and encourages customers to choose the most cost-saving way.



2. The nucleolus of cooperative game

An important issue in the cooperative game settlement is to find one or a group of assignments, so that each player in the group gets their payment according to this group of assignments, and everyone has no opposition. Among the various solutions of the cooperative game, only the nucleolus solution must exist and be unique, and it also has symmetry, which satisfies the individual rationality and collective rationality. The individual rationality conditions indicates that the income of each player in each group is at least not less than the income of his individual. The collective rationality indicates that the sum of the incomes distributed by each player is exactly equal to the total maximum income of various alliance forms. Therefore, the nucleolus solution can always give the customer cost allocation with a personal maximal income.

Define a model $\Gamma(N, v)$ of cooperative game with n players, v is a feature function, $N = \{1, 2, \dots, n\}$ is a set of n players, $S \subset N$ called an alliance. For each subset S of the game, the function value $v(S)$ is the maximum return value that alliance S can achieved. When the players in S become an alliance, regardless of the strategy adopted by the players outside S , the players in S can always achieved this maximum return. Let $\mathbf{x}(I) = \{\mathbf{x} \in R^n \mid x_i \geq v(\{i\}), i=1, 2, \dots, n; x(N) = v(N)\}$, $\mathbf{x} \in \mathbf{x}(I)$ is the column vector, \mathbf{x} is called an allocation, and the allocation of this game is denoted as $E(v)$.

The idea of nucleolus solution is as follows. First, the allocation is prior determined in the most dissatisfied alliances. This allocation must minimize the complaints of such alliances. Then, this allocation is used for less dissatisfied alliances, and it also should minimize the dissatisfaction as small as possible of such less dissatisfied alliances. And so on, at last all alliance dissatisfaction is as small as possible [3]. The attitude of the union S to the allocation \mathbf{x} is expressed by the excess value $e(S, \mathbf{x}) = v(S) - \mathbf{x}(S)$. And the larger the excess value, the more unsatisfied S is to \mathbf{x} . List the 2^n sub-alliances $S \subseteq N$ about the excess values of \mathbf{x} and arrange them in descending order to get a dimension vector.

$$\beta(\mathbf{x}) = (\beta_1(\mathbf{x}), \beta_2(\mathbf{x}), \dots, \beta_{2^n}(\mathbf{x})) \tag{1}$$

$$\beta_i(\mathbf{x}) = e(S_i, \mathbf{x}) (i = 1, 2, \dots, 2^n) \tag{2}$$

Where S_1, S_2, \dots, S_{2^n} is an arrangement of all subsets of N , satisfying

$$e(S_1, \mathbf{x}) \geq e(S_2, \mathbf{x}) \geq \dots \geq e(S_{2^n}, \mathbf{x}) \tag{3}$$

As for

$$\left\{ \begin{array}{l} \beta(\mathbf{x}) = (\beta_1(\mathbf{x}), \beta_2(\mathbf{x}), \dots, \beta_{2^n}(\mathbf{x})) \end{array} \right. \tag{4}$$

$$\beta(\mathbf{y}) = (\beta_1(\mathbf{y}), \beta_2(\mathbf{y}), \dots, \beta_{2^n}(\mathbf{y})) \tag{5}$$

If $1 \leq h \leq 2^n$ exists, make

$$\beta_i(\mathbf{x}) = \beta_i(\mathbf{y}), i=1, 2, \dots, h-1 \tag{6}$$

$$\beta_h(\mathbf{x}) < \beta_h(\mathbf{y}) \tag{7}$$

Then the vector $\beta(\mathbf{x})$ is said to be less than the vector $\beta(\mathbf{y})$ in lexicographic order, and is denoted as $\beta(\mathbf{x}) \stackrel{L}{<} \beta(\mathbf{y})$. Then $\beta(\mathbf{x}) \stackrel{L}{\leq} \beta(\mathbf{y})$ means either $\beta(\mathbf{x}) \stackrel{L}{<} \beta(\mathbf{y})$ or $\beta(\mathbf{x}) = \beta(\mathbf{y})$ [5, 6]. The nucleolus is the whole allocations that makes $\beta(\mathbf{x})$ lexicographically minimized, ie

$$\{\mathbf{x} \in E(v) \mid \forall \mathbf{y} \in E(v), \beta(\mathbf{x}) \stackrel{L}{\leq} \beta(\mathbf{y})\} \quad (8)$$

3. The fixed cost allocation with the nucleolus

The fixed cost of the transmission grid reflects the investment in the construction line such as transmission lines, substations on both sides, series compensators, static compensators, etc., including debt service, depreciation, return, taxes, inflation rates, insurance premiums, and so on. Some fixed costs of grid also include the operation and maintenance costs of the grid. In this paper, fixed costs mainly include the recycling of equipment investment, that is, one-time investment in the transmission grid and debt service.

The fixed cost allocation is expressed by the n persons cooperative game model $\Gamma(N, v)$. The grid customers are the players in the game, and the benefits caused by the formation of the alliance are characteristic functions. The calculation can be divided into the following steps.

(1) Discount the fixed cost of the transmission network to the trading period according to a certain economic calculation period and a certain discount rate, and determine the fixed cost shared by all grid customers per unit of current flow during each trading period. In the electricity market environment, the reactive service itself is billed separately as a type of service. Therefore, in the allocation of fixed costs, the customer's load is measured by active power. The fixed cost of the transmission grid allocated to each trade period is K . And the total load for all transactions in this period is P_{total} . So the unit fixed cost λ of the grid active power flow is [4].

$$\lambda = \frac{K}{P_{total}}$$

(2) Analyze and calculate the fixed cost C_i for each customer when they do not participate in any alliance. Let P_i be the active power flow of the grid when only transmitting power to customer i , then $C_i = \lambda P_i$. And in the same way, the total fixed cost C_S taken by the alliance S is $C_S = \lambda P_S$. P_S is the sum of the active powers of transmission for all members of the alliance.

(3) Due to the economies of scale of the transmission grid, all grid customers form a major alliance when all customers are participating in fixed cost allocation. And all players participate in this major alliance. The existence of reverse power flow on the transmission line makes the power flow generated by the grid transmission to the entire alliance smaller than the total power flow caused by the individual transmission to each customer. Therefore, the fixed cost incurred by the customer after participating in the alliance is also reduced. The difference is the participating income of each customer and each alliance, that is, the feature function $v(S)$.

$$v(S) = \sum_{i=1}^{n_S} C_i - C_S \quad (9)$$

Where n_S is the number of people participating in the alliance S .

(4) Applying the nucleolus solution of the cooperative game to determine the participating income of each customer. That is the reduced fixed cost amount x_i . Then the fixed cost allocated to customer i is

$$\left\{ \begin{array}{l} \{\mathbf{x} \in E(v) \mid \forall \mathbf{y} \in E(v), \beta(\mathbf{x}) \stackrel{L}{\leq} \beta(\mathbf{y})\} \\ C'_i = C_i - x_i, \quad C_i \geq x_i \\ C'_i = 0, \quad C_i < x_i \end{array} \right. \quad (10)$$

4. Economic incentive factor for cost allocation

For the transmission network in stable operation, due to the non-storability of electrical energy, the power generating by all generator should be just the sum of power using by all loads, transmission losses and other electrical energy maintaining the stability of the grid. When the power customer

increases the load, the generator must generating the corresponding power. In the environment of power monopoly, the generators are uniformly dispatched, the customers cannot select the generator that supplies power, and the same type of customer pays the power at a uniform price. Power deregulation, except for the lack of capacity of the unit and the safety of the grid, the customer can select the generator at any bus. This will inevitably produces different power flow in the grid. So the fixed cost of the customer sharing will also be different. According to economic incentive signal of the fixed cost allocation, the customer choose the power generator to enjoy the electricity service in the most cost-effective way.

The economy signal of fixed cost allocation based on nucleolus means that if the customer increase 1kW power and the generator choosed increase 1kW power, the corresponding increase of fixed cost allocated to this customer through nucleolus. The larger the fixed cost shared by the customer, the less economically the customer chooses to the generator at this bus, and the lower the selectivity of the generator at this bus.

When the grid is in normal operation, according to the nucleolus, the fixed cost of the transmission network undertaken by the power customer of bus j is C'_j . When this customer load increases Δp_j and the fixed cost allocated to it is C''_{jk} under the choose of the generator at bus k , the fixed cost increment of the customer's share is ΔC_{jk} , then $\Delta C_{jk} = C''_{jk} - C'_j$. The economic incentive of fixed cost based on the nucleolus for this customer is γ_{jk} , which is called an economic incentive factor. And $\gamma_{jk} = \Delta C_{jk} / \Delta p_j$. The physical meaning of this factor is the increment of fixed cost allocated to the increment of load, reflecting the economic benefit of choosing the generator at bus k by the customer. The smaller the value, the more economical the customer chooses the generator here.

5. Economic incentive factor algorithm based on DC power flow and nucleolus

Fixed costs have two economic incentives for customers. In the first case, there is a generator at the bus where the customer is also located, that is, $j=k$. When the customer load increases, the generator of the same bus has the ability to meet the load demand. At this time, the customer selects the same bus generator is obviously the most economical. And the increased load is digestion inside the bus, which will not cause an increase in the power flow in grid, and the power transmitted in grid will not increase. Therefore, the customer does not need to share additional fixed cost of the transmission grid for the increased load, γ_{jk} is zero.

In the second case, the customer bus does not have a generator or the power generation of the same bus is saturated. If the customer load increases, the generator supply of other bus must be selected to provide power, which will inevitably increase the power flow on the grid. In the power flow calculation, the customer load is active. According to the DC power flow model [5]

$$\mathbf{P} = \mathbf{B}\boldsymbol{\theta}$$

j, k are the grid buses, \mathbf{P} is injection power vector at bus, where the element $P_j = P_{Gj} - P_{Dj}$ ($j = 1, 2, \dots, n-1$, n is the equilibrium bus), P_{Gj} and P_{Dj} are the generator output and load respectively at the bus j . \mathbf{B} is the imaginary part of the bus admittance matrix, where the element $B_{jk} = -\frac{1}{x_{jk}}$,

$B_{jj} = \sum_{\substack{k \in j \\ k \neq j}} \frac{1}{x_{jk}}$. $\boldsymbol{\theta}$ is the bus voltage phase angle vector, in which θ_j and θ_k are the voltage phase angle

of bus j and k . They can be regarded as the bus voltage in the DC power flow equation. The active power of the branch jk is

$$P_{jk} = -B_{jk}\theta_{jk} = \frac{\theta_j - \theta_k}{x_{jk}}$$

The power increment of customer at bus j is Δp_j , the generator at bus k selected to provides the increased power. The increment of the j th element of the bus injection power vector P_j is $-\Delta p_j$, the increment of the k th element P_k is Δp_j , then the voltage phase angle increments at bus j' and k' are

$$\begin{aligned}\Delta\theta_{j'} &= \Delta p_j ([\mathbf{B}^{-1}]_{j'k} - [\mathbf{B}^{-1}]_{j'j}) \\ \Delta\theta_{k'} &= \Delta p_j ([\mathbf{B}^{-1}]_{k'k} - [\mathbf{B}^{-1}]_{k'j})\end{aligned}\quad (11)$$

The active increment of the branch $j'k'$ is

$$\Delta P_{j'k'} = -B_{j'k'} \Delta\theta_{j'k'} = \frac{\Delta\theta_{j'} - \Delta\theta_{k'}}{x_{j'k'}} = \Delta p_j B_{j'k'} ([\mathbf{B}^{-1}]_{k'k} - [\mathbf{B}^{-1}]_{k'j} - [\mathbf{B}^{-1}]_{j'k} + [\mathbf{B}^{-1}]_{j'j}) \quad (12)$$

When the active power flow distribution of the entire transmission network changes, L represents the branch of the power grid, and the total active power flow increment is

$$\sum_{j'k' \in L} \Delta P_{j'k'}$$

The expression of the active increment of the branch $j'k'$ indicates that this value is only related to the branch parameter containing bus j and bus k and the load power increment at the bus j . Therefore, except for the customer j , the fixed cost C_i allocated to each customer does not change when the customer does not participate in any alliance. And except for the alliance in which customer j participates, the total fixed cost C_S shared by alliance S also does not change. Then the incremental cost shared by the customer or the alliance that this customer participates in caused by the load increases the customer j is as follow.

$$\lambda \sum_{j'k' \in L} \Delta P_{j'k'}$$

According to the formula (9), if the customer j is included in the alliance S , $\sum_{i=1}^{n_S} C_i$ and C_S are the same as the fixed cost increment. The value of the feature function does not change. And the profit x_i of the customer in the alliance S determined by the nucleolus solution of the cooperative game does not change. Therefore, the fixed cost increment of customer j is $\lambda \sum_{j'k' \in L} \Delta P_{j'k'}$, and the economic incentive factor is

$$\gamma_{jk} = \lambda \sum_{j'k' \in L} B_{j'k'} ([\mathbf{B}^{-1}]_{k'k} - [\mathbf{B}^{-1}]_{k'j} - [\mathbf{B}^{-1}]_{j'k} + [\mathbf{B}^{-1}]_{j'j}) \quad (13)$$

Only when the nucleolus model is used to allocate the fixed cost of the customer based on the DC tidal model, the economic incentive factor can be simplified to the expression of the unit fixed cost containing only the branch parameters and the grid active power flow, without the power value of the grid load and the power source.

6. A five-bus example

Take a simple 5-bus system shown in Figure 1 as an example to illustrate the increase in customer load and the change in the nucleolus distribution of fixed costs in the transmission grid. In the figure, bus 1

is a balanced bus, and the remaining buses are PQ buses. The data of each bus is shown in Table 1. Symbols ①-⑦ represent the branch number, and the branch parameters are indicated. Set the fixed cost in the hourly trading market to a certain period of time, the energy transaction per MW should bear 50 yuan.

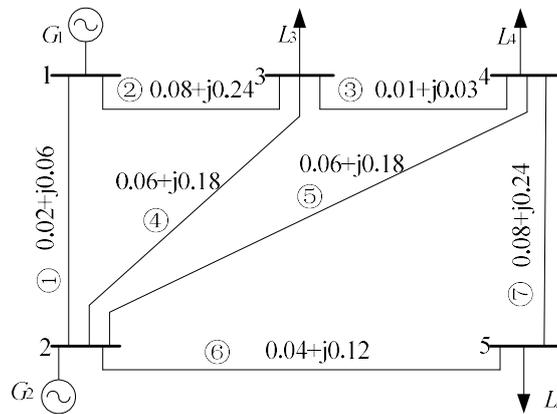


Figure 1. Five-bus system.

Table 1. Data of buses.

bus	Bus voltage		Generator power		Load power	
	Magnitude(p.u.)	Phase	Active(MW)	Reactive(Mvar)	Active(MW)	Reactive(Mvar)
1	1.06	0.00	130.5	26.5	0.0	0.0
2	1.04	-2.64	20.0	20.0	0.0	0.0
3	1.01	-4.81	0.0	0.0	45.0	15.0
4	1.01	-5.13	0.0	0.0	40.0	5.0
5	1.00	-5.98	0.0	0.0	60.0	10.0

Based on the nucleolus algorithm, the fixed cost allocation of the original load structure of the power grid as follows: the customers on buses 3, 4, and 5 share fixed costs of 3405, 3665, and 6105 yuan, respectively. The customer of bus 3 increases the load of 50 MW. If the generator of bus 1 is selected, the customers on buses 3, 4, and 5 share the fixed costs of 7710, 3843.25, and 5526, respectively, and $\gamma_{31} = 8.607$. If the generator of bus 2 is selected, the customers on the buses 3, 4, and 5 share fixed costs of 6464, 3352.25, and 4251.25, respectively, and $\gamma_{32} = 6.1165$. According to the formula (13), $\gamma_{31} = 8.47$ and $\gamma_{32} = 5.82$, which are close to the method according to the definition of the economic incentive factor after determining the fixed cost shared by the customer sequentially. So the economic incentive factor can be directly calculated by formula (13) when the fixed cost allocation based on the DC tidal model using the nucleolus solution. Comparing the value of the economic incentive factor, it is economical for the customer at bus 3 to select the generator at bus 2 under the premise of safe operation of the grid.

7. Conclusion

Although there are many ways to motivate, economic means are indispensable. Effective incentives are inseparable from funds. The fixed cost allocation method that customers charge indiscriminately by load cannot achieve effective incentives. The fixed cost allocation of the transmission network based on the cooperative game nucleolus solution encourages the grid customers to actively participate in the power market from the perspective of funds, and urges customers to work closely with the grid companies to share economic benefits on the basis of mutual benefit and achieve win-win results. The nucleolus solution not only gives an economically reasonable fixed cost allocation, but also has

influence on the customer's choice of the power supply generator. For the power grid, it is conducive to the economical rational allocation of transmission resources. For power customers, it can reduce transmission consumption, reduce production and operation costs of enterprises, reduce electricity expenditures, and improve the energy efficiency of enterprises and the competitiveness of products.

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