

# Modeling and identification of the error in eccentric axis follow-up grinding machine

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**Abstract.** Apply the theory of multi-body system, this paper establishes the follow-up error analysis model of grinding machine, and focuses on the study of its error identification method to improve the accuracy of the eccentric shaft servo grinder, providing a theoretical basis for error compensation of CNC grinding machine. And by using nine-line identification method to identify the all 21 items of geometric errors grinding machine translation system three translational axis of, By using the nine line identification method of the translation system to identify the all 21 items of geometric errors about the three translational axis of the grinding machine. Based on the three indicators, axial runout, radial runout and totating error of the rotating shaft, to arrive at the basic geometric error in the direction of six degrees of freedom of the rotating shaft. Motion model of multibody system error is analyzed, and based on the traditional geometry description method of multibody system, putting forward more practical description method on engineering body motion. The ideal movement of engineering object and the motion model under the condition of error are reasonably integrated and analyzed.

**Keywords:** Multibody theory; follow-up grinding machine; error analysis; error identification.

## 1. Introduction

Grinding technology is an important area in advanced manufacturing technology. It has been widely used for rough and fine machining of metals and other materials. It is a very important cutting method. It has the advantages of high precision and good surface quality. It is usually used as The final processing method of parts, grinding is the use of abrasives on the grinding tools at high cutting speeds

A machining process that cuts fine chips from the surface of the workpiece [1]. It has become the most effective and widely used basic process technology for realizing precision and ultra-precision machining in the field of modern machinery manufacturing, providing people with the development and development of high-precision, high-quality, highly automated technical equipment [2].

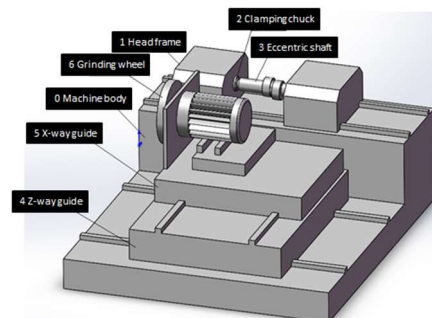
The comprehensive error parameter identification method is to measure the positioning error of a specific point in the work area of the machine tool. The mathematical model is used to identify the parameters of the measurement point and indirectly obtain the discrete values of each geometric error of the machine tool, which is more efficient than the traditional single item measurement efficiency.



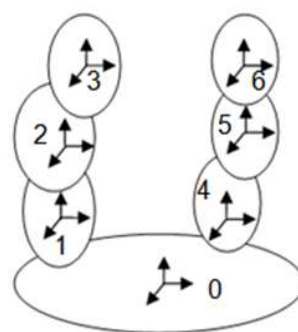
Gao was deeply concerned by scholars. At present, there are many researches on error modeling and error identification both at home and abroad. Zhang proposed a 22-line method to identify the 21 error parameters of a three-axis machine tool and conducted experiments on a coordinate measuring machine. Liu Youwu et al. of Tianjin University proposed a spatial error model for NC machine tools based on the multi-body system theory, proposed a 9-line method for identifying displacement errors, and performed a software error compensation experiment in a CNC machining center. Su Shiping established a universal accuracy model for multi-axis CNC machine tools based on the multi-body system theory; Ren Yongqiang and others established a spatial error model based on the homogeneous coordinate transformation theory; Yang Chengxu et al. based on the multi-body system theory, expounded the comprehensive spatial error of the four-axis motion platform and the modeling process. In this paper, by modeling and identification of the error of the eccentric shaft follower grinding machine, to improve the eccentric shaft grinding machining efficiency and machining accuracy, and promote the practical application of the technology.

## 2. The topological structure of the follow-up grinding machine and the low sequence array

The grinding wheel frame adopts the rear movement type. In order to display the movement relationship, the original machine tool is reversed. 0-machine body, 1-headframe, 2-clamping chuck, 3-eccentric shaft, 4-Z guide, 5-X guide, 6-grinding wheel, each moving body passes different connections make up a multibody system. According to the movement relationship between the various structures of the follow-up grinding machine (as shown in Figure 1), the topological relationship of the machine tool is shown in Figure 2:



**Figure 1.** Follow-up Eccentric Axis CNC Grinding Machine Structure



**Figure 2.** Follow-up grinding machine topology

The low-sequence array of the eccentric shaft grinder is shown in Table 1. The motion of the degrees of freedom in each space between the moving bodies is shown in Table 2. In the table, 0 represents no relative movement between each other, and 1 represents a single degree of freedom between each other. In the sports relations, the corresponding X represents a relative movement relationship in the X

direction, and the corresponding movement represents a rotational movement around the X axis, and the other movement forms are similar.

**Table 1.** Machine low sequence array

body ( <i>j</i> )	1	2	3	4	5	6
$L^0(j)$	1	2	3	4	5	6
$L^1(j)$	0	1	2	0	4	5
$L^2(j)$	0	0	1	0	0	4
$L^3(j)$	0	0	0	0	0	0
$L^4(j)$	0	0	0	0	0	0

**Table 2.** Degree of freedom between moving parts of machine tools

Degree of freedom	X	Y	Z	$\alpha$	$\beta$	$\gamma$
0-1	0	0	0	0	0	0
1-2	0	0	0	0	0	1
2-3	0	0	0	0	0	0
0-4	0	0	1	0	0	0
4-5	1	0	0	0	0	0
5-6	0	0	0	0	0	0

### 3. Establishing follow-up grinding precise motion equation

Establish the C-X-Z axis motion constraint equation. Machine-workpiece branch, any point in the workpiece coordinate position vector in the inertial coordinate system:

$$\begin{aligned}
 P_w &= [AOW_1][AW_1W_2][AW_2W_3]\{r_w\} \\
 &= [AOW_1]_{pl}[AOW_1]_{pe}[AOW_1]_{sl}[AOW_1]_{se} \\
 &\quad [AW_1W_2]_{pl}[AW_1W_2]_{pe}[AW_1W_2]_{sl} \\
 &\quad [AW_1W_2]_{se}[AW_2W_3]_{pl}[AW_2W_3]_{pe} \\
 &\quad [AW_2W_3]_{sl}[AW_2W_3]_{se}\{r_w\}
 \end{aligned} \tag{1}$$

Among them,  $\{r_w\} = [\omega_x \ \omega_y \ \omega_z \ 1]^T$ , The conversion matrix in the formula is shown in Table 3

Substituting into (1), the position vector of the arbitrary point on the eccentric shaft in the inertial coordinate system can be obtained.

In the same way, in the machine tool-knife kinematic chain, any point on the wheel coordinate system is the position vector on the inertial coordinate system:

$$\begin{aligned}
 P_t &= [AOW_4][AW_4W_5][AW_5W_6]\{r_t\} \\
 &= [AOW_4]_{pl}[AOW_4]_{pe}[AOW_4]_{sl}[AOW_4]_{se} \\
 &\quad [AW_4W_5]_{pl}[AW_4W_5]_{pe}[AW_4W_5]_{sl} \\
 &\quad [AW_4W_5]_{se}[AW_5W_6]_{pl}[AW_5W_6]_{pe} \\
 &\quad [AW_5W_6]_{sl}[AW_5W_6]_{se}\{r_t\}
 \end{aligned} \tag{2}$$

Among them,  $\{r_t\} = [t_x \ t_y \ t_z \ 1]^T$ , the conversion matrix in the formula is shown in Table 4

**Table 3.** Conversion Matrix (1)

Neighbors	Ideal position matrix	Error position matrix	Ideal displacement matrix	Error displacement matrix
0-1	$[AOW_1]_{pl} = I_{4 \times 4}$	$[AOW_1]_{pe} = I_{4 \times 4}$	$[AOW_1]_{sl} = I_{4 \times 4}$	$[AOW_1]_{se} = I_{4 \times 4}$
1-2	$[AW_1W_2]_{pl} =$ $\begin{bmatrix} 1 & 0 & 0 & q_{2x} \\ 0 & 1 & 0 & q_{2y} \\ 0 & 0 & 1 & q_{2z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$[AW_1W_2]_{pe} =$ $\begin{bmatrix} 1 & 0 & \varepsilon_{xc} & 0 \\ 0 & 1 & -\varepsilon_{yc} & 0 \\ -\varepsilon_{xc} & \varepsilon_{yc} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$[AW_1W_2]_{sl} = I_{4 \times 4}$	$[AW_1W_2]_{se} = I_{4 \times 4}$
2-3	$[AW_2W_3]_{pl} =$ $\begin{bmatrix} 1 & 0 & 0 & q_{3x} \\ 0 & 1 & 0 & q_{3y} \\ 0 & 0 & 1 & q_{3z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$[AW_2W_3]_{pe} = I_{4 \times 4}$	$[AW_2W_3]_{sl} =$ $\begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$[AW_2W_3]_{se} =$ $\begin{bmatrix} 1 & -\varepsilon_z(\theta) & \varepsilon_y(\theta) & \delta_x(\theta) \\ \varepsilon_z(\theta) & 1 & -\varepsilon_x(\theta) & \delta_y(\theta) \\ -\varepsilon_y(\theta) & \varepsilon_x(\theta) & 1 & \delta_z(\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$

**Table 4.** Conversion Matrix (2)

Neighbors	Ideal position matrix	Error position matrix	Ideal displacement matrix	Error displacement matrix
0-4	$[AOW_4]_{pl} =$ $\begin{bmatrix} 1 & 0 & 0 & q_{4x} \\ 0 & 1 & 0 & q_{4y} \\ 0 & 0 & 1 & q_{4z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$[AOW_4]_{pe} =$ $\begin{bmatrix} 1 & 0 & \varepsilon_{xz} & 0 \\ 0 & 1 & -\varepsilon_{yc} & 0 \\ -\varepsilon_{xz} & \varepsilon_{yc} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$[AOW_4]_{sl} =$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$[AOW_4]_{se} =$ $\begin{bmatrix} 1 & -\varepsilon_z(z) & \varepsilon_y(z) & \delta_x(z) \\ \varepsilon_z(z) & 1 & -\varepsilon_x(z) & \delta_y(z) \\ -\varepsilon_y(z) & \varepsilon_x(z) & 1 & \delta_z(z) \\ 0 & 0 & 0 & 1 \end{bmatrix}$
4-5	$[AW_4W_5]_{pl} =$ $I_{4 \times 4}$	$[AW_4W_5]_{pe} =$ $\begin{bmatrix} 1 & 0 & \varepsilon_{xz} & 0 \\ 0 & 1 & 0 & 0 \\ -\varepsilon_{xz} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$[AW_4W_5]_{sl} =$ $\begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$[AW_4W_5]_{se} =$ $\begin{bmatrix} 1 & -\varepsilon_z(x) & \varepsilon_y(x) & \delta_x(x) \\ \varepsilon_z(x) & 1 & -\varepsilon_x(x) & \delta_y(x) \\ -\varepsilon_y(x) & \varepsilon_x(x) & 1 & \delta_z(x) \\ 0 & 0 & 0 & 1 \end{bmatrix}$
5-6	$[AW_5W_6]_{pl} =$ $\begin{bmatrix} 1 & 0 & 0 & q_{6x} \\ 0 & 1 & 0 & q_{6y} \\ 0 & 0 & 1 & q_{6z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$[AW_5W_6]_{pe} =$ $I_{4 \times 4}$	$[AW_5W_6]_{sl} =$ $I_{4 \times 4}$	$[AW_5W_6]_{se} = I_{4 \times 4}$

Substituting into (2), the position vector of any point on the grinding wheel in the inertial coordinate system can be obtained.

(1)(2) is the position vector expression of any point of the machine-workpiece kinematic chain and the machine tool-knife kinematic chain in the eccentric shaft follow-up CNC grinding machine. The former is usually called the tool course, the latter is usually called tool path. Make

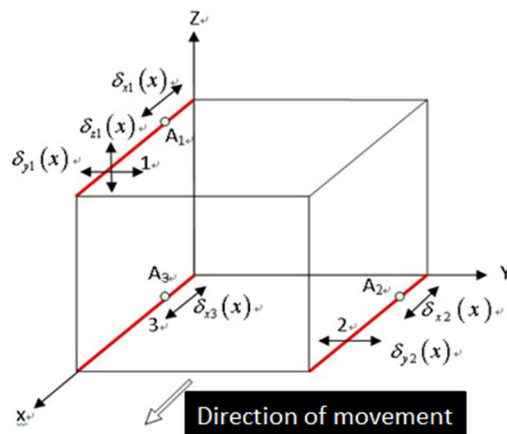
$$\begin{aligned}
 P_{r_w} &= P_{r_t} \\
 &[AOW_1]_{pl}[AOW_1]_{pe}[AOW_1]_{sl}[AOW_1]_{se} \\
 &[AW_1W_2]_{pl}[AW_1W_2]_{pe}[AW_1W_2]_{sl} \\
 &[AW_1W_2]_{se}[AW_2W_3]_{pl}[AW_2W_3]_{pe} \\
 &[AW_2W_3]_{sl}[AW_2W_3]_{se}\{r_w\} \\
 &=[AOW_4]_{pl}[AOW_4]_{pe}[AOW_4]_{sl}[AOW_4]_{se} \\
 &[AW_4W_5]_{pl}[AW_4W_5]_{pe}[AW_4W_5]_{sl} \\
 &[AW_4W_5]_{se}[AW_5W_6]_{pl}[AW_5W_6]_{pe} \\
 &[AW_5W_6]_{sl}[AW_5W_6]_{se}\{r_w\}
 \end{aligned} \tag{3}$$

Formula (3) is the precision machining constraint equation of the eccentric shaft follow-up CNC grinding machine. This equation reveals the essence of precision grinding. Through the control of this equation, the tool trajectory coincides with the tool path under the actual machining conditions of the machine tool, to meet the precision machining requirements.

#### 4. Follow-up grinding machine error parameter identification

##### 4.1. Nine-line identification method for translational systems

The following describes the process of the 9-line identification method by taking the X-axis as an example.



**Figure 3.** Measurement of translation along the X axis

As shown in Figure 3, the moving part moves in the X-axis direction. Three straight lines 1, 2, and 3 are selected on the table coordinate system, and then three points A1, A2, and A3 are optionally selected to measure the displacement  $\delta_{x1}(x), \delta_{x2}(x), \delta_{x3}(x)$ . The straightness error in the Y-axis and Z-axis directions of the straight line 1 is measured at the point A1, and the straightness error in the Y-axis direction of the straight line 2 is measured at the point A2. According to the geometrical characteristics of the six basic errors, it can be seen that the  $\varepsilon_x(x)$  in the Y and Z directions, the  $\varepsilon_y(x)$  in the X and Z directions, the displacement errors  $\varepsilon_z(x)$  also occur in the X and Y directions, and the following 6 relationships can be obtained:

$$\begin{cases} \delta_{x1}(x) = \delta_x(x) - \varepsilon_z(x)y_1 + \varepsilon_y(x)z_1 \\ \delta_{y1}(x) = \delta_y(x) - \varepsilon_x(x)z_1 + \varepsilon_z(x)x_1 \\ \delta_{z1}(x) = \delta_z(x) - \varepsilon_y(x)x_1 + \varepsilon_x(x)y_1 \\ \delta_{x2}(x) = \delta_x(x) - \varepsilon_z(x)y_2 + \varepsilon_y(x)z_2 \\ \delta_{y2}(x) = \delta_y(x) - \varepsilon_x(x)z_2 + \varepsilon_z(x)x_2 \\ \delta_{x3}(x) = \delta_x(x) - \varepsilon_z(x)y_3 + \varepsilon_y(x)z_3 \end{cases} \quad (4)$$

Among them,  $y_1 = y_3 = z_2 = z_3 = 0$ , therefore:

$$\begin{cases} \delta_{x1}(x) = \delta_x(x) + \varepsilon_y(x)z_1 \\ \delta_{y1}(x) = \delta_y(x) - \varepsilon_x(x)z_1 + \varepsilon_z(x)x_1 \\ \delta_{z1}(x) = \delta_z(x) - \varepsilon_y(x)x_1 \\ \delta_{x2}(x) = \delta_x(x) - \varepsilon_z(x)y_2 \\ \delta_{y2}(x) = \delta_y(x) + \varepsilon_z(x)x_2 \\ \delta_{x3}(x) = \delta_x(x) \end{cases} \quad (5)$$

In order to facilitate matrix expression, now orders are:

$$\{\delta(x)\} = [\delta_{x1}(x), \delta_{y1}(x), \delta_{z1}(x), \delta_{x2}(x), \delta_{y2}(x), \delta_{x3}(x)]^T \quad (6)$$

$$\{\Delta(x)\} = [\delta_x(x), \delta_y(x), \delta_z(x), \varepsilon_x(x), \varepsilon_y(x), \varepsilon_z(x)]^T \quad (7)$$

$$A_x = \begin{bmatrix} 1 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 1 & 0 & -z_1 & 0 & x_1 \\ 0 & 0 & 1 & 0 & -x_1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -y_2 \\ 0 & 1 & 0 & 0 & 0 & x_2 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

Thus the matrix expression is

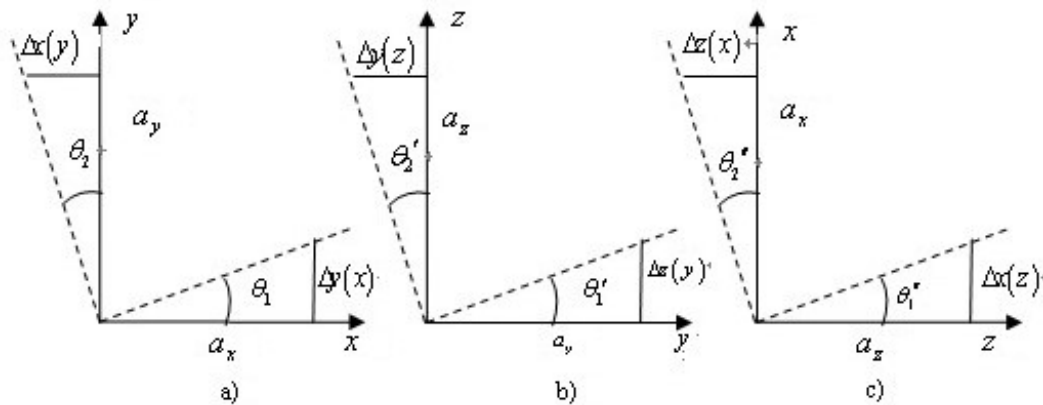
$$\{\delta(x)\} = [A_x] \{\Delta(x)\} \quad (9)$$

When  $Z1, Y2$  are non-zero,  $A_x$  is reversible, the above equation can be converted to:

$$\{\Delta(x)\} = [A_x]^{-1} \{\delta(x)\} \quad (10)$$

From this, six geometric errors that occur when moving in the direction of the X axis and Y axis can be obtained. Similarly, the geometric errors when moving in the direction of the axis and the axis can be obtained.

The three perpendicularity errors between the translation axes of the machine tool can be calculated by measuring the straightness errors of the three axes. The identification process is as follows:



**Figure 4.** Calculation of verticality error of triaxial axes

Therefore:

$$\theta_1 = \arctan \frac{\Delta y(x)}{a_x}; \theta_2 = \arctan \frac{\Delta x(y)}{a_y} \quad (11)$$

$$\varepsilon_{xy} = \theta_2 - \theta_1 \quad (12)$$

$$\theta'_1 = \arctan \frac{\Delta z(y)}{a_y}; \theta'_2 = \arctan \frac{\Delta y(z)}{a_z} \quad (13)$$

$$\varepsilon_{yz} = \theta'_2 - \theta'_1 \quad (14)$$

$$\theta''_1 = \arctan \frac{\Delta x(z)}{a_z}; \theta''_2 = \arctan \frac{\Delta z(x)}{a_x} \quad (15)$$

$$\varepsilon_{xz} = \theta''_2 - \theta''_1 \quad (16)$$

In the formula,  $a_x, a_y, a_z$  express the long of the X, Y, Z axis.  $\Delta x(y), \Delta x(z), \Delta y(x), \Delta y(z), \Delta z(x), \Delta z(y)$  are the measured values. So the verticality errors among the X, Y, Z axis are obtained by (12), (14) and (16).

## 5. Conclusion

(1) Analyze the motion structure, motion form, and motion principle of the CNC grinding machine with the eccentric shaft to facilitate the subsequent modeling of the error of the grinder using the kinematic theory of the multi-body system.

(2) According to the relative body movement of the machine tool, establish rack-workpiece kinematic chain and rack-knife kinematic chain, analyze its topological structure diagram and

corresponding low-sequence body array, and produce the common in the body movement process. The six-degree-of-freedom error parameter is described to facilitate the establishment of a precision error equation.

(3) Based on the multi-body system kinematics theory, describe the position vector of any machining point of the workpiece coordinate system in inertial coordinate system in the rack-workpiece kinematic chain, and describe the tool coordinate system in the rack-knife specific kinematic chain. The position vector of the arbitrary grinding point in the inertial coordinate system. Based on the principle of coincidence between tool path and tool path, the precision error equation of follow-up grinding machine was established and the essence of precision machining of eccentric shaft was revealed.

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