Gyro-Free Satellite Attitude Determination Based on Special Orthogonal Group

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Abstract. The attitude determination problem of Earth Orientation gyro-less spacecraft is studied in this paper. Considering the sensitivity of the initial attitude error for EKF, a new non linear attitude determination filter is developed by using the Special Orthogonal Group SO (3) which avoids any algebraic reconstruction of the attitude from reference vectors. Lyapunov analysis results for the proposed the filter is derived that ensure global stability of the error. The simulation results show that the estimation of attitude and angular velocity converge to the real in the Earth Orientation mode.

1. Introduction

The attitude of the satellite is usually determined by its rotation matrix relative to the reference coordinate system. The accuracy and reliability of the attitude determination system will significantly affect the accuracy of the attitude control system. With the wide application of microsatellites, the gyro-free attitude-angular velocity estimation has become an important research field. Moreover, even satellites equipped with gyroscopes need to use an estimated angular rate. When the satellite rolls over or some undesired motion occurs, the angular velocity will exceed the range of the gyro. Some satellites that exceed the gyro's service life also need to adopt a gyro-free attitude determination method in order to continue working.

Varieties of methods for obtaining the angular velocity of the gyro-free satellite have been developed, and are roughly classified into two types. One is to use the attitude kinematics to differentiate the attitude to obtain the angular velocity. This method introduces high-frequency noise, and the angular velocity sensitivity of the attitude sensor is more obvious. The other is a Kalman-like filtering method, including Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), etc. [1-4]. Although Kalman filtering is widely used, global convergence cannot be guaranteed when applied to nonlinear dynamics, and random assumptions of measurement and process noise distribution usually do not satisfy the actual conditions.

Recently, nonlinear filters based on special orthogonal group SO (3) and non-Euclidean spatial topological obstacles have been used for attitude and position determination [5-11], and such filters are often used for single vector attitude determination [12, 13]. The nonlinear filter based on SO (3) can also be applied to the field of gyro-free attitude determination.

During the operation of the satellite, there is uncertainty in the moment of inertia, and the disturbance torque and other perturbation factors are often not accurately known. For most Earth

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imaging satellites, the angular velocity and angular acceleration during operation are small. Therefore, only the use of kinematics for attitude and angular velocity estimation can meet the accuracy requirements. In this paper, a gyro-free attitude determination method based on SO (3) and vector observation is designed, and the stability proof is given.

This article consists of five sections. The section II covers the mathematical formulas that need to be used. In section III, the gyro-free attitude and angular velocity estimation algorithm are designed in detail, and the stability proof is given. The section IV is the numerical simulation of the algorithm. Finally section V concludes the work.

2. Mathematical Preliminaries

For any matrix $\boldsymbol{A},\boldsymbol{B}\in\boldsymbol{R}^{n\times n}$, $[\bullet]$ defined as $[\boldsymbol{A},\boldsymbol{B}]=\boldsymbol{AB}-\boldsymbol{BA}$. Cross product matrix can be defined as

$$\boldsymbol{\Omega}_{\times}=\begin{bmatrix}0 & -\Omega_z & \Omega_y\\ \Omega_z & 0 & -\Omega_x\\ -\Omega_y & \Omega_x & 0\end{bmatrix} \tag{1}$$

Where $\boldsymbol{\Omega}\in\boldsymbol{R}^3$, and inverse operation of the cross product matrix is defined as $\mathrm{vex}(\boldsymbol{\Omega}_{\times})=\boldsymbol{\Omega}$. Defining matrix inner product and F norm

$$\begin{aligned}&\langle\langle\boldsymbol{A},\boldsymbol{B}\rangle\rangle=tr(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B})\\ &\|\boldsymbol{A}\|=\sqrt{\langle\langle\boldsymbol{A},\boldsymbol{A}\rangle\rangle}\end{aligned} \tag{2}$$

Other formulas used in this article are as follows:

$$\begin{aligned}&(\boldsymbol{Rv})_{\times}=\boldsymbol{Rv}_{\times}\boldsymbol{R}^{\mathrm{T}},\quad \boldsymbol{R}\in SO(3),\boldsymbol{v}\in\boldsymbol{R}^3\\ &\boldsymbol{v}^{\mathrm{T}}\boldsymbol{w}=\langle\boldsymbol{v},\boldsymbol{w}\rangle=\frac{1}{2}\langle\langle\boldsymbol{v}_{\times},\boldsymbol{w}_{\times}\rangle\rangle,\quad \boldsymbol{v},\boldsymbol{w}\in\boldsymbol{R}^3\\ &\boldsymbol{v}^{\mathrm{T}}\boldsymbol{v}=|\boldsymbol{v}|^2=\frac{1}{2}\|\boldsymbol{v}_{\times}\|^2,\qquad \boldsymbol{v}\in\boldsymbol{R}^3\\ &\langle\langle\boldsymbol{A},\boldsymbol{v}_{\times}\rangle\rangle=0,\quad \boldsymbol{A}^{\mathrm{T}}=\boldsymbol{A}\in\boldsymbol{R}^{3\times3},\boldsymbol{v}\in\boldsymbol{R}^3\\ &tr([\boldsymbol{A},\boldsymbol{B}])=0,\qquad \boldsymbol{A},\boldsymbol{B}\in\boldsymbol{R}^{3\times3}\\ &P_s(\boldsymbol{A})=\frac{1}{2}(\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}),\quad P_a(\boldsymbol{A})=\frac{1}{2}(\boldsymbol{A}-\boldsymbol{A}^{\mathrm{T}})\end{aligned} \tag{3}$$

The conversion relationship between the Direction cosine matrix $\boldsymbol{R}$ and Euler axis angle $(\theta,\boldsymbol{a})$ is

$$\boldsymbol{R}=\exp(\theta\boldsymbol{a}_{\times}),\quad \cos(\theta)=\frac{1}{2}(tr(\boldsymbol{R})-1) \tag{4}$$

3. Filter design and analysis

3.1. Measurement equations and error criteria

Let $\boldsymbol{R}$ be the Direction cosine matrix (DCM) of inertial frame i with respect to body frame b. The attitude sensor is a vector measurement sensor (solar sensor, magnetometer, earth sensor, etc.), and measures the component of the reference vector expressed in body frame b. $\boldsymbol{v}_{b_r}$ and $\boldsymbol{v}_I$ are vector $\boldsymbol{v}$

of frame b with respect to frame i expressed in frame b and frame i respectively, and $\boldsymbol{v}_{b_m}$ is measurement of $\boldsymbol{v}_{b_r}$ expressed in frame b, then

$$\boldsymbol{v}_{b_m} = \boldsymbol{R}^{\mathrm{T}}\boldsymbol{v}_I + \boldsymbol{b}_0 + \boldsymbol{\mu} = \boldsymbol{v}_{b_r} + \boldsymbol{b}_0 + \boldsymbol{\mu} \tag{5}$$

With $\boldsymbol{b}_0$ as bias and $\boldsymbol{\mu}$ as zero mean Gaussian white noise. Usually the bias $\boldsymbol{b}$ is far less than noise, so we assume $\boldsymbol{b}_0 = \boldsymbol{0}$.

Therefore, we define the multiplicative error DCM representation as $\tilde{\boldsymbol{R}}$ with $\boldsymbol{R}$ as the real DCM and $\hat{\boldsymbol{R}}$ as the estimate DCM. For one vector, error criteria is given by

$$E_i = 1-\cos(\angle \boldsymbol{v}_{bi}, \hat{\boldsymbol{v}}_{bi}) = 1-\langle \boldsymbol{v}_{bi}, \hat{\boldsymbol{v}}_{bi}\rangle = 1-tr(\hat{\boldsymbol{R}}^{\mathrm{T}}\boldsymbol{v}_{Ii}\boldsymbol{v}_{Ii}^{\mathrm{T}}\boldsymbol{R}) = 1-tr(\tilde{\boldsymbol{R}}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{v}_{Ii}\boldsymbol{v}_{Ii}^{\mathrm{T}}\boldsymbol{R}) \tag{6}$$

And for more vectors,

$$E_{mes} = \sum_{i=1}^{n} k_i E_i = \sum_{i=1}^{n} k_i - tr(\tilde{\boldsymbol{R}}\boldsymbol{M}), \quad k_i > 0 \tag{7}$$

Where $\boldsymbol{M} = \boldsymbol{R}^{\mathrm{T}}\boldsymbol{M}_0\boldsymbol{R}$, $\boldsymbol{M}_0 = \sum_{i=0}^{n} k_i \boldsymbol{v}_{Ii}\boldsymbol{v}_{Ii}^{\mathrm{T}}$. $\boldsymbol{M} > 0$ when $\boldsymbol{v}_{Ii}$ are linearly independent and $n \geq 3$, $\boldsymbol{M} \geq 0$ when $\boldsymbol{v}_{Ii}$ are linearly independent and n=2. According to the double vector attitude determination principle, it can be known that measurement information ensures completeness when $n \geq 2$.

3.2. Filter structure

When the satellite is in the earth orientation mode, the angular velocity and angular acceleration are small, and body dynamics model is not required. Similar to the literature [1], the filter is as follows

$$\begin{cases} \dot{\hat{\boldsymbol{R}}} = \hat{\boldsymbol{R}}(\hat{\boldsymbol{\Omega}} + k_P\boldsymbol{\omega})_{\times} \\ \dot{\hat{\boldsymbol{\Omega}}} = k_I\boldsymbol{\omega} \\ \boldsymbol{\omega} = \sum_{i=1}^{n} k_i(\boldsymbol{v}_i \times \hat{\boldsymbol{v}}_i) \end{cases} \tag{8}$$

Where $\hat{\boldsymbol{R}}$ is the estimate of DCM of inertial frame i with respect to body frame b, $\hat{\boldsymbol{\Omega}}$ is the estimate of angular velocity of frame b with respect to frame i expressed in frame b, k_P, k_I are proportional and integral coefficient greater than 0, $\boldsymbol{v}_i$ is the measurement of the i-th vector expressed in frame b, $\hat{\boldsymbol{v}}_i$ is the estimate of the i-th vector expressed in frame b, $\boldsymbol{\omega}$ is the nonlinear residual of the estimated value and the measurement, k_i is weight coefficient and $\Sigma k_i = 1$.

For this problem, $\boldsymbol{v}_i$ can be selected as $\boldsymbol{v}_s$ (sun vector), $\boldsymbol{v}_m$ (geomagnetic field vector), $\boldsymbol{v}_g$ (geocentric vector), etc. The weight coefficients are based on the accuracy of each sensor. The approximate method is $k_i : k_j = \sigma_j^2 : \sigma_i^2$, where σ_i is the variance of the i-th vector measurement error.

3.3. Convergence Analysis

The rotational kinematic equations written in terms of DCM can be written as follows

$$\begin{cases} \dot{\boldsymbol{R}} = \boldsymbol{R}\boldsymbol{\Omega}_{\times} \\ \dot{\boldsymbol{\Omega}} \approx \boldsymbol{0} \end{cases} \tag{9}$$

Let $\hat{\boldsymbol{\Omega}} = \boldsymbol{\Omega} + \Delta\boldsymbol{\Omega}$, then

$$\begin{aligned} \dot{\tilde{\boldsymbol{R}}} &= \dot{\hat{\boldsymbol{R}}}^{\mathrm{T}}\boldsymbol{R} + \hat{\boldsymbol{R}}^{\mathrm{T}}\dot{\boldsymbol{R}} \\ &= (\boldsymbol{\Omega} + \Delta\boldsymbol{\Omega} + k_P\boldsymbol{\omega})_{\times}^{\mathrm{T}}\hat{\boldsymbol{R}}^{\mathrm{T}}\boldsymbol{R} + \hat{\boldsymbol{R}}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\Omega}_{\times} \\ &= -(\boldsymbol{\Omega} + \Delta\boldsymbol{\Omega} + k_P\boldsymbol{\omega})_{\times}\tilde{\boldsymbol{R}} + \tilde{\boldsymbol{R}}\boldsymbol{\Omega}_{\times} \\ &= [\tilde{\boldsymbol{R}}, \boldsymbol{\Omega}_{\times}] - k_P\boldsymbol{\omega}_{\times}\tilde{\boldsymbol{R}} - \Delta\boldsymbol{\Omega}_{\times}\tilde{\boldsymbol{R}} \\ \dot{\boldsymbol{M}} &= \dot{\boldsymbol{R}}^{\mathrm{T}}\boldsymbol{M}_0\boldsymbol{R} + \boldsymbol{R}^{\mathrm{T}}\boldsymbol{M}_0\dot{\boldsymbol{R}} + \boldsymbol{R}^{\mathrm{T}}\dot{\boldsymbol{M}}_0\boldsymbol{R} \\ &= -\boldsymbol{\Omega}_{\times}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{M}_0\boldsymbol{R} + \boldsymbol{R}^{\mathrm{T}}\boldsymbol{M}_0\boldsymbol{R}\boldsymbol{\Omega}_{\times} + \boldsymbol{R}^{\mathrm{T}}\dot{\boldsymbol{M}}_0\boldsymbol{R} \\ &= [\boldsymbol{R}^{\mathrm{T}}\boldsymbol{M}_0\boldsymbol{R}, \boldsymbol{\Omega}_{\times}] + \boldsymbol{R}^{\mathrm{T}}\dot{\boldsymbol{M}}_0\boldsymbol{R} \\ &= [\boldsymbol{M}, \boldsymbol{\Omega}_{\times}] + \boldsymbol{R}^{\mathrm{T}}\dot{\boldsymbol{M}}_0\boldsymbol{R} \\ \Delta\dot{\boldsymbol{\Omega}} &= k_I\boldsymbol{\omega} \end{aligned} \tag{10}$$

When the reference vector is a sun vector, a geocentric vector, or a geomagnetic vector, the vector changes slowly in the inertial space, so choose Lyapunov function as

$$\begin{aligned} V &= E_{mes} + \frac{1}{k_I}\Delta\boldsymbol{\Omega}^2 \\ &= \sum_{i=1}^{n} k_i - tr(\tilde{\boldsymbol{R}}\boldsymbol{M}) + \frac{1}{k_I}\Delta\boldsymbol{\Omega}^2 \end{aligned} \tag{11}$$

The differential equation for the Lyapunov function is

$$\begin{aligned} \dot{V} &= -tr(\dot{\tilde{\boldsymbol{R}}}\boldsymbol{M} + \tilde{\boldsymbol{R}}\dot{\boldsymbol{M}}) + \frac{2}{k_I}\Delta\boldsymbol{\Omega}^{\mathrm{T}}\Delta\dot{\boldsymbol{\Omega}} \\ &= -tr([\tilde{\boldsymbol{R}}, \boldsymbol{\Omega}_{\times}]\boldsymbol{M} - k_P\boldsymbol{\omega}_{\times}\tilde{\boldsymbol{R}}\boldsymbol{M} - \Delta\boldsymbol{\Omega}_{\times}\tilde{\boldsymbol{R}}\boldsymbol{M} + \tilde{\boldsymbol{R}}[\boldsymbol{M}, \boldsymbol{\Omega}_{\times}]) + \frac{2}{k_I}\Delta\boldsymbol{\Omega}^{\mathrm{T}}\Delta\dot{\boldsymbol{\Omega}} \\ &= -tr([\tilde{\boldsymbol{R}}\boldsymbol{M}, \boldsymbol{\Omega}_{\times}] - (\Delta\boldsymbol{\Omega} + k_P\boldsymbol{\omega})_{\times}\tilde{\boldsymbol{R}}\boldsymbol{M}) + \frac{2}{k_I}\Delta\boldsymbol{\Omega}^{\mathrm{T}}\Delta\dot{\boldsymbol{\Omega}} \\ &= k_P tr(\boldsymbol{\omega}_{\times}\tilde{\boldsymbol{R}}\boldsymbol{M}) + tr(\Delta\boldsymbol{\Omega}_{\times}\tilde{\boldsymbol{R}}\boldsymbol{M}) + \frac{2}{k_I}\Delta\boldsymbol{\Omega}^{\mathrm{T}}\Delta\dot{\boldsymbol{\Omega}} \\ &= k_P tr(\boldsymbol{\omega}_{\times}P_a(\tilde{\boldsymbol{R}}\boldsymbol{M})) + tr(\Delta\boldsymbol{\Omega}_{\times}P_a(\tilde{\boldsymbol{R}}\boldsymbol{M})) + \frac{1}{k_I}tr(\Delta\boldsymbol{\Omega}_{\times}^{\mathrm{T}}\Delta\dot{\boldsymbol{\Omega}}_{\times}) \\ &= -k_P tr(\boldsymbol{\omega}_{\times}^{\mathrm{T}}P_a(\tilde{\boldsymbol{R}}\boldsymbol{M})) + tr(\Delta\boldsymbol{\Omega}_{\times}(P_a(\tilde{\boldsymbol{R}}\boldsymbol{M}) - \frac{1}{k_I}\Delta\dot{\boldsymbol{\Omega}}_{\times})) \\ &= -2k_P\left|\boldsymbol{\omega}\right|^2 + tr(\Delta\boldsymbol{\Omega}_{\times}(\boldsymbol{\omega}_{\times} - \frac{1}{k_I}(k_I\boldsymbol{\omega}_{\times}))) \\ &= -2k_P\left|\boldsymbol{\omega}\right|^2 \le 0 \end{aligned} \tag{12}$$

When $\theta = \pi$, $\boldsymbol{\omega} = \boldsymbol{0}$, the filter is at an unstable equilibrium point, but will not stabilize at that point due to the presence of measurement noise. When $\theta = 0$, $\boldsymbol{\omega} = \boldsymbol{0}$, the equilibrium point is stabilized [12]. So when the satellite is in earth orientation mode, the filter estimation $(\hat{\boldsymbol{R}}, \hat{\boldsymbol{\Omega}})$ converges globally.

4. Simulation and Discussions

This section gives two sets of numerical simulations to verify the convergence process of attitude and angular velocity.

(1) Earth orientation mode, large initial error simulation. Mainly verify the global convergence of the filter. Select two reference vectors with no measurement noise, $\boldsymbol{v}_1 = [0,0,1]^{\mathrm{T}}, \boldsymbol{v}_2 = [0,1,0]^{\mathrm{T}}$. The initial Euler angle error is $\delta\boldsymbol{\Psi} = [20^\circ, 60^\circ, 150^\circ]^{\mathrm{T}}$, corresponding to $\theta \approx 145^\circ$.

(2) Earth orientation mode, small initial error simulation. At this time, the attitude of the satellite is relatively stable, mainly verifying the steady-state accuracy of the filter. Select two reference vectors $\boldsymbol{v}_1 = [0,0,1]^{\mathrm{T}}, \boldsymbol{v}_2 = [0,1,0]^{\mathrm{T}}$, and measure the noise as $3\sigma_1 = 0.1^\circ$, $3\sigma_2 = 0.2^\circ$. Orbital angular velocity $\omega_o = 0.0011\mathrm{rad/s}$, external torque interference additional angular velocity is

$$\boldsymbol{\Omega}_d = 10^{-3} \bullet [\sin(0.5t) \quad \sin(0.5t + \frac{2}{3}\pi) \quad \sin(0.5t + \frac{4}{3}\pi)]^{\mathrm{T}} \ ^\circ/\mathrm{s}$$

The filter gains of the two sets of simulations are selected as (1) $k_P = 1$, $k_I = 0.5$; (2) $k_P = 1$, $k_I = 1$. The simulation algorithm is a DCM normalized fourth-order Runge-Kutta method with a data sampling rate of 10 Hz.

It can be seen from Fig. 1 that although the initial error is large, the attitude estimation error and the angular velocity estimation error converge to zero within 10s.

As can be seen from Fig. 2, the filter can significantly improve the accuracy of the attitude determination. The error is about 0.05°, which is less than the measurement error of two reference vectors. In the case of measuring noise with a reference vector of 0.1°, the attitude stability of $0.001^\circ/\mathrm{s}$ has no influence on the estimation accuracy of the angular velocity, and is basically stable at $0.03^\circ/\mathrm{s}$.

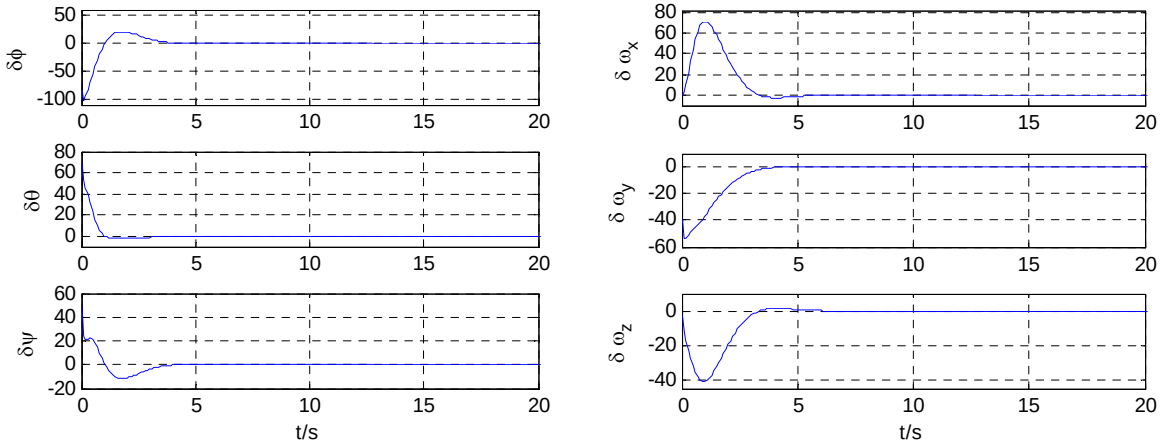


Figure 1. Attitude and angular velocity error without noise (°, °/s)

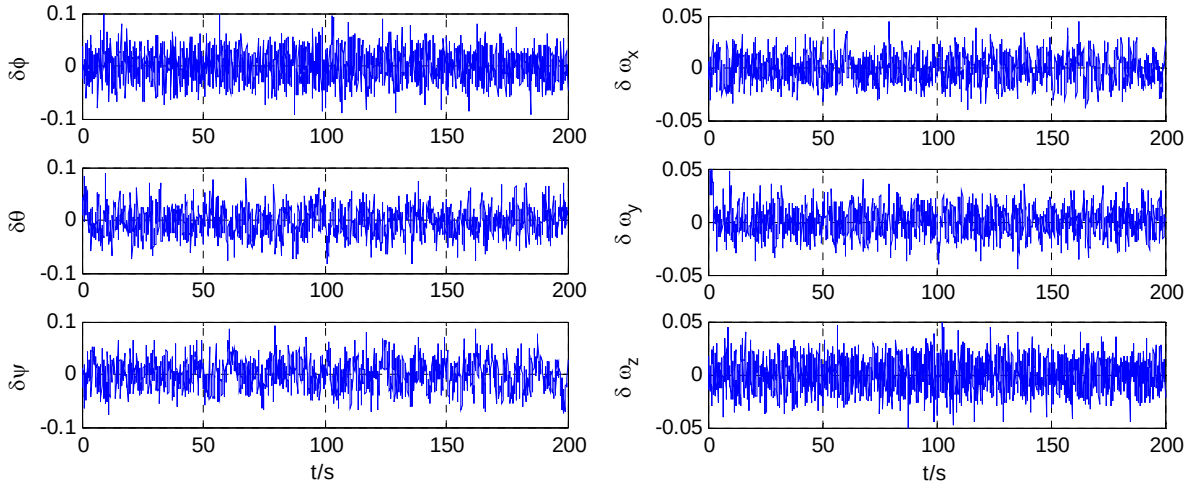


Figure 2. Attitude and angular velocity error with noise (°, °/s)

In summary, for low-cost satellites with low-precision vector sensors (earth sensors, magnetometers, etc.), the gyro-free attitude determination method can obtain an angular velocity estimation with a certain accuracy when the angular acceleration is relatively small.

5. Conclusion

For earth orientation satellite with no gyro configuration, a nonlinear filter based on special orthogonal group is designed and the stability of the filter is proved. The filter not only achieves high accuracy at steady state, but also has global convergence to the initial error. Compared with EKF, the calculation amount is small, and it is more advantageous in real-time. This method has certain reference value for theoretical research and engineering application of small low-cost satellite attitude determination.

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