Optimization of installing angles of photovoltaic panels to maximize solar radiation by simulated annealing algorithm

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Abstract. In this study, simulated annealing (SA) algorithm was used to optimize the installing angles, specifically the tilt angle and surface azimuth angle, to maximize the solar radiation on photovoltaic (PV) panels in Wuhan, China. The KT model was adopted to calculate the monthly average solar radiation on the tilt PV panels and the SA algorithm was chosen to do the optimization. The optimized installing angles obtained by the KT method was compared with those obtained by the experimental solar radiation information, and the results agreed quite well. It is found that the summer in Wuhan has the largest monthly solar radiation. The optimized tilt angle varies in different months but the optimized surface azimuth angle is approximate 0 degree. It is implied that the tilt angle is the key parameter in the PV installation rather than the azimuth angle. For the yearly total solar radiation, the optimized tilt angle is 15°, which is about 15.52° less than the latitude.

Keywords. Photovoltaics, Power generation, solar energy, simulated annealing algorithm.

1. Introduction

Due to the increasing demand of energy consumption and environmental protection, renewable energy has been developed fast in these decades, like solar energy, wind power, hydropower, biomass, geothermal energy, etc [1-4]. Among the many types, solar energy is considered as the most promising since all other kinds of energy comes from solar after all. Photovoltaic (PV) technology is the most widely used approaches to utilize the solar energy and has advanced rapidly in recent years. PV panels convert part of the absorbed solar energy into electrical power; thus they are clean energy devices which do not release hazardous pollutants into the environment. One of the key factors when installing a PV panel is to achieve the maximum solar energy irradiance and to avoid shading [4]. One can imagine that the PV panel must be positioned so that the sun rays arrive at the panel vertically. This is due to the decrease of reflected energy. If not, the solar reflection increases and the output power is not as much as it could. In order to maximize the possible daily solar energy irradiance on the PV panels, one solution is to use sun tracking system. A tracker is a mechanical device that follows the direction of the sun on its daily sweep across the sky. However, these systems are expensive and fail to apply in some case. The most preferable way is to use fixed installation with optimized installing angles, like tilt angle and surface azimuth angle, etc.

As the installing angles are important to maximize solar irradiance, many people have spared no effort in the optimization of these installing angles recently based on observation of specific diagrams, empirical relationships, or based on some optimization methodologies. Rhodes et al. [5] proposed a mutli-objective assessment method to analyze the effect of PV array orientation and tilt on energy production in US. Seme and Stumberger [6] used the Differential Evolution algorithm to predict the solar angles for a dual-axis sun tracking purpose. Chang [7-9] developed an ant direction hybrid differential evolution algorithm to determine the tilt angle for PV modules in Taiwan. Celik and Muneer [10] used artificial neural network (ANN) algorithm to optimize the tilt angle to increase the solar incident on the PV surface. Talebizadeh et al. [11] adopted the genetic algorithm (GA) to optimize the slop and surface azimuth angles to maximize the solar radiation in Iran.

As far as we know, though people have done much work in terms of the tilt angle's optimization, but other installing angles' optimization is scarcely referred. It is sure that the tilt angle is the most important one, but other angles also influence the incident solar irradiance. One acceptable practical optimization should include all the installing angles simultaneously and comply with the local characteristics of the installation sites.

In this study, we adopted the simulated annealing (SA) algorithm to optimize the installing angles of PV panels in Wuhan, China. The detailed algorithm was introduced and the optimization results were compared with experimental data for verification. The yearly optimized installing angles were presented.

2. Mathematic Modeling

As shown in Fig. 1, the total monthly average daily radiation $\overline{H}_T$ is the sum of beam, diffuse, and reflecting solar radiations:

$$\overline{H}_T = \overline{H}_b + \overline{H}_d + \overline{H}_r \tag{1}$$

where $\overline{H}_b$ is the monthly average daily beam radiation on a horizontal surface, $\overline{H}_d$ is the monthly average daily diffuse radiation on the horizontal surface, $\overline{H}_r$ is the monthly average daily ground reflected radiation on the horizontal surface.

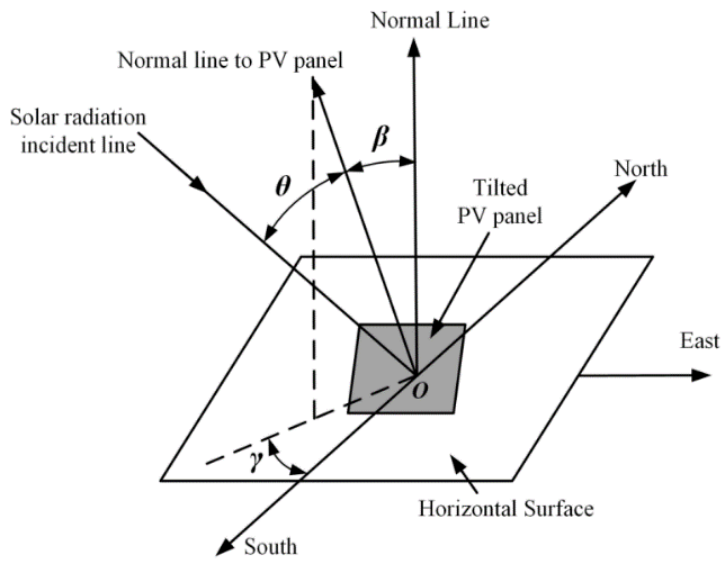

Figure 1. Geometrical relationships

To calculate Eq. (1), Liu and Jordan [12] first developed the isotropic method which is later extended by Klein [13]. But this method just can be applied when the azimuth angle (γ) of the PV panel is zero. To overcome this limitation, Klein and Theilacker [14] developed another alternative model, which is known as KT method. According to this method, the total monthly average daily solar radiation $\overline{H}_T$ can be expressed as:

$$\overline{H}_T = \overline{H} \cdot \overline{R} \tag{2}$$

With

$$\overline{R} = D + \frac{\overline{H}_d}{\overline{H}}\left(\frac{1+\cos\beta}{2}\right) + \rho_g\left(\frac{1-\cos\beta}{2}\right) \tag{3}$$

Where

$$D = \begin{cases} \max\{0, G(\omega_{ss}, \omega_{sr})\} & if\ \omega_{ss} \ge \omega_{sr} \\ \max\{0, [G(\omega_{ss}, -\omega_{sr}) + G(\omega_s, \omega_{sr})]\} & if\ \omega_{ss} \le \omega_{sr} \end{cases} \tag{4}$$

$$G(\omega_1, \omega_2) = \frac{1}{2d}\left[\left(\frac{bA}{2} - aB\right)(\omega_1 - \omega_2)\frac{\pi}{180} + (aA - bB)(\sin\omega_1 - \sin\omega_2) - aC(\cos\omega_1 - \cos\omega_2) - \frac{bA}{2}(\sin\omega_1\cos\omega_1 - \sin\omega_2\cos\omega_2) + \frac{bC}{2}(\sin^2\omega_1 - \sin^2\omega_2)\right] \tag{5}$$

Sunrise angle ωsr and sunset angle ωss are calculated as follows.

$$|\omega_{sr}| = \min\left[\omega_s, \cos^{-1}\frac{AB + C\sqrt{A^2 - B^2 + C^2}}{A^2 + C^2}\right] \tag{6}$$

$$\omega_{sr} = \begin{cases} -|\omega_{sr}| & if\ (A > 0\ and\ B > 0)\ or\ (A \ge B) \\ +|\omega_{sr}| & otherwise \end{cases} \tag{7}$$

$$|\omega_{ss}| = \min\left[\omega_s, \cos^{-1}\frac{AB - C\sqrt{A^2 - B^2 + C^2}}{A^2 + C^2}\right] \tag{8}$$

$$\omega_{ss} = \begin{cases} +|\omega_{ss}| & if\ (A > 0\ and\ B > 0)\ or\ (A \ge B) \\ -|\omega_{sr}| & otherwise \end{cases} \tag{9}$$

The necessary coefficients are A=cosβ+tanϕcosγsinβ, B=cosωscosβ+tanδsinβcosγ, C=sinβsinγ/cosϕ, $a = 0.4090 + 0.5016\sin(\omega_s - 60) - \overline{H}_d / \overline{H}$, b=0.6609-0.4767sin(ωs-60), and d=sinωs –πcosωs/180, where β is the tilt angle (0≤β≤180°), ρg is the ground albedo (=0.14 in this study), ϕ is the latitude, γ is the surface azimuth angle, δ is the declination angle, ωs is the hour angle.

From above equations, it is seen that we need the monthly average daily radiation on a horizontal surface ($\overline{H}$) and the monthly average daily diffuse radiation on the horizontal surface ($\overline{H}_d$) first. Usually, the information of solar radiation on the horizontal surface ($\overline{H}$) is available, but $\overline{H}_d$ is not available in some sites. If we fail to obtain $\overline{H}_d$, KT method also involves a way to estimate it. The monthly average clearness index KT is defined as the ratio of $\overline{H}$ and the monthly average daily extraterrestrial radiation ($\overline{H}_0$). For latitude of +60° to -60°, $\overline{H}_0$ can be calculated for the average day of month as:

$$\overline{H}_0 = \frac{24\times 3600 G_{SC}}{\pi}\left(1+0.033\cos\frac{360n}{365}\right)\times(\cos\phi\cos\delta\sin\omega_s + \frac{\pi\omega_s}{180}\sin\phi\sin\delta) \tag{10}$$

Where GSC is the solar constant and n is the day of the year from January 1st. When KT falls in the range of between 0.3 and 0.8, $\overline{H}_d$ can be calculated

$$\overline{H}_d = \begin{cases}(1.391-3.56K_T+4.189K_T^2-2.317K_T^3)\overline{H} & if\ \omega_s \le 81.4° \\ (1.311-3.022K_T+3.427K_T^2-1.821K_T^3)\overline{H} & otherwise\end{cases} \tag{11}$$

3. Simulated Annealing Algorithm

Simulated snnealing (SA) algorithm, introduced in 1983 by Kirkpatrick et al., is a random-search technique which exploits an analogy of the metal annealing process [15]. SA's major advantage over other methods is an ability to avoid becoming trapped in local minima, thus SA is a generic probabilistic metaheuristic for the global optimization problem. SA not only accepts changes the decrease the objective function, but also accepts some changes that increases the objective function with certain probability

$$p = \exp\left(-\Delta f \ / \ T_{Anneal}\right) \tag{12}$$

Where Δf is the increase of the objective function and TAnneal is the annealing temperature.

For the problem stated in this paper, the flowchart is shown in Fig. 2. The key steps of the SA procedure in this study are:

(1). Randomly choose an initial solution set D (all the installing angles), calculate its solar radiation $\overline{H}_T$; set the annealing temperature is t0= $\overline{H}_T$;

(2). Generate a new solution set Dnew, calculate its solar radiation $\overline{H}_{T\_new}$;

(3). If $\overline{H}_{T\_new} > \overline{H}_T$ or $\exp\left((\overline{H}_{T\_new} - \overline{H}_T)/t_{anneal}\right)$ is less than a random number X (0<X<1), substitute the present solution set D with the new solution set D_{new};

(4). If the stop condition is satisfied, stop the loops; otherwise decrease the annealing temperature by αt0. Go to step (2).

The initial solution is generated by randomly choose a reasonable installing angles (β, γ, δ). The random number X is generated by a uniform distribution random number generator. The annealing coefficient, α, is set as 0.9. The stop conditions have two forms: one is that the iteration times equal to K where K is a rather large (like 1000); the other is that the solar radiation $\overline{H}_T$ doesn't change for K' times.

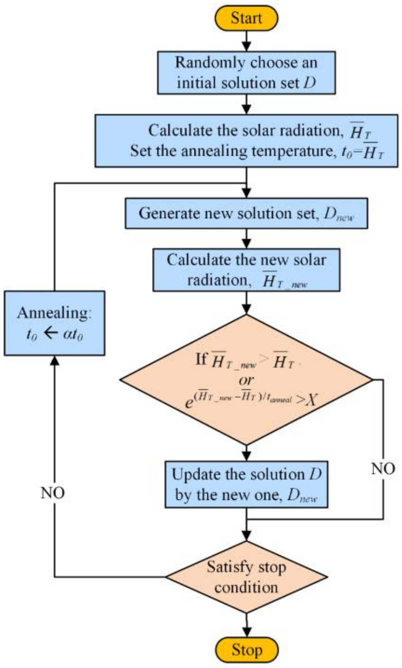

Figure 2. Flowchart of the SA algorithm for installing angles' optimization. X is a random number between 0 and 1. α is the annealing coefficient.

Table 1. Solar radiation information (MJ/m2)

Month	Date	$\overline{H}$	$\overline{H}_d$
Jan.	17	5.375	3.763
Feb.	16	5.514	4.277
Mar.	16	7.949	5.587
Apr.	15	9.722	5.946
May	15	12.843	7.422
Jun.	11	14.593	6.638
Jul.	17	18.422	5.972
Aug.	16	15.00	7.920
Sep.	15	12.793	6.690
Oct.	15	10.397	5.594
Nov.	14	7.812	4.627
Dec.	10	7.444	4.400

4. Results and Discussions

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Wuhan is located in central Hubei province, China (E114°20', N30°31'). Wuhan has a subtropical monsoon climate with abundant rainfall and distinctive four seasons. It is known for its oppressive sultry summer, when the temperature is often as high as 35° or more known. The average temperature in July is 37.2°C (99°F) and the maximum of often above 40°C (104°F). Spring and autumn are generally mild, and winters are cool with occasional snow. In the past thirty years, the average annual rainfall of 1269 mm, mainly from June to August; annual temperature is -17.5°~15.8°, annual frost-free period lasts 211~272 days, and the annual sunshine duration in 1810~2100 hours.

To calculate the solar radiation on the tilt surface, we should first obtain the solar radiation on the horizontal surface. These experimental monthly horizontal radiation data employed in this study were borrowed from a book edited by Chinese Meteorological Bureau [16], as shown in Table 1. Together with the radiation information, the recommended average days for months, which can be used to calculate the n in a year, are also listed in Table 1.The annual collectible radiation was obtained by summing monthly collectible radiation of each month in a year.

The calculated maximum monthly average solar radiation on the tilt PV panels $\overline{H}_T$ with the optimized installing angles is shown in Fig. 3. The trend is in accordance with the monthly average solar radiation on the horizontal surface $\overline{H}$ in Wuhan, where the summer has larger solar radiation. The optimized tilt angle β and surface azimuth angle at different months of a year are shown in Fig. 4. We compared the optimized installing angles obtained by the KT method and the experimental data borrowed from Ref. [16]. As shown in Fig. 4, the present results are in good agreement with each other, which implied that KT method can be used to predict the $\overline{H}_d$ and $\overline{H}_T$ well. The optimized tilt angle β decreases From January, and becomes smallest in June, and then begins to increase until December. While the optimized surface azimuth angle is approximate zero, which is consistent with Duffie and Beckman [17]. It also proves that the optimized tilt angle is the key parameter for PV panels to receive the maximum input solar energy. The optimized tilt angle β on yearly total solar radiation on tilt PV panels is shown in Fig. 5. It is seen that with the increase of tilt angle, the yearly total solar radiation increase first before reaching a peak and then decreases sharply. The peak point corresponds to the yearly optimized tilt angle of 15° in Wuhan, which is about 15.52° less than the latitude.

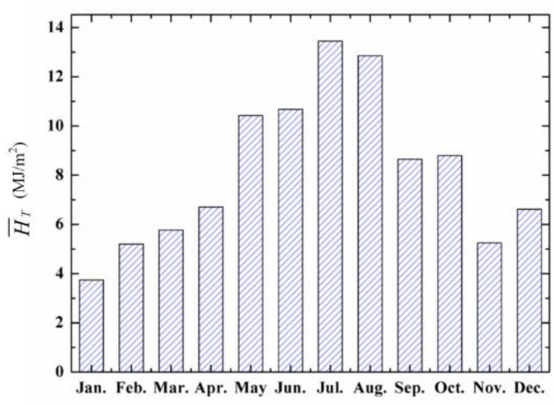

Figure 3. Monthly solar radiation on the tilt PV panels

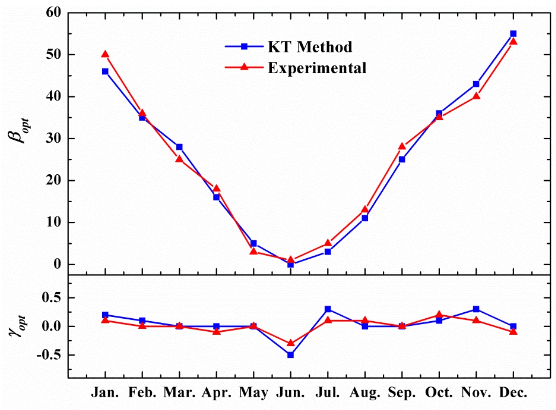

Figure4. Optimized tilt angle β and azimuth angle γ in each month

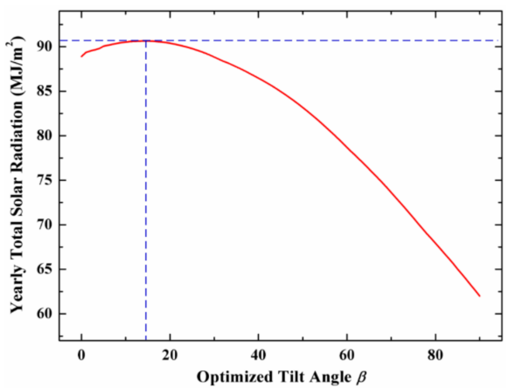

Figure 5. Optimized tilt angle β on yearly total solar radiation on tilt PV panels.

5. Conclusion

In this study, we applied simulated annealing (SA) algorithm to optimize the installing angles, specifically the tilt angle and surface azimuth angle, to maximize the solar radiation on photovoltaic

(PV) panels so as to maximize the electrical power generation in Wuhan, China. Detailed mathematic modeling process (KT method) and the SA algorithm procedure were introduced. The optimized installing angles calculated by the KT method were compared with those obtained by the experimental solar radiation information, and the results agreed quite well. It is found that the optimized tilt angle varies in different months but the optimized surface azimuth angle is approximate 0 degree. It is implied that the tilt angle is the key parameter in the PV installation rather than the azimuth angle. For the yearly total solar radiation, the optimized tilt angle is 15°, which is about 15.52° less than the latitude.

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