

Sliding mode observer-based active fault tolerant control for UAVs formation

Lei Yin ^a, Jianwei Liu ^{b, *}, Pu Yang ^c and Junpeng Shi ^d

College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, 211100, China.

^ayinlei@nuaa.edu.cn, ^{*}, ^b Corresponding author E-mail: ljw301@nuaa.edu.cn

^cppyang@nuaa.edu.cn, ^dshijunpeng@nuaa.edu.cn

Abstract. An active fault tolerant controller for unmanned aerial vehicles (UAVs) formation in the presence of actuator fault is developed in this paper. First, an outer loop controller is designed to ensure the stability of the whole UAVs formation stable, and an inner loop controller is designed to make UAV tracking the desired outer loop signal. Considering the occurrence of actuator fault, an active fault tolerant controller is developed based on the nominal controller to eliminate the influence of the actuator faults and guarantee the stability of the UAVs formation system. A sliding mode observer is used to reconstruct the states and actuator faults. At last, the effect of actuator faults on the system is analysed, and the validity of the method is proved by MATLAB.

Keywords: UAVs formation, sliding mode observer, fault diagnosis, fault tolerant control.

1. Introduction

Unmanned aerial vehicle (UAV) has received a considerable attention and development for several decades. It has huge value as it has been widely used into both military and civilian fields. However, a single UAV is unable to accomplish complex tasks such as forest fire, border patrol, nature resources exploration [1]. To deal with this problem, formation flight of multiple unmanned aerial vehicles (UAVs) [2] technique has been developed to play the better role of UAV. At present, the structure of UAVs formation can be generally classified as leader-follower, virtual-leader and behavior based [3]. But in most studies, simple first-order or second-order kinetic model is considered. Therefore, the ideal of combination of inner loop and outer loop is considered in this paper [4]. This method can reflect the influence of UAV's fault to the whole formation and describe dynamic behavior better.

More UAVs mean more faults, a small actuator fault can not only affect the performance of tasks, but also lead to more serious consequences. Therefore, fault tolerant capability has become more and more necessary for UAVs formation to guarantee that when failure happens, UAVs formation can still complete the task [5].

In UAVs formation control, there are two main faults, communication faults (dropout, delay, failure, etc.) and UAV faults [6]. Aiming at the communication fault, reference [7] proposes a new extraction algorithm that simplifies the design of formation control laws with delayed communication for this class of under-actuated systems. Reference [8] develops a fault tolerant decentralized receding horizon



controller, which uses a safe protection zone called a tube around the trajectory of faulty neighboring vehicles to ensure safety.

UAV faults mainly include actuator fault, sensor fault and engine fault. Actuator fault is studied in this paper, because it is the most common and serious fault. In previous researches, robust control, linear quadratic (LQ) control, feedback linearization, sliding mode control (SMC) [9-11] have been adopted in this problem. The above-mentioned FTC methods are almost passive FTC, which need neither fault detection and diagnosis (FDD) nor controller reconfiguration, but its fault tolerant capacity is limited [12]. Unlike passive FTC method, active FTC method needs an FDD mechanism to identify the unknown fault in real time [13]. Until now, active FTC for UAVs formation has not been fully studied yet [14]. The contribution of this paper is that, an active FTC method using the sliding mode observer to reconstruct the actuator faults is presented. The inner loop controller is reconfigured according to the online fault information provided by the sliding mode observer to realize that the UAV can track the desired outer loop signal. Through this, the UAVs formation will be stable. Finally, the influence of the actuator faults on the system is analyzed, and a simulation example is given to verify the effectiveness of the proposed method.

This paper is organized as follows. In Section 2, necessary preliminaries are given, including UAVs formation model and UAV's own kinematic model. In Section 3, the detail of the proposed nominal controller scheme is described. The influence of the actuator faults on the system is in Section 4. In Section 5, simulation results are analyzed. In Section 6, some conclusions are discussed.

2. Problem Description

2.1. Outer loop model

In this paper, the leader-follower structure is considered as the flight formation, which consists of $i(i \geq 2)$ UAVs. The kinematic model of UAV i is:

$$\begin{cases} \dot{x}_i = v_i \cos \psi_i \\ \dot{y}_i = v_i \sin \psi_i \\ \dot{\psi}_i = w_i \end{cases} \quad (1)$$

Where x_i, y_i represent the position of the i th UAV, and v_i is forward velocity, ψ_i is angle between v_i and x-axis, w_i is angular velocity, respectively.

For the designed communication topology, the i th UAV only has one leader, the j th UAV. As illustrated in Fig. 1, the actual forward and lateral distance between the leader and the follower f_{ij} and l_{ij} can then be described as:

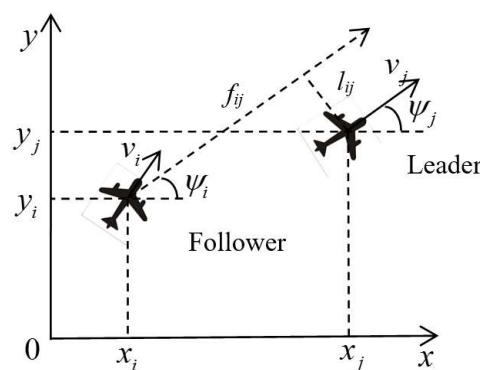


Fig 1. Formation geometry

$$\begin{aligned} f_{ij} &= (x_i - x_j) \cos \psi_j + (y_i - y_j) \sin \psi_j + d \cos(\psi_i - \psi_j), \\ l_{ij} &= (x_i - x_j) \sin \psi_j - (y_i - y_j) \cos \psi_j - d \sin(\psi_i - \psi_j). \end{aligned} \quad (2)$$

Where d is the distance between the control point and the center of mass [15].

Then, define the forward error and the lateral error as $e_{f_{ij}} = f_{ij} - f_{ij}^d$ and $e_{l_{ij}} = l_{ij} - l_{ij}^d$, where f_{ij}^d and l_{ij}^d represent the desired forward and lateral distance. The derivation of the error model can then be obtained:

$$\begin{bmatrix} \dot{e}_{f_{ij}} \\ \dot{e}_{l_{ij}} \end{bmatrix} = \begin{bmatrix} -v_j - l_{ij} w_j \\ f_{ij} w_j \end{bmatrix} + \begin{bmatrix} \cos(\psi_i - \psi_j) & -d \sin(\psi_i - \psi_j) \\ -\sin(\psi_i - \psi_j) & -d \cos(\psi_i - \psi_j) \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix} \quad (3)$$

2.2. Inner loop model

The general dynamic model of the UAV considered in this paper is as following:

$$\begin{cases} \dot{x} = Ax(t) + G(x, u) + D\phi(x, t) + Ef(u, t) \\ y = Cx \end{cases} \quad (4)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $y(t) \in R^r$ is the output vector. $A \in R^{n \times n}$, $D \in R^{n \times r}$, $E \in R^{n \times q}$, $C \in R^{p \times n}$ ($q \leq p < n$) are all constant matrices, and the matrices D, C are full rank. The known nonlinear term $G(x, u)$ is assumed to be Lipschitz about x and can be decoupled into $Bu + g(x)$. $\phi(x, t)$ mean the nonlinear perturbation and parameter uncertainties. $f(u, t)$ is the unknown actuator fault with a known upper bound γ_2 .

And in this paper, following assumptions are required.

Assumption 1: In (4), the matrix pair (A, C) is detectable.

It can be concluded from Assumption 1 that there exists a matrix L such that $A - LC$ is stable, and the following Lyapunov equation will be satisfied.

$$(A - LC)^T P + P(A - LC) = -Q \quad (5)$$

where $Q > 0$, and the equation has an unique solution $P > 0$.

Assumption 2: There exists a known $\gamma_1(x, t)$ which is Lipschitz about x uniformly, such that

$$\|\phi(x, t)\| \leq \gamma_1(x, t) \quad (6)$$

Assumption 3: There exist Matrices $F_1 \in R^{r \times p}$ and $F_2 \in R^{q \times p}$ which satisfy the following equation:

$$\begin{bmatrix} D^T \\ E^T \end{bmatrix} P = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} C. \quad (7)$$

2.3. Problem proposed

This paper takes the leader-follower structure as the outer loop, and a proportional-integral (PI) controller is designed to maintain the outer loop to be stable. To realize this, an active FTC method is designed to ensure UAV can track the desired outer loop signal. Then, the combination of outer loop and inner loop is realized.

3. Formation Controller Design

3.1. Outer loop controller design

First, reconstruct the equation (3):

$$\begin{bmatrix} \dot{e}_{f_{ij}} \\ \dot{e}_{l_{ij}} \end{bmatrix} = \begin{bmatrix} -v_j - l_{ij}w_j \\ f_{ij}w_j \end{bmatrix} + \begin{bmatrix} \cos(\psi_i - \psi_j) & -d \sin(\psi_i - \psi_j) \\ -\sin(\psi_i - \psi_j) & -d \cos(\psi_i - \psi_j) \end{bmatrix} \begin{bmatrix} v_i^* \\ w_i^* \end{bmatrix} + \begin{bmatrix} \cos(\psi_i - \psi_j) & -d \sin(\psi_i - \psi_j) \\ -\sin(\psi_i - \psi_j) & -d \cos(\psi_i - \psi_j) \end{bmatrix} \begin{bmatrix} v_i - v_i^* \\ w_i - w_i^* \end{bmatrix} \quad (8)$$

It can be found that forward velocity and angular velocity of the inner loop UAV are the inputs of outer loop formation model. Define v_i^*, w_i^* to be the expected forward velocity and angular velocity, then outer loop model will be stable by designing properly.

$$\begin{bmatrix} \dot{e}_{f_{ij}} \\ \dot{e}_{l_{ij}} \end{bmatrix} = \begin{bmatrix} -v_j - l_{ij}w_j \\ f_{ij}w_j \end{bmatrix} + \begin{bmatrix} \cos(\psi_i - \psi_j) & -d \sin(\psi_i - \psi_j) \\ -\sin(\psi_i - \psi_j) & -d \cos(\psi_i - \psi_j) \end{bmatrix} \begin{bmatrix} v_i^* \\ w_i^* \end{bmatrix} \quad (9)$$

To eliminate the steady-state error, a PI controller is designed as follows:

$$\begin{bmatrix} v_i^* \\ w_i^* \end{bmatrix} = \begin{bmatrix} \cos e_{\psi_{ij}} & -\sin e_{\psi_{ij}} \\ -\frac{1}{d} \sin e_{\psi_{ij}} & -\frac{1}{d} \cos e_{\psi_{ij}} \end{bmatrix} \times \begin{bmatrix} -(K_{P1}e_{f_{ij}} + K_{I1} \int e_{f_{ij}} dt) + v_j + l_{ij}w_j \\ -(K_{P2}e_{l_{ij}} + K_{I2} \int e_{l_{ij}} dt) - f_{ij}w_j \end{bmatrix} \quad (10)$$

Where $e_{\psi_{ij}} = \psi_i - \psi_j$, $K_{P1}, K_{P2}, K_{I1}, K_{I2} > 0$ are the feedback gains.

Apply (10) to (8):

$$\begin{bmatrix} \dot{e}_{f_{ij}} \\ \dot{e}_{l_{ij}} \end{bmatrix} = \begin{bmatrix} -(K_{P1}e_{f_{ij}} + K_{I1} \int e_{f_{ij}} dt) \\ -(K_{P2}e_{l_{ij}} + K_{I2} \int e_{l_{ij}} dt) \end{bmatrix} + \begin{bmatrix} \cos(\psi_i - \psi_j) & -d \sin(\psi_i - \psi_j) \\ -\sin(\psi_i - \psi_j) & -d \cos(\psi_i - \psi_j) \end{bmatrix} \begin{bmatrix} v_i - v_i^* \\ w_i - w_i^* \end{bmatrix} \quad (11)$$

The forward error and the lateral error will converge to 0, if $v_i(w_i)$ can track the expected $v_i^*(w_i^*)$.

3.2. Inner loop controller design

In order to make the inner loop output y track the expected y_r without $f(u, t)$ and $\phi(x, t)$ (they will be reconstructed in Section IV), a sliding mode controller is designed.

The sliding surface is defined as:

$$S = e_y + K_1 \int e_y dt \quad (12)$$

where $e_y = y_r - L_1 y$ and L_1 is a known constant matrix used to extract $v_i(w_i)$ from output to ensure y_r and $L_1 y$ have the same dimension. K_1 is a gain diagonal matrix with right dimension.

Consider the following Lyapunov function candidate:

$$V(t) = \frac{1}{2} S^T S > 0 \quad (13)$$

The derivative of (13) is identified as:

$$\dot{V} = S^T \dot{S} = S^T (\dot{y}_r - L_1 C \dot{x} + K_1 e_y) = S^T (\dot{y}_r - L_1 C (Ax + Bu + g(x)) + K_1 e_y) \quad (14)$$

Where $f(u, t)$ and $\phi(x, t)$ will be considered later in Part C, Section IV. To ensure $\dot{V} \leq 0$, the controller is designed as follows:

$$u = B^T (BB^T)^{-1} ((L_1 C)^T ((L_1 C)(L_1 C)^T)^{-1} (\dot{y}_r + K_1 e_y + \rho S^T \text{sign}(S)) - A\hat{x} - g(\hat{x})) \quad (15)$$

Where $\rho > 0$, $g(\hat{x})$ is known and the estimated state vector \hat{x} can be observed in Section 4. Apply (15) to (14), it can get

$$\dot{V} = -\rho S^T \text{sign}(S) \leq 0 \quad (16)$$

From (16), e_y converges to 0 in finite time. Then the inner loop output can track the expected outer loop input, and the UAVs formation system will be stable.

4. Active Fault Tolerant Controller Design

4.1. Sliding Mode Observer Design

In this part, a sliding mode observer will be presented to ensure the state estimation can be stable on the sliding surface.

Consider system (4), because the output matrix C is full rank and $p < n$, the output matrix C can always be transformed into the following form:

$$C = \begin{bmatrix} 0 & I_p \end{bmatrix} \quad (17)$$

Then the system (4) can be rewritten as:

$$\begin{cases} \dot{x}_1 = A_{11}x_1 + A_{12}x_2 + G_1(x, u) + D_1\phi(x, t) + E_1f(u, t) \\ \dot{x}_2 = A_{21}x_1 + A_{22}x_2 + G_2(x, u) + D_2\phi(x, t) + E_2f(u, t) \\ y = x_2 \end{cases} \quad (18)$$

Where $x_1 \in R^{n-p}$, $x_2 \in R^p$ are respectively components of the state matrix x . And $G_1(x, u)$, $G_2(x, u)$ are the same as x_1, x_2 . Also, the matrix A, D, E are divided into blocks with right dimension, like $A_{11} \in R^{(n-p) \times (n-p)}$.

Here, a linear transformation $z = Tx$ will be introduced.

$$T = \begin{bmatrix} I_{n-p} & P_1^{-1}P_2 \\ 0 & I_p \end{bmatrix} \quad (19)$$

Where

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} \quad (20)$$

Are in the equation (5). Before transform the system (18), a lemma will be introduced.

Lemma 1: If P, Q can satisfy the equation (7) and (20), then $P_1^{-1}P_2E_2 + E_1 = 0$ and $P_1^{-1}P_2D_2 + D_1 = 0$.

Proof:

$$E^T P = \begin{bmatrix} E_1^T P_1 + E_2^T P_2^T & E_1^T P_2 + E_2^T P_3 \end{bmatrix} = \begin{bmatrix} (E_1^T + E_2^T P_2^T P_1^{-1})P_1 & E_1^T P_2 + E_2^T P_3 \end{bmatrix} = F_2 C \quad (21)$$

Apply (17) to (21):

$$\begin{aligned} (E_1^T + E_2^T P_2^T P_1^{-1})P_1 &= 0 \\ (P_1(E_1 + P_1^{-1}P_2E_2))^T &= 0 \\ E_1 + P_1^{-1}P_2E_2 &= 0 \end{aligned} \quad (22)$$

The proof is achieved. $P_1^{-1}P_2D_2 + D_1 = 0$ is similar, so omit.

Then transform the system (18) into the new system z :

$$\begin{cases} \dot{z}_1 = (A_{11} + KA_{21})z_1 + (A_{12} + KA_{22} - (A_{11} + KA_{21})K)y + G_1(T^{-1}z, u) + KG_2(T^{-1}z, u) \\ \dot{z}_2 = A_{21}z_1 + (A_{22} - A_{21}K)z_2 + G_2(T^{-1}z, u) + D_2\phi(T^{-1}z, t) + E_2f(u, t) \\ y = z_2 \end{cases} \quad (23)$$

Where $K = P_1^{-1}P_2$, $z = \text{col}(z_1, z_2)$ with $z_1 \in R^{n-p}$. And Lemma 1 is used in (23).

Theorem 1: Consider the UAV model (4), Assumptions 1-3 are hold. After the transformation (19), the following sliding mode observer (24) of the UAV model is asymptotic stable on the sliding surface (28) if $\lambda_{\min}(Q_1) > 2L_g \| \begin{bmatrix} P_1 & P_2 \end{bmatrix} \|$.

$$\begin{cases} \dot{\hat{z}}_1 = (A_{11} + KA_{21})\hat{z}_1 + (A_{12} + KA_{22} - (A_{11} + KA_{21})K)y + G_1(T^{-1}\hat{z}, u) + KG_2(T^{-1}\hat{z}, u) \\ \dot{\hat{z}}_2 = A_{21}\hat{z}_1 + (A_{22} - A_{21}K)\hat{z}_2 + G_2(T^{-1}\hat{z}, u) + v \\ \hat{y} = \hat{z}_2 \end{cases} \quad (24)$$

Where $\|G(T^{-1}z, u) - G(T^{-1}\hat{z}, u)\| \leq L_g \|T^{-1}z - T^{-1}\hat{z}\|$,

$$v = (\|A_{22} - A_{21}K\| \|y - \hat{y}\| + \|D_2\| \gamma_1(T^{-1}\hat{z}, u) + \|E_2\| \gamma_2 + w) \text{sgn}(y - \hat{y}) \quad (25)$$

And W is a positive constant

Proof: Let $e_1 = z_1 - \hat{z}_1$, $e_y = y - \hat{y}$, the error dynamical equations are

$$\dot{e}_1 = (A_{11} + KA_{21})e_1 + \tilde{G}_1 + K\tilde{G}_2 \quad (26)$$

$$\dot{e}_y = A_{21}e_1 + (A_{22} - A_{21}K)e_y + \tilde{G}_2 + D_2\phi(T^{-1}z, t) + E_2f(u, t) - v \quad (27)$$

Where $\tilde{G}_i = G_i(T^{-1}z, u) - G_i(T^{-1}\hat{z}, u)$.

Consider the sliding surface

$$S = \{(e_1, e_y) | e_y = 0\} \quad (28)$$

And the Lyapunov candidate function is

$$V_1 = e_1^T P_1 e_1 \quad (29)$$

The derivative of (29) is identified as:

$$\dot{V} = e_1^T \left[P_1 (A_{11} + KA_{21}) + (A_{11} + KA_{21})^T P_1 \right] e_1 + 2e_1^T P_1 \begin{bmatrix} I_{n-p} & K \end{bmatrix} \begin{bmatrix} \tilde{G}_1 & \tilde{G}_2 \end{bmatrix}^T \quad (30)$$

Apply (20) to (5), and the first block is

$$P_1 (A_{11} + KA_{21}) + (A_{11} + KA_{21})^T P_1 = -Q_1 \quad (31)$$

From (30) and (31)

$$\begin{aligned} \dot{V} &= -e_1^T Q_1 e_1 + 2e_1^T \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} \tilde{G}_1 & \tilde{G}_2 \end{bmatrix}^T \leq -e_1^T Q_1 e_1 + 2L_g \|e_1^T\| \left\| \begin{bmatrix} P_1 & P_2 \end{bmatrix} \right\| \|T^{-1}(z - \hat{z})\| \\ &\leq -\lambda_{\min}(Q_1) \|e_1\|^2 + 2L_g \|e_1^T\| \left\| \begin{bmatrix} P_1 & P_2 \end{bmatrix} \right\| \|e_1\| \leq -\|e_1\|^2 (\lambda_{\min}(Q_1) - 2L_g \left\| \begin{bmatrix} P_1 & P_2 \end{bmatrix} \right\|) \leq 0 \end{aligned} \quad (32)$$

The proof is achieved.

At last, W will be designed to ensure the system can reach the sliding surface in finite time.

$$\begin{aligned} e_y^T \dot{e}_y &= e_y^T (A_{21}e_1 + (A_{22} - A_{21}K)e_y + \tilde{G}_2 + D_2\phi(T^{-1}z, t) + E_2f(u, t) - v) \\ &\leq \|e_y^T\| (\|A_{21}\| + L_g + \|D_2\| L_\phi) \|e_1\| - w \end{aligned} \quad (33)$$

Where $\|\gamma_1(T^{-1}z, t) - \gamma_1(T^{-1}\hat{z}, t)\| \leq L_\phi \|T^{-1}z - T^{-1}\hat{z}\|$.

From equation (31), $A_{11} + KA_{21}$ is stable. Therefore, in equation (26), because \tilde{G}_i is bounded, so e_1 is bounded. For convenience, assume that $\|e_1\| \leq \eta$. Then W can be designed as follows:

$$W > (\|A_{21}\| + L_g + \|D_2\| L_\phi) \eta + \sigma \quad (34)$$

Where σ is a positive constant. Then the reachability is proved.

4.2. Outer loop controller design

During the sliding motion, $e_y = 0, \dot{e}_y = 0$. Then equation (27) will be changed as follows:

$$0 = A_{21}e_1 + \tilde{G}_2 + D_2\phi(T^{-1}z, t) + E_2f(u, t) - v_{eq} \quad (35)$$

And because $\lim_{t \rightarrow \infty} e_1 = 0$

$$v_{eq} = \begin{bmatrix} D_2 & E_2 \end{bmatrix} \begin{bmatrix} \phi(T^{-1}z, t) \\ f(u, t) \end{bmatrix} \quad (36)$$

Assumption 4: There exists a nonsingular matrix $M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \in R^{p \times p}$ such that

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \begin{bmatrix} D_2 & E_2 \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \\ 0 & H_3 \end{bmatrix} \quad (37)$$

Where $H_1 \in R^{(p-q) \times r}$, $H_3 \in R^{q \times r}$.

Multiplying by M on both sides of the equation (36)

$$Mv_{eq} = \begin{bmatrix} H_1 & H_2 \\ 0 & H_3 \end{bmatrix} \begin{bmatrix} \phi(T^{-1}z, t) \\ f(u, t) \end{bmatrix} \quad (38)$$

Then the fault can be reconstructed by

$$\hat{f} = H_3^{-1} M_2 v_{eq} \quad (39)$$

And $\phi(x, t)$ can be reconstructed by

$$\hat{\phi} = H_1^{-1} (M_1 - H_2 H_3^{-1} M_2) v_{eq} \quad (40)$$

Where

$$v_{eq} = (\|A_{22} - A_{21}K\| \|y - \hat{y}\| + \|D_2\| \gamma_1 (T^{-1}\hat{z}, u) + \|E_2\| \gamma_2 + W) \frac{y - \hat{y}}{\|y - \hat{y}\| + \delta} \quad (41)$$

Where δ is a small positive constant.

4.3. Fault tolerant Controller Design

The system (4) without $f(u, t)$ and $\phi(x, t)$ is stable under the control law (15), and $f(u, t)$, $\phi(x, t)$ are both reconstructed in part B, so an additional compensate controller is designed.

Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2} S^T S > 0 \quad (42)$$

The derivative of (42) is identified as:

$$\dot{V} = S^T \dot{S} = S^T (\dot{y}_r - L_1 C \dot{x} + K_1 e_y) = S^T (\dot{y}_r - L_1 C (Ax + Bu + g + D\phi + Ef) + K_1 e_y) \quad (43)$$

To ensure $\dot{V} \leq 0$, the controller is designed as follows:

$$u^* = u + u_f \quad (44)$$

Where $u_f = -B^T (BB^T)^{-1} (D\hat{\phi} + E\hat{f})$

Apply (44) to (43), it can get:

$$\dot{V} = -\rho S^T \text{sgn}(S) \leq 0 \quad (45)$$

5. Simulation Results

In this part, 5 UAVs are used in the simulation. The formation structure is shown in Fig 2.



Fig 2. Formation structure

The model of inner loop UAV [16] is given as follows:

$$A = \begin{bmatrix} A_{lon} & 0 \\ 0 & A_{lat} \end{bmatrix}, B = \begin{bmatrix} B_{lon} & 0 \\ 0 & B_{lat} \end{bmatrix}, C = \begin{bmatrix} C_{lon} & C_{lat} \end{bmatrix}$$

$$A_{lon} = \begin{bmatrix} -0.0334 & -2.9770 & 0 & -9.8100 \\ -0.0016 & -4.133 & 0.986 & 0 \\ 0.0077 & -140.200 & -4.435 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_{lat} = \begin{bmatrix} -0.732 & 0.0143 & -0.996 & 0.0706 \\ -893.000 & -9.0590 & 2.044 & 0 \\ 101.637 & 0.0186 & -1.283 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B_{lon} = \begin{bmatrix} -1.075 & 0.2453 \\ -0.347 & 0 \\ -140.22 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_{lat} = \begin{bmatrix} 0 & 0.244 \\ 328.653 & 308.498 \\ 47.528 & -102.891 \\ 0 & 0 \end{bmatrix}$$

$$C_{lon} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C_{lat} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The desired v^* and w^* of the leader UAV are $36m/s$ and $0.1rad/s$. And $|f_{ij}^d| = 20m$, $|l_{ij}^d| = 20m$.

Take any one follower UAV as an example, when there are no $f(u,t)$ and $\phi(x,t)$, results are as follows:

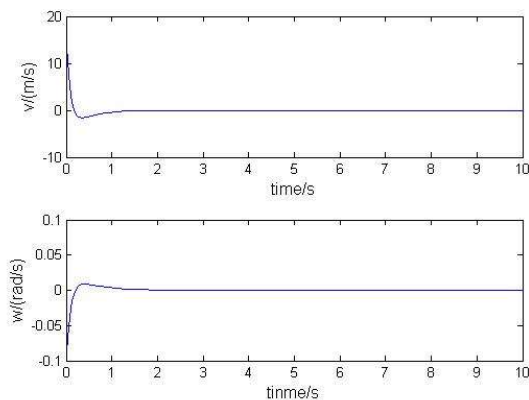


Fig 3. forward velocity error and angular velocity error

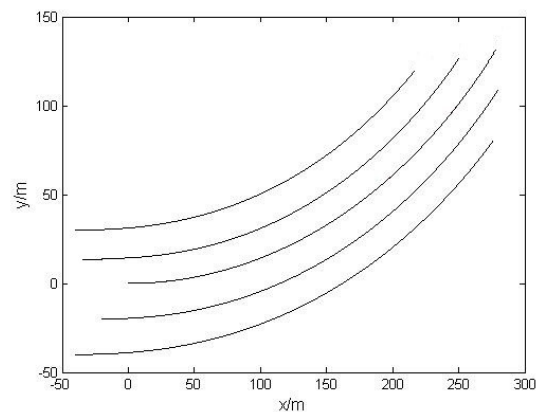


Fig 4. formation flight trajectory

From Fig 3 and Fig 4 above, it is obvious that forward velocity and angular velocity can track the expected v_i^* and w_i^* in 2 seconds, and the formation performance is quite good. This also proved that the inner loop controller and the outer loop controller are effective and the whole formation is stable.

Take the longitudinal equation of any one UAV as an example, $f(u, t)$ and $\phi(x, t)$ will be considered. And the fault happens at 5 seconds. In input channel 1, fault has the following form

$$f_1(u, t) = \begin{cases} 0 & , 0 \leq t < 5 \\ 0.3 \sin t, & 5 \leq t < 10 \end{cases}$$

And in input channel 2, fault is

$$f_2(u, t) = \begin{cases} 0 & , 0 \leq t < 5 \\ 0.1, & 5 \leq t < 10 \end{cases}$$

And in the whole simulation process, $\phi(x, t) = 0.05 \cos t$ and $D = [1 \ -2.7727 \ 0 \ 5]^T$. Then choose

$$L = \begin{bmatrix} -2.9323 & 0.0643 & 0.0570 \\ 7.3146 & -0.5569 & 1.4470 \\ -0.5796 & 1.7905 & -2.4022 \\ 7.1042 & -21.9799 & 154.0866 \end{bmatrix} \quad Q = \begin{bmatrix} 27.6202 & 9.8562 & 3.5970 & -3.7471 \\ 9.8562 & 8.3607 & 1.9534 & -1.2967 \\ 3.5970 & 1.9534 & 14.6729 & 1.1950 \\ -3.7471 & -1.2967 & 1.1950 & 6.8819 \end{bmatrix}$$

After calculation,

$$P = \begin{bmatrix} 3.9562 & 1.4078 & 0.0835 & -0.0106 \\ 1.4078 & 1.1454 & 0.0706 & -0.0094 \\ 0.0835 & 0.0706 & 0.9777 & 0.0116 \\ -0.0106 & -0.0094 & 0.0116 & 0.0220 \end{bmatrix}$$

Let

$$F_1 = [-1.8153 \ -0.0542 \ 0.1255] \quad F_2 = \begin{bmatrix} 0.1189 & -1.3601 & -3.0674 \\ -0.0107 & -4.0236 & -0.0502 \end{bmatrix}$$

Then

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1.8033 & 0 & 1 \end{bmatrix} \quad H = \begin{bmatrix} -2.7727 & -1.075 & 0.2453 \\ 0 & 0.347 & -4.133 \\ 0 & -142.1585 & 0.4424 \end{bmatrix}$$

Results are as follows:

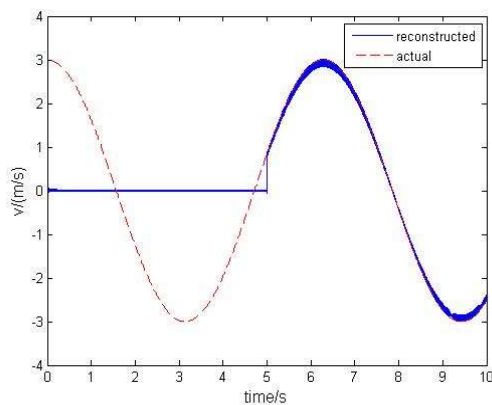


Fig 5. reconstructed fault 1

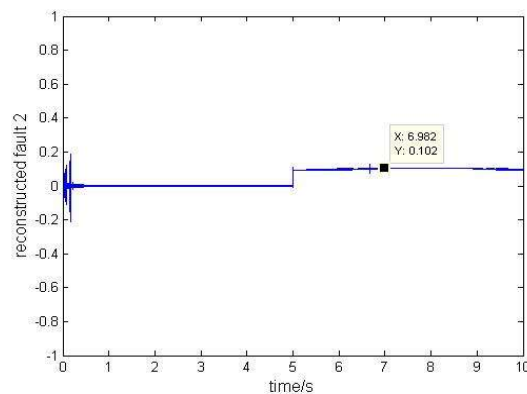
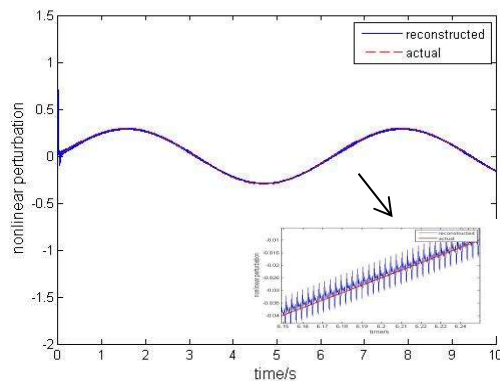
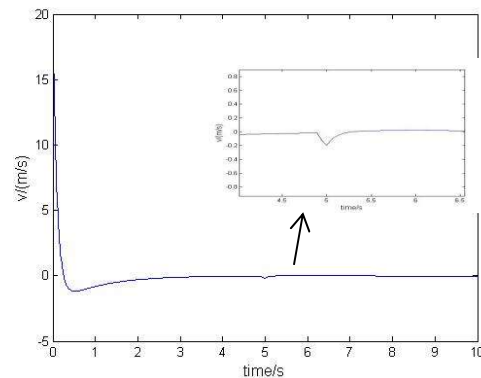


Fig 6. reconstructed fault 2

**Fig 7.** reconstructed nonlinear perturbation**Fig 8.** forward velocity error with fault tolerant control

From Fig 5, Fig 6 and Fig 7 above, it can be found that the $f(u,t)$ and $\phi(x,t)$ added can be constructed by the sliding mode observer exactly. And in this simulation, all the sign function is replaced with saturation function to reduce chattering, and the boundary layer is 0.05. Then from Fig 8, the forward velocity error is influenced slightly when fault happens at 5 seconds. Also, angular velocity error is similar. At last, formation flight trajectory is nearly the same. Generally speaking, the inner loop controller and the outer loop controller is effective, the formation is stable. And the sliding mode observer can reconstruct the fault in a quite short time.

6. Conclusion

In this paper, an active FTC method for UAVs formation with actuator faults is developed. The inner loop controller ensures every single UAV track the desired signal and the outer loop controller maintains the stability of UAVs formation. And in the FTC part, a sliding mode observer is designed to reconstruct the actuator faults and then eliminate the influence of the actuator faults. Simulation results verify that the active FTC scheme proposed for the UAVs formation system with actuator faults is effective. At last, the validity of all methods is proved by MATLAB.

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