

PI Sliding Mode Control and Vibration Suppression of a Space Robot with Elastic Base and Two Flexible Arms

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Abstract. The trajectory tracking and vibration suppression of elastic base and two flexible arms for a free-floating space robot system with flexible arms and elastic base is discussed. Using singular perturbation theory of two kinds of time scale, a slow-subsystem describing the rigid motion and a fast-subsystem corresponding to vibration of base elastic and two flexible arms are obtained. For the slow-subsystem, neural network is used to approximate the two uncertainties in the dynamic model of the space robot. The integral term of proportional integral (PI) sliding mode control makes initial state of the system falls on the sliding surface, which can offset the unknown disturbance at the beginning of control and eliminate the chattering of controller output, so the robustness of the control law is enhanced. The PI sliding mode neural network controller is designed to improve the tracking performance of the rigid point-to-point movement. For the fast-subsystem, an optimal linear quadratic regulator(LQR) controller is adopted to damps out the vibration of the two flexible links and base elastic, which guarantees the stability and tracking accuracy of the system. Finally, computer simulation results show the effectiveness of the compound control method.

1. Introduction

Free-floating space robot (FFSR) can instead of astronauts to perform a variety of space tasks on orbit. With the deepening of research work, more and more attention to FFSR with the flexible-arm because of its characteristics of light weight, long arm, heavy load and so on [1~3]. J Cheong et al [4] proposed a direct parameter updating law to suppress the flexible vibration of FFSR quickly. X -Y Yu et al [5] designed an adaptive controller for FFSR of multi-flexible-arm.

In order to expand the working range of the FFSR on space station, it is installed on the mobile base which can move along the guide rails assembled by truss. The FFSR will arouse the elastic vibration of guide rail inevitably in the operation process [6]. The control precision of the space robot is affected. Therefore, the base elasticity should be considered as a factor in the high-precision control of FFSR.



For a FFSR with elastic base and two flexible arms, the non-integrity of the dynamics and the coupling of rigid motion and flexible vibration make difficulties of its dynamics analysis and control scheme design. This kind space robot is closer to actual working condition, but study this kind model less [7], so it is necessary to carry out research on the dynamic analysis and control of this highly nonlinear multi-coupled system.

2. Establishing dynamic equation

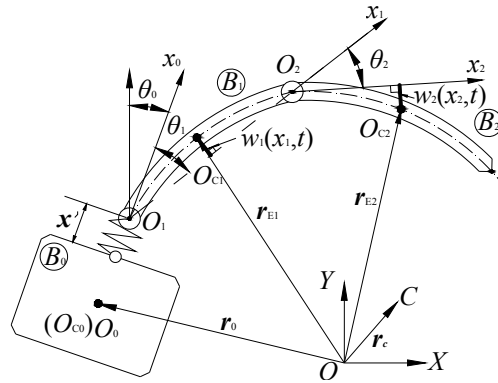


Figure 1. Space robot system with elastic base and two flexible arms

Fig.1 shows the structure model of the FFSR with elastic base and two flexible arms. The system consists of the free-floating base B_0 , and two flexible arms B_1 , B_2 . According to Lagrange equation of the second kind, the dynamic equation is expressed as follows

$$M(\theta, \eta) \begin{bmatrix} \ddot{\theta} \\ \ddot{\eta} \end{bmatrix} + H(\theta, \eta, \dot{\theta}, \dot{\eta}) \begin{bmatrix} \dot{\theta} \\ \dot{\eta} \end{bmatrix} + \begin{bmatrix} \theta_{3 \times 1} \\ K_{\delta} \delta \\ k_{x'} x' \end{bmatrix} = \begin{bmatrix} \tau \\ \theta \end{bmatrix} \quad (1)$$

Where $\theta = [\theta_0 \ \theta_1 \ \theta_2]^T$ is the generalized coordinate vector of base and the two-arm joints' relative angles, $\delta = [\delta_{11} \ \delta_{12} \ \delta_{21} \ \delta_{22}]^T$ is the generalized coordinate vector of flexible modal coordinates, $q = [\theta^T \ \eta^T]^T$, $\eta = [\delta^T \ x'^T]^T$. $M(\theta, \eta) \in \mathbb{R}^{8 \times 8}$ is the symmetric positive inertia matrix, $H(\theta, \eta, \dot{\theta}, \dot{\eta}) \begin{bmatrix} \dot{\theta}^T & \dot{\eta}^T \end{bmatrix}^T \in \mathbb{R}^8$ is The column vector containing the Coriolis and centrifugal forces, $K_{\delta} \in \text{diag}(k_{11}, k_{12}, k_{21}, k_{22})$, $k_{ij} = (EI)_i \int_0^{l_i} \phi_{ij}''^T \phi_{ij}'' dx_i$ is the stiffness matrix of two flexible arms, $\tau \in \mathbb{R}^3$ is the vector of base and two joint torques. Fq. (1) with block matrix writing as follows:

$$\begin{bmatrix} M_{rr} & M_{rf} \\ M_{fr} & M_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\eta} \end{bmatrix} + \begin{bmatrix} H_{rr} & H_{rf} \\ H_{fr} & H_{ff} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\eta} \end{bmatrix} + \begin{bmatrix} \theta_{3 \times 1} \\ K_{\delta} \delta \\ K_{x'} x' \end{bmatrix} = \begin{bmatrix} \tau \\ \theta \end{bmatrix} \quad (2)$$

Where $M_{rr} \in \mathbb{R}^{3 \times 3}$, $M_{ff} \in \mathbb{R}^{5 \times 5}$ and $M_{rf} = M_{fr}^T \in \mathbb{R}^{3 \times 5}$ are the sub-matrix of $M(\theta, \eta)$. $H_{rr} \in \mathbb{R}^{3 \times 3}$, $H_{rf} \in \mathbb{R}^{3 \times 5}$, $H_{fr} \in \mathbb{R}^{5 \times 3}$ and $H_{ff} \in \mathbb{R}^{5 \times 5}$ are the sub-matrix of $H(\theta, \eta, \dot{\theta}, \dot{\eta})$.

Since M is a symmetric, positive definite matrix, its inverse is present and can be written:

$$\mathbf{D} = \mathbf{M}^{-1} = \begin{bmatrix} \mathbf{M}_{rr} & \mathbf{M}_{rf} \\ \mathbf{M}_{fr} & \mathbf{M}_{ff} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{D}_{rr} & \mathbf{D}_{rf} \\ \mathbf{D}_{fr} & \mathbf{D}_{ff} \end{bmatrix} \quad (3)$$

The following combination control law is designed:

$$\boldsymbol{\tau} = \bar{\boldsymbol{\tau}} + \boldsymbol{\tau}_f \quad (4)$$

Where $\bar{\boldsymbol{\tau}}$ to control slow-subsystem to achieve the desired trajectory tracking; $\boldsymbol{\tau}_f$ is to control fast-subsystem to suppress the vibration.

Define singular perturbation scale factor $\varepsilon = 1 / \sqrt{\min\{k_{11}, k_{12}, k_{21}, k_{22}, k_{x'}\}}$, new state variables $\mathbf{z} = \boldsymbol{\eta} / \varepsilon^2$ and $\mathbf{K} = \text{diag}\{k_{11}, k_{12}, k_{21}, k_{22}, k_{x'}\}$, new matrix $\tilde{\mathbf{K}} = \varepsilon^2 \mathbf{K}$. The slow subsystem is obtained:

$$\bar{\mathbf{M}}_{rr} \ddot{\boldsymbol{\theta}} + \bar{\mathbf{H}}_{rr} \dot{\boldsymbol{\theta}} = \bar{\boldsymbol{\tau}} \quad (5)$$

Where the matrix or variable with a crossed "-" means the corresponding slow variable when $\varepsilon = 0$.

Then, in order to obtain the fast sub-system, new variables are defined: $\mathbf{p}_1 = \mathbf{z} - \bar{\mathbf{z}}$, $\mathbf{p}_2 = \varepsilon \dot{\mathbf{z}}$. Join the fast variable time scale $\varpi = t / \varepsilon$, and the fast-subsystem is:

$$\frac{d\mathbf{p}_1}{d\varpi} = \mathbf{p}_2, \quad \frac{d\mathbf{p}_2}{d\varpi} = -\bar{\mathbf{D}}_{ff} \tilde{\mathbf{K}} \mathbf{p}_1 + \bar{\mathbf{D}}_{fr} \boldsymbol{\tau}_f \quad (6)$$

It describes the vibration of elastic base and two flexible arms.

3. Combined control law design

3.1. The slow subsystem controller design

The desired trajectory of rigid motion $\boldsymbol{\theta}_d$, the system tracking error is $\mathbf{e} = \boldsymbol{\theta}_d - \boldsymbol{\theta}$. Define the filter trajectory error:

$$\mathbf{s} = \boldsymbol{\Lambda} \mathbf{e} + \dot{\mathbf{e}} \quad (7)$$

Where $\boldsymbol{\Lambda} \in \mathbf{R}^{3 \times 3}$ is symmetric and positive definite constant matrix.

The auxiliary reference vector is defined: $\mathbf{q}_r = \boldsymbol{\Lambda} \mathbf{e} + \dot{\boldsymbol{\theta}}_d$. The slow subsystem control law is designed:

$$\bar{\boldsymbol{\tau}} = \bar{\mathbf{f}} + \mathbf{K}_1 \mathbf{s} + \mathbf{K}_2 \int \mathbf{s} dt + \mathbf{K}_s \frac{\mathbf{s}}{\|\mathbf{s}\|} \quad (8)$$

Where $\bar{\mathbf{f}} = \hat{\mathbf{M}}_{rr} \dot{\mathbf{q}}_r + \hat{\mathbf{H}}_{rr} \mathbf{q}_r$, \mathbf{K}_1 , \mathbf{K}_2 and \mathbf{K}_s are positive constant matrices, respectively.

$\bar{\mathbf{f}}$ is model uncertainty, the Gauss radial neural network can be used to approximate it. $\bar{\mathbf{f}}_e$ is an approximation of $\bar{\mathbf{f}}$:

$$\bar{\mathbf{f}}_e = \hat{\mathbf{W}}^T \boldsymbol{\Phi} \quad (9)$$

Where $\hat{W}^T = [w_{ij}](i=1, \dots, n; j=1, \dots, m)$ is the network weight matrix; $\Phi(z) = [\phi_1(z), \dots, \phi_m(z)]^T$ is the basis function column vector; m is the number of the basis function center points; $z = [e^T \quad \dot{e}^T]^T$ is the neural network inputs; $\phi_j(z)$ can be represented as:

$$\phi_j(z) = \exp\left(-\|z - c_j\|^2 / \sigma_j^2\right) \quad (10)$$

Where c_j is the center of the j th basis function, and σ_j is the width of the corresponding basis function.

The optimal approximation of \bar{f} is:

$$\bar{f} = \bar{f}^* = W^{*T} \Phi + \varepsilon \quad (11)$$

Where W^* is the optimal value of W , ε is the optimal approximation error, $\|\varepsilon\| \leq \rho$.

The control law is rewritten:

$$\bar{\tau} = \bar{f}_e + K_1 s + K_2 \int s dt + K_s \frac{s}{\|s\|} \quad (12)$$

Theorem: for Eq. (5), slow subsystem of the FFSR with elastic base and two flexible arms, the control law of the Eq. (12) and adaptive regulation law are used as follows:

$$\dot{\hat{W}} = -\dot{\tilde{W}} = \eta_0 \Phi s^T - \mu \eta_0 \|s\| \hat{W} \quad (13)$$

Where $\eta_0 \in R^{5 \times 5}$, $\tilde{W} = W^* - \hat{W}$, μ is positive constant.

It will make the filtering error s eventually consistent bounded, that is, the tracking error e converges to zero in an arbitrary small neighborhood.

3.2. Linear Quadratic Controller of Fast Subsystem

The linear quadratic regulator to control the fast-subsystem. Eq. (6) is expressed as a state equation, the state variable is $\varsigma = [p_1^T \quad p_2^T]^T$, and Eq. (6) is changed to:

$$\dot{\varsigma} = A\varsigma + B\tau_f \quad (14)$$

$$\text{Where } A = \begin{bmatrix} 0 & I \\ -\bar{D}_{ff} \tilde{K} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \bar{D}_{ff} \end{bmatrix}.$$

As shown in Eq. (14), the fast subsystem is a linear system, and the system state variable ς can be adjusted to zero by the optimal control method, so that the base elasticity and the two arms flexible vibration can be suppressed.

The linear quadratic optimal performance index function is introduced as follows:

$$J = \frac{1}{2} \int_0^\infty (\varsigma^T Q \varsigma + \tau_f^T R \tau_f) dt \quad (15)$$

Where Q is semi-definite weighted symmetric matrix, R is positive definite weighted symmetric matrix.

By linear quadratic optimal control theory, in order to minimize J , the controller should be designed as follows:

$$\tau_f = -R^{-1}B^T P \zeta \quad (16)$$

Where P satisfies the following Riccati equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (17)$$

4. Simulation example

Taking the model system shown in Fig. 1 as an example, numerical simulations are carried out. The desired angles of base and two flexible arms respectively are: $\theta_{0d} = 0$, $\theta_{1d} = \pi/3$, $\theta_{2d} = -\pi/6$. The initial angles are taken as: $\theta_0(0) = 0.1$, $\theta_1(0) = \frac{\pi}{3} - 0.2$, $\theta_2(0) = -\frac{\pi}{6} - 0.15$. Simulation time $t = 50s$.

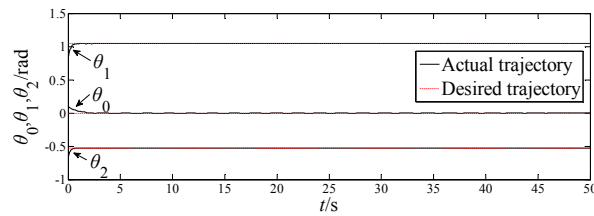


Figure 2. Trajectory tracking of the base's attitude and the two arms' joints

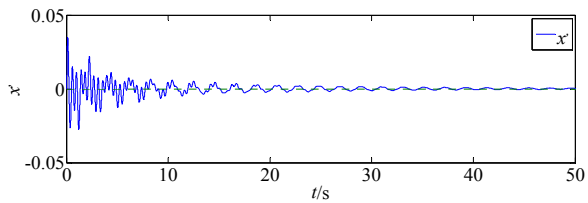


Figure 3. Elastic displacement of the base

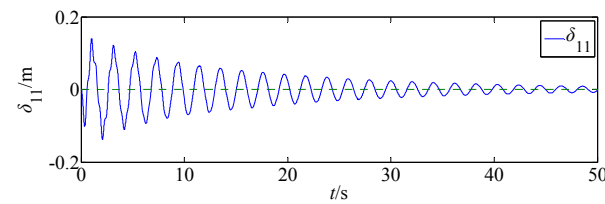


Figure 4. First mode of flexible link B_1

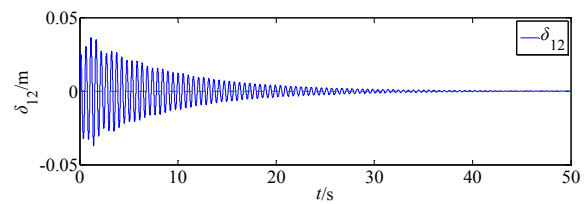


Figure 5. Second mode of flexible link B_1

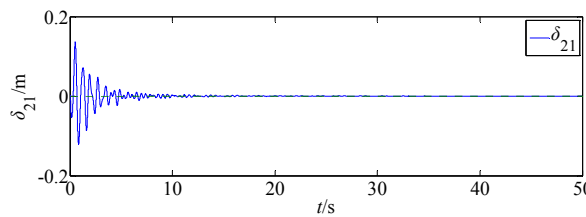


Figure 6. First mode of flexible link B_2

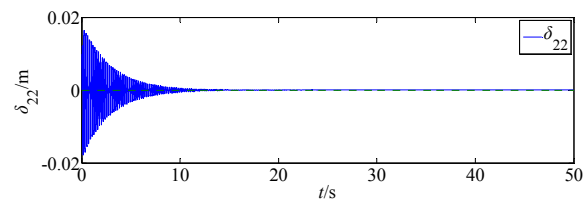


Figure 7. Second mode of flexible link B_2

Fig.2 shows the actual trajectory and the desired trajectory of base and two arms. Fig. 3 shows the elastic displacement of the base. Fig.4 - Fig.7 show the first and second order modal of the two flexible arms. From the simulation results, it can be seen that the actual trajectories tracked the desired trajectories well and LQR method suppressed the vibration of the base and two flexible arms.

5. Conclusion

Considering the multiple coupling between the elastic base and the two flexible arms, dynamic model of the FFSR is derived. Based on the singular perturbation method, an integral sliding mode neural

network control law of the slow-subsystem and an optimal LQR controller of the fast-subsystem are designed. It can improve the tracking performance and damp out the vibration.

Acknowledgments

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