Nonlinear Dynamic Modeling and Simulation Analysis of 6DOF Underwater Vehicles

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Abstract. According to the translation in three directions, pitch, sway and roll motion of underwater vehicles, and dividing the force of underwater vehicles into three independent parts, such as hull hydrodynamic force, oar force and rudder hydrodynamic force, the motion equations of underwater vehicles are deduced. Then the nonlinear kinematics model of underwater vehicles with six DOF is established in Matlab/Simulink, and the simulation analysis of horizontal rotation and spiral floating is carried out. The results show that the model can accurately reflect the characteristics of underwater vehicles motion, and can realize various motion modes of different heading control modes, which lays a foundation for the research of underwater vehicles trajectory prediction and trajectory control.

Keywords: Six DOF, underwater vehicles, nonlinear, kinematics modelling.

1. Introduction

In the course of sailing, underwater vehicles are often disturbed by the natural environment such as wind and waves. How to ensure the safety of underwater vehicles navigation is the research focus of underwater vehicles design stage. The simulation study of underwater vehicles motion characteristics depends on the accurate underwater vehicles' kinematics model [1].

There are many forms of underwater vehicles kinematics model used, such as American Edgar equation, Swedish Lorebin equation, other scholars proposed models by Japanese, Russian, etc. [2-3]. These models can reflect the nature of underwater vehicles underwater motion and the basic laws of operating response, but they are divided into mathematical description and theory. Based on the above model, Michael and Gerrit et al. studied the stability control of the linear model of vertical plane in underwater vehicles [4-5]. Wang Zheng Guo studied the decoupling control of vertical plane motion while ignoring the nonlinear factors in underwater vehicles motion characteristics [6]. These studies have achieved great results.

In regular waves, the holistic modeling is used in the analysis of underwater vehicles motion in the past. The underwater vehicles, the paddle and the rudder is regarded as a unified whole in this approach. Although the interference between the three is ignored, the model is too complex, and there is a coupling relationship between the various items of the model, and the physical meaning is vague. In order to analyze the underwater vehicles motion and facilitate the transformation between the mathematical model and the solid underwater vehicles, underwater vehicles, oars, and rudders are regarded as separate

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parts of each other in this paper. According to the horizontal motion of the underwater vehicles in three degrees of freedom, it combines vertical, vertical, and horizontal motion. The kinematics model of the underwater vehicles with six degrees of freedom is established based on the correlation theory, and the simulation analysis of the typical conditions such as horizontal plane rotation and spiral floating is carried out.

2. Underwater vehicles coordinate system
There are two coordinate systems in the course of underwater vehicles movement. Ground coordinate system is regarded as fixed coordinate system and Hull coordinate system as motion coordinate system, as shown in Figure 1.
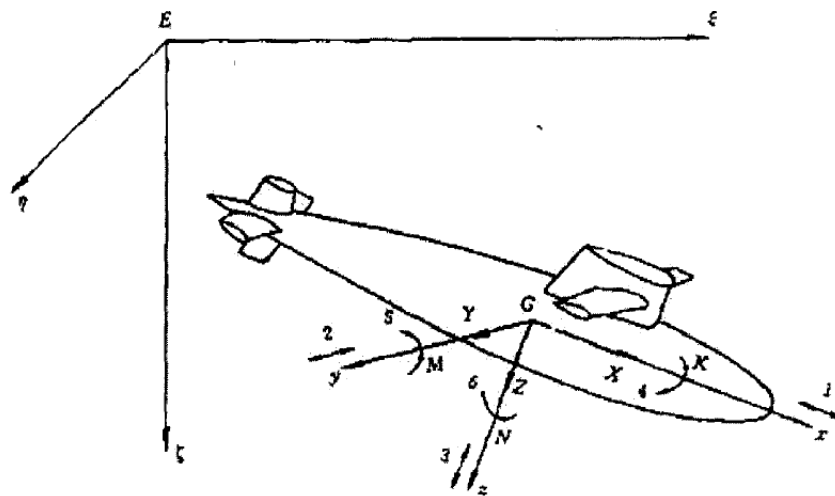
Figure 1. Underwater vehicles motion coordinate system.

To facilitate the description of the hull motion, fixed coordinate system $E-\eta\xi\zeta$ is assumed in the earth surface. It does not change with time and the movement of the hull. Generally, in the time of t=0 the center of gravity of the underwater vehicles is located at the origin E of the fixed coordinate system. ξ axis is in the horizontal plane whose positive direction is usually the general direction of motion of the underwater vehicles. Rotate the ξ axis clockwise 90° in the hydrostatic plane, which is the positive direction of η axis. The positive direction of ς is towards the center of the earth. The dynamic coordinate system with the hull movement is $G-xyz$. The center of mass is usually the origin G of the coordinates. The vertical line of the transverse section is the X axis. Its direct direction points to the bow. The vertical line of the longitudinal section is the Y-axis, and the coordinate system obeys the right-hand rule.

In a fixed coordinate system $E-\eta\xi\zeta$, hull movement is expressed by the swinging angle ψ, the horizontal inclination φ, inclination θ and underwater vehicles center of gravity position X_G, Y_G, Z_G. Meanwhile, ψ turn right to positive, φ lean to the right and θ lean to the end.

3. Six degrees of freedom space motion equation for underwater vehicles
The motion of a underwater vehicles can be represented by a velocity vector u、v、w and angle speed vector p、q、r in a moving coordinate system. It also can be represented by using the derivative of the position vector and the derivative of the attitude vector $\dot{\psi}$、$\dot{\varphi}$、$\dot{\theta}$. The influence of hull hydrodynamic force, propeller thrust and rudder hydrodynamic in underwater vehicles motion is considered, and the influence of subsidiary control, fluid power, and external forces is also considered.

The "slow motion" hypothesis is introduced [7], in this way, the underwater vehicles can be approximated as a rigid body when moving in space. According to the momentum theorem $m\dfrac{dV_c}{dt}=F$

and momentum moment theorem $dM = R \times dm \dfrac{dV_c}{dt}$. The kinematics equation of the underwater vehicles' six degrees of freedom is derived as follows.

a. Axial equation

$$
\begin{aligned}
&m(\dot{u} - rv + wq) \\
&= \frac{1}{2}\rho L^4 (X'_{qq}q^2 + X'_{rr}r^2 + X'_{rp}rp) + \frac{1}{2}\rho L^3 \\
&(X'_{\dot{u}}\dot{u} + X'_{vr}vr + X'_{wq}wq) + \frac{1}{2}\rho L^2(X'_{uu}u^2 + X'_{vv}v^2 \\
&+ \frac{1}{2}\rho L^2 u^2 (X'_{\delta_r \delta_r}\delta_r^2 + X'_{\delta_s \delta_s}\delta_s^2 + X'_{\delta_b \delta_b}\delta_b^2) + F_{xp} + F_x
\end{aligned}
\tag{1}
$$

Where different L power terms represent different fluid forces and moments.

$$
F_{xp} = \begin{cases} X_T - X_R \\ \dfrac{\rho}{2}L^2[a_T u^2 + b_T u u_c + c_T u_c^2] \end{cases}
\tag{2}
$$

XT and XR is the longitudinal component of thrust and resistance, respectively. u_c is command speed. F_x is the longitudinal component of the thrust generated by the subsidiary control. In additional, when there is a thrust characteristic curve, F_{xp} is the first item. Conversely, F_{xp} is the second item.

b. Horizontal equation

$$
\begin{aligned}
&m(\dot{v} - wp + ur) \\
&= \frac{1}{2}\rho L^4 (Y'_r \dot{r} + Y'_p \dot{p} + Y'_{pq}pq) + \frac{1}{2}\rho L^3 (Y'_{\dot{v}}\dot{v} + Y'_r ur + \\
&Y'_p up + Y'_{wp}wp + Y'_{v|r|}\frac{v}{|v|}(v^2 + w^2)^{\frac{1}{2}}|r|) + \frac{1}{2}\rho L^2 (Y'_v uv \\
&+ Y'_{vw}vw + Y'_{v|v|}(v^2 + w^2)^{\frac{1}{2}}) + \frac{1}{2}\rho L^2 (Y'_{\delta_r}u^2\delta_r) + F_y
\end{aligned}
\tag{3}
$$

c. Vertical equation

$$
\begin{aligned}
&m(\dot{w} - qu + pv) \\
&= \frac{1}{2}\rho L^4 (Z'_q \dot{q} + Z'_{rr}r^2 + Z'_{rp}rp) + \frac{1}{2}\rho L^3 (Z'_{\dot{w}}\dot{w} + Z'_q uq + \\
&Z'_{vr}vr + Z'_{vp}vp + Z'_{w|q|}\frac{w}{|w|}(v^2 + w^2)^{\frac{1}{2}}|q|) + \frac{1}{2}\rho L^2 (Z'_* u^2 + \\
&Z'_w uw + Z'_{w|w|}w(v^2 + w^2)^{\frac{1}{2}} + Z'_{vv}v^2) + \frac{1}{2}\rho L^2 (Z'_{\delta_s}u^2\delta_s + Z'_{\delta_b}u^2\delta_b)
\end{aligned}
\tag{4}
$$

d. Horizontal equation

$$
\begin{aligned}
&I_x \dot{p} + (I_x - I_y)qr \\
&= \frac{1}{2}\rho L^5 (K'_{\dot{p}}\dot{p} + K'_{\dot{r}}\dot{r} + K'_{|p|p}p|p| + K'_{qr}qr) \\
&+ \frac{1}{2}\rho L^4 (K'_{\dot{v}}\dot{v} + K'_p up) + \frac{1}{2}\rho L^3 (K'_v uv + K'_{vw}vw + \\
&K'_{|v|v}v(v^2 + w^2)^{\frac{1}{2}}) + \frac{1}{2}\rho L^3 (K'_{\delta_r}u^2 \delta_r) - mgh\cos\theta\sin\varphi + M_x
\end{aligned}
\tag{5}
$$

e. Longitudinal equation

$$
\begin{aligned}
&I_y \dot{q} + (I_x - I_z)rp \\
&= \frac{1}{2}\rho L^5 (M'_{\dot{q}}\dot{q} + M'_{rr}r^2 + M'_{rp}rp) + \\
&\frac{1}{2}\rho L^4 (M'_{\dot{w}}\dot{w} + M'_{vr}vr + M'_{vp}vp + M'_{|w|q}(v^2 + w^2)^{\frac{1}{2}}q \\
&+ M'_q uq) + \frac{1}{2}\rho L^3 (M'_* u^2 + M'_w uw + M'_{w|w|}(v^2 + w^2)^{\frac{1}{2}} + \\
&M'_{vv}v^2) + \frac{1}{2}\rho L^3 (M'_{\delta_s}u^2 \delta_s + M'_{\delta_b}u^2 \delta_b) - mgh\sin\theta + M_y
\end{aligned}
\tag{6}
$$

f. Yaw equation

$$
\begin{aligned}
&I_z \dot{r} + (I_y - I_x)pq \\
&= \frac{1}{2}\rho L^5 (N'_{\dot{p}}\dot{p} + N'_{\dot{r}}\dot{r} + N'_{pq}pq) + \frac{1}{2}\rho L^4 (N'_{\dot{v}}\dot{v} + N'_p up \\
&+ N'_r ur + N'_{wp}wp) + N'_{|v|r}(v^2 + w^2)^{\frac{1}{2}}) + \frac{1}{2}\rho L^3 (N'_v uv \\
&+ N'_{vw}vw + N'_{|v|v}v(v^2 + w^2)^{\frac{1}{2}}) + \frac{1}{2}\rho L^3 (N'_{\delta_r}u^2 \delta_r) + M_z
\end{aligned}
\tag{7}
$$

The right side of the above equal signs is the general representation of the inertial hydrodynamic, viscous hydrodynamic, and rudder hydrodynamic forces of the underwater vehicles. $X_{qq'}$ and $X_{\dot{r}'}$ hydrodynamic derivative. m is the hull quality. L is chief of underwater vehicles. H is underwater vehicles stability. Ix, Iy, Iz represent the moment of inertia of the X, Y, Z coordinate axis indicating the underwater vehicles' mass in the hull motion coordinate system.

In order to determine the position and posture of the underwater vehicles in a global coordinate system, six differential equations of motion need to be added.

$$
\dot{\psi} = q\frac{\sin\varphi}{\cos\theta} + r\frac{\cos\varphi}{\cos\theta}
\tag{8}
$$

$$
\dot{\varphi} = p + q\tan\theta\sin\varphi + r\tan\theta\cos\varphi
\tag{9}
$$

$$
\dot{\theta} = q\cos\varphi - r\sin\varphi
\tag{10}
$$

$$
\begin{aligned}
\dot{\xi} &= u\cos\psi\cos\theta + v(\cos\psi\sin\theta\sin\varphi - \sin\psi \\
&\cos\varphi) + w(\cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi)
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
\dot{\eta} &= u\sin\psi\cos\theta + v(\sin\psi\sin\theta\sin\varphi + \cos\psi\cos\varphi) \\
&+ w(\sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi)
\end{aligned}
\tag{12}
$$

$$\dot{\zeta} = -u\sin\theta + v\cos\theta\sin\varphi + w\cos\theta\cos\varphi \tag{13}$$

The above equation represents the various directions of the underwater vehicles in the water, which serves as an important basis for forecasting and controlling the underwater vehicles motion posture.

4. Six degrees of freedom model establishment based on Simulink

The above kinematics equations are all first-order differential equations. Under the platform of Matlab/Simulink. In order to facilitate the modification of the model, the construction of the model is carried out in a modular manner. The hydrodynamic derivatives in modeling are measured by experiments. At the same time, table 1 shows the basic inertial parameters of the underwater vehicles. The established model is shown in Figure 2. The input of the model is the tail rudder angle δ_r, head and tail lift rudder angle δb, δs and command speed u_c.

Table 1. Inertial parameters of underwater vehicles.

variables	m (kg)	L (m)	Ix (kg·m^2)	Iy (kg·m^2)	Iz (kg·m^2)	h (m)
value	9071945	126.5	1.213e8	9.440e9	9.440e9	0.3
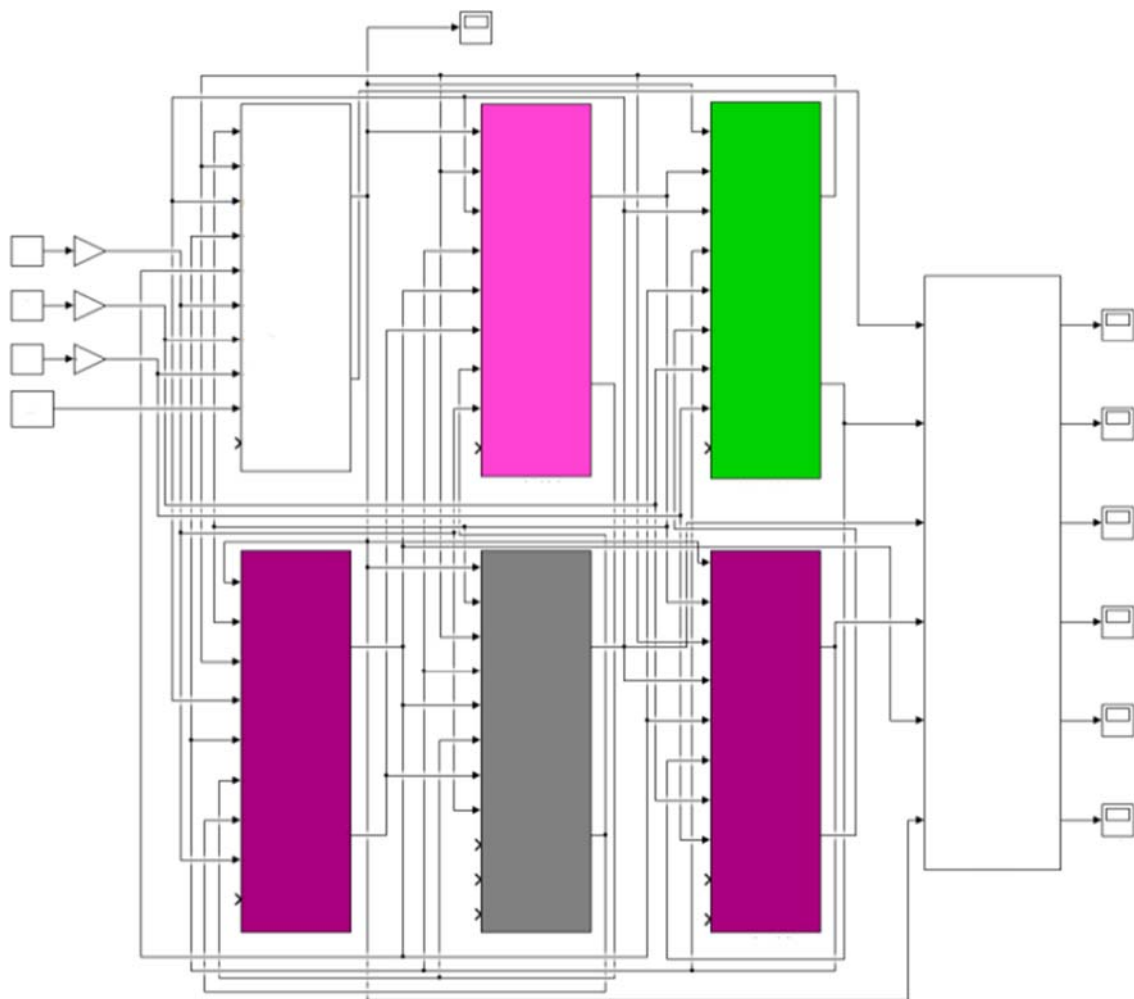
**Figure 2.** Six degrees of freedom model based on Simulink

5. Motion characteristic simulation

Based on the simulation model established, simulation of two typical working conditions of low speed plane rotation and bolt floating. The simulation conditions and results are as follows.

The outside interference is none. Initial speed is 0. After given command speed arrives 2kn, the speed increases gradually. When speed reach a certain value, start steering. The rudder angle is increased to 35 ° in a short period of time. The navigation curve is got from start to steady state direct flight to turn, as shown in Figure 3.

The outside interference is none. Two underwater vehicles raised and lowered rudder angles δb and δs is -10°. Turning rudder corner is 10°. The trajectory of the underwater vehicles space is shown in Figure 4.

Simulation results show that under horizontal rotation conditions, underwater vehicles are growing at full speed. The initial phase is a straight line voyage. When starting steering, the ship's resistance has changed and its speed has dropped. The speed is stable at last, into steady state rotation. The radius of rotation is approximately 170m. The helix floats at different depths and the size of the gyre is basically the same. Comparing simulation results with published literature, it is shown that the six degrees of freedom simulation model can effectively reflect the motion state of underwater vehicles.
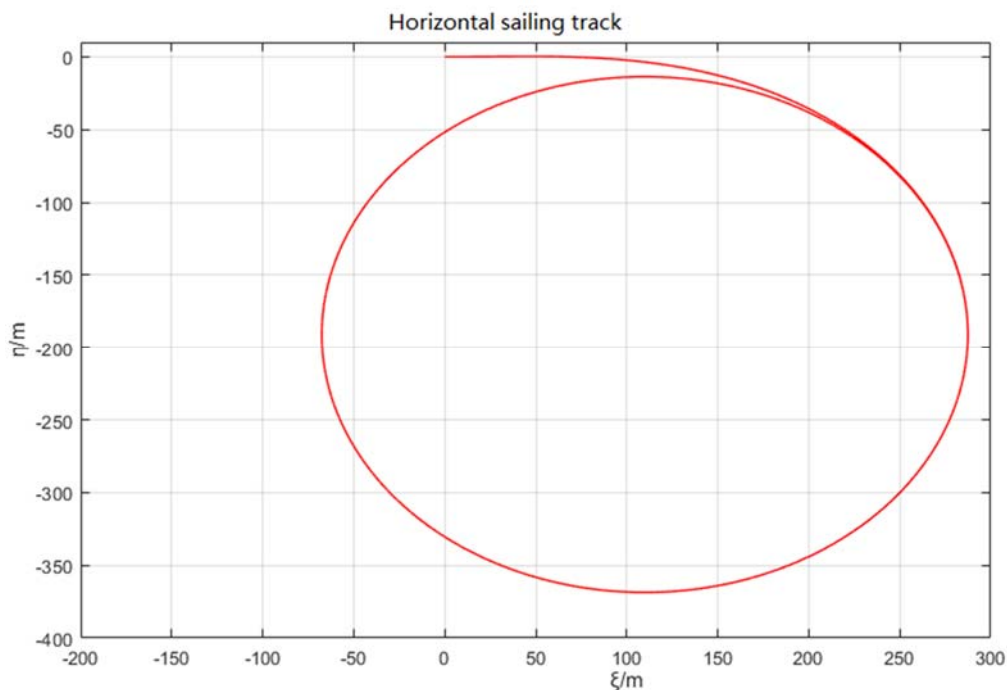
Figure 3. Horizontal rotation trajectory
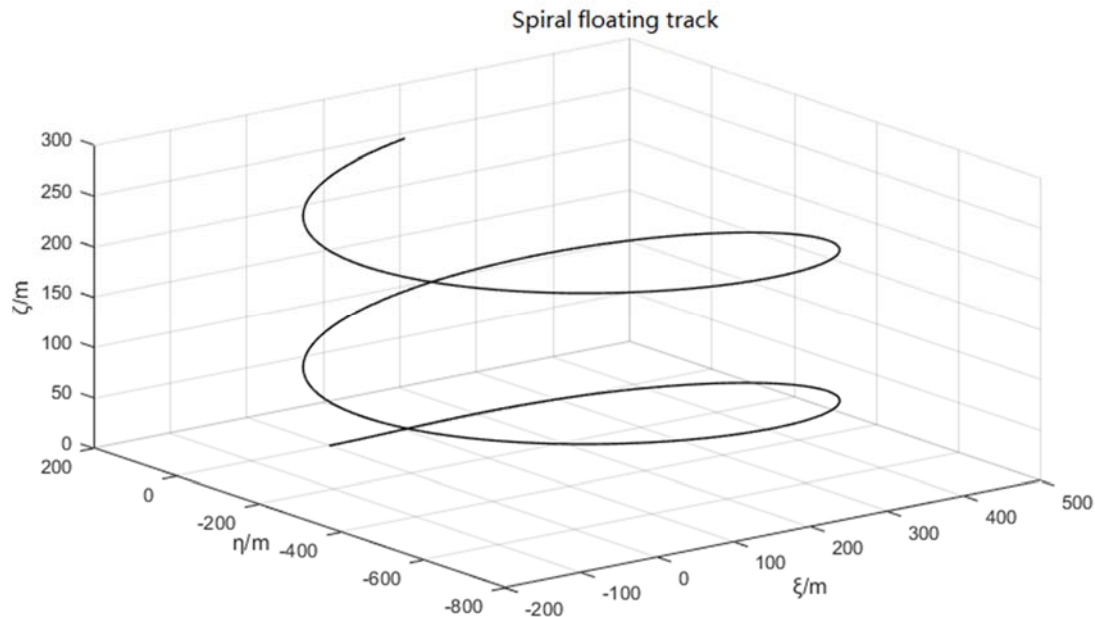
Figure 4. Spiral upward trajectory

6. Conclusion

In this paper, three parts of hydrodynamic force of underwater vehicles hull, rudder and paddle are considered separately. Combining three flat and pitch, roll, vertical motion and their respective coupling terms, and considering the role of subsidiary control, a modular six degree of freedom kinematics model of underwater vehicles is established in the Matlab/Simulink platform. The model can accurately reflect the motion characteristics of underwater vehicles. It can realize various motion modes of different navigation control modes and lay a model foundation for the research of trajectory prediction and trajectory control of underwater vehicles.

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