

# A Method for Calculating Wind Power Penetration Limit Considering Wake Effect and Wind Speed Correlation

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**Abstract.** With the development of wind power technology, accurate analysis of wind power penetration limit is conducive to large-scale wind power safely connected to the power grid. At present, the related research does not consider the influence of wake effect. Therefore, this paper proposes a wind power penetration limit analysis method considering the wake effect and wind speed correlation. Firstly, this paper uses a stochastic simulation technique based on the Latin hypercube sampling (LHS) to simulate the wind speed of multiple wind farms with correlation, and establishes a wind power active output model considering the wake effect, thus the calculation model of wind power penetration limit is established based on the chance constrained programming (CCP) theory and solved by the particle swarm optimization algorithm. Taking the IEEE-30 node system as an example, the feasibility and effectiveness of the above model and algorithm in evaluating wind power access capability are verified.

## 1. Introduction

Wind energy is a renewable green energy source, and its inherent randomness, intermittence and volatility have a serious impact on the safe and stable operation of the power system [1]. The wind power penetration limit is the major indicator for evaluating wind power access capability. It can be expressed as the percentage of the largest installed wind farms capacity to the system load [2].

Early studies used rough estimation methods and dynamic simulation methods to calculate, but these methods have limitations [3]. Reference [4] first proposed a wind power penetration limit calculation model based on stochastic optimization technology. This paper also first introduced the theory of chance constrained programming (CCP), transforming deterministic constraints of the system into probability constraints that satisfy a certain level of confidence. At the same time, the research on wind power penetration limit has gradually evolved from considering only the randomness of wind speed to considering the randomness and correlation of wind speed [5-8]. Studies have shown that wind speed between multiple adjacent wind farms has a strong correlation [9]. Reference [6] evaluated the influence of wind speed correlation on wind power penetration limit from the perspective of voltage constraint, but it ignored the constraints such as line flow and system rotation reserve. Reference [5, 7] consider the above constraints, but ignore the ramp rate constraint. Reference [8] considers the influence of wind speed randomness and correlation on wind power penetration limit, but ignored the influence of wake effect. According to reference review, there are relatively few studies on wind power penetration limit considering the wake effect, which need to be further deepened and improved.

This paper further improves the calculation model of wind power penetration limit, considering the wake effect and wind power correlation. The model aims at maximizing the wind power capacity of the system, and



simulates the correlation wind speed based on the Latin hypercube sampling (LHS). Based on the sampled wind speed, we establish a probabilistic analytical model of the active output of the wind farm considering the wake effect. Under the system AC flow constraint and safe operation, we can apply the optimal probabilistic power flow method based on CCP, and use the particle swarm optimization (PSO) algorithm based on stochastic simulation technology to solve the model. Finally, we take the IEEE-30 node system as an example to verify the feasibility and effectiveness of the proposed model and algorithm.

## 2. Wind farm output model

### 2.1. Correlated wind speed simulation based on Latin hypercube sampling

The wind speed approximation obeys the two-parameter Weibull distribution [5], and the correlation coefficient matrix is commonly used in engineering to describe the correlation of multiple random variables [9]. The relevant wind speed simulation process based on LHS is shown in Fig.1.

The correlated wind speed simulation based on LHS consists of two steps: one is to generate initial sample data based on the probability distribution, and the other is to sort the samples according to the correlation of multiple random variables [10].

Assuming that there are  $m$  wind farms in the system, the sampling frequency is  $N$ , the correlation coefficient matrix is  $C_v$ . The cumulative distribution function of wind speed  $v$  is as follows:

$$y_i = F_i(v_i), i=1,2,\dots,n \quad (1)$$

The sampling process is as follows: first, the value interval  $[0,1]$  of the cumulative distribution function  $F_i(v_i)$  is uniformly divided into  $N$  equal parts. Then a number is extracted from each subinterval as the sample value of  $y_i$ . Finally, the  $N$  sample values of  $v$  are obtained by inverse function  $v=F^{-1}(y)$ .

The sorting process is as follows: firstly, the Cholesky decomposition technique is used to construct a normal distribution sample matrix  $Z_{m \times N}$  with a correlation coefficient matrix of  $C_v$ ; then calculate the order matrix  $L_S$  of  $Z$  ( $L_S$  is a matrix of  $m \times N$ , each row is an arrangement of integers  $1$  to  $N$ , indicating the order of the elements of the corresponding rows in  $Z$ ); finally, the initial sample data of wind speed is sorted according to  $L_S$  to obtain wind speed sample with a correlation coefficient matrix of  $C_v$ .

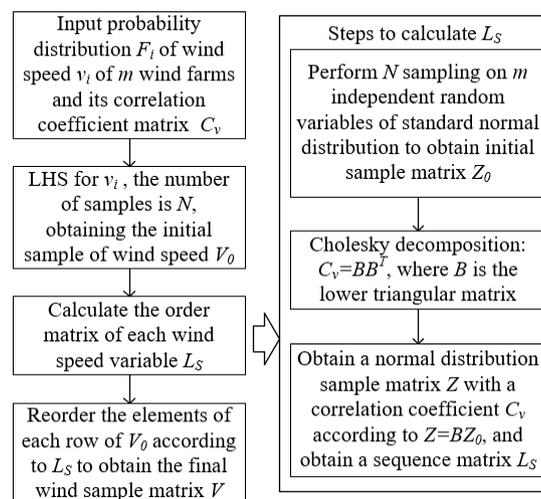


Figure 1. The relevant wind speed simulation process.

## 2.2. Wind farm output model considering wake effect

The actual wind farm is usually built in a checkerboard shape, and the interval between columns is relatively large, and the influence between each other is negligible [11]. Therefore, the wind turbines in the same row can be equivalent to one. This paper proposes a wind farm output probability model considering the wake effect, the number of rows and columns in the wind farm are calculated as follows:

$$n_{row} = \text{floor}(\text{sqr}(n_w)) \quad (2)$$

$$n_i = \begin{cases} \text{ceil}(n_w / n_{row}) & i \neq n_{row} \\ n_w - (n_{row} - 1) \times \text{ceil}(n_w / n_{row}) & i = n_{row} \end{cases} \quad (3)$$

Where  $n_w$  is the number of installed wind turbines,  $n_{row}$  is the number of rows of the wind farm,  $n_i$  is the number of columns corresponding to the  $i$ -th row,  $i=1, \dots, n_{row}$ .  $\text{floor}(\cdot)$  represents the down-round function,  $\text{sqr}(\cdot)$  represents the square root function,  $\text{ceil}(\cdot)$  represents the up-round function.

In the process of wind farm modeling, the influence of the wake effect on the reliability of the wind farm should also be considered. In this paper, the Jensen model is taken as an example to study the wake effect [12]. The wind speed of the  $i$ -th row of wind turbines can be obtained as follows:

$$v_i = v_1 \left[ 1 - (1 - \sqrt{1 - C_T}) \left( \frac{R_0}{R_0 + k_w D_{row}(i-1)} \right)^2 \right] \quad (4)$$

Where  $C_T$  is the thrust coefficient of the wind turbines,  $k_w$  is the wake attenuation coefficient,  $D_{row}$  is the row spacing,  $v_1$  and  $v_i$  respectively represent the initial wind speed perpendicular to the first row of the wind turbines and the wind speed of the  $i$ -th row of the wind turbines,  $i=2, \dots, n_{row}$ ,  $R_0$  is the radius of the wind wheel.

The active output of a wind turbine is proportional to the wind receiving area and is approximately proportional to the cube of the wind speed [4], as follows:

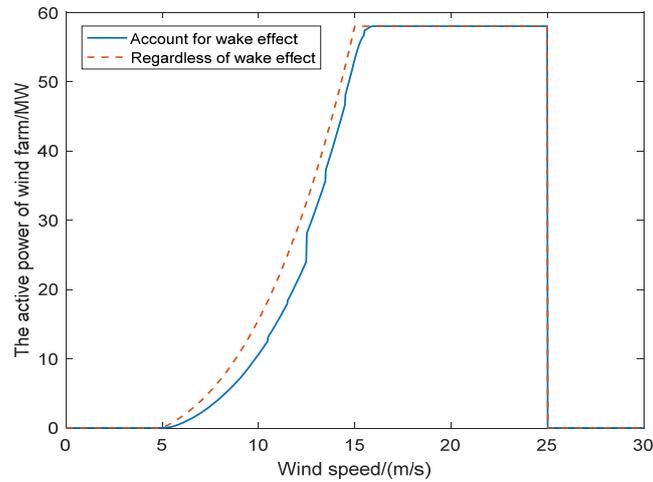
$$p(v) = \begin{cases} 0 & v < v_{in} \text{ or } v > v_{out} \\ \frac{v^3 - v_{in}^3}{v_N^3 - v_{in}^3} p_N & v_{in} \leq v \leq v_N \\ p_N & v_N < v \leq v_{out} \end{cases} \quad (5)$$

Where  $v_{in}$ ,  $v_{out}$ ,  $v_N$  are the cut-in, cut-out, rated wind speed of the wind turbine, respectively.  $p$  and  $p_N$  are the actual and rated output power of the wind turbine respectively.

The wind farm output model considering the wake effect is as follows:

$$P_W = \begin{cases} 0 & v_1 > v_{out} \\ \sum_{i=1}^{n_{row}} p(v_i) \cdot n_i & v_1 \leq v_{out} \end{cases} \quad (6)$$

Assuming that the rated capacity of the wind turbine is 1 MW, the maximum installed capacity  $n_w$  of the wind farm at a certain node is 58, then the active output of the wind farm considering the wake effect or not are shown in Fig.2. We can see that the wake effect has a significant influence on the active output of the wind farm, and ignoring it will affect the objectivity of the result.



**Figure 2.** The active output of the wind farm.

### 3. Wind power penetration power limit model

#### 3.1. Objective function

The model is targeted at maximizing the sum installed capacity of each wind farm that the system can accept. The objective function is as follows:

$$\max \sum_{i=1}^m n_i P_{NWi} \quad (7)$$

Where  $m$  is the number of wind farms,  $n_i$  is the number of wind turbines in the  $i$ -th wind farm,  $P_{NWi}$  is the rated power of the wind turbine in the  $i$ -th wind farm.

#### 3.2. Equality constraint

The equality constraint is the power flow equation of the system, as follows:

$$\begin{cases} P_{Gi} + P_{Wi} - P_{Li} - U_i \sum_{j \in i} U_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \\ Q_{Gi} + Q_{Wi} - Q_{Li} - U_i \sum_{j \in i} U_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \end{cases} \quad (8)$$

Where  $i \in C_{PQ} \cup C_{PV}$ ,  $P_{Gi}$ ,  $Q_{Gi}$  are the active and reactive outputs of the conventional generator units set at node  $i$ , respectively.  $P_{Wi}$ ,  $Q_{Wi}$  are the active and reactive outputs of the wind farm at node  $i$ , respectively.  $P_{Li}$ ,  $Q_{Li}$  are the active and active load at node  $i$ , respectively.  $U_i$ ,  $U_j$ ,  $\theta_{ij}$  are the voltage amplitude and phase angle difference of node  $i$  and node  $j$ , respectively.  $G_{ij}$ ,  $B_{ij}$  are the real and imaginary parts in the system admittance matrix, respectively.  $C_{PQ}$ ,  $C_{PV}$  are respectively a set of PQ and PV nodes.

#### 3.3. Inequality constraint

Inequality constraints include constraints on decision variables and state variables. The decision variables are the number of wind turbines and the output of conventional generator units. The constraints are as follows:

$$\begin{cases} 0 \leq n_i \leq n_{i\max} & n_i \in N^*, i=1,2,\dots,m \\ P_{Gi\min} \leq P_{Gi} \leq P_{Gi\max} & i \in C_G \end{cases} \quad (9)$$

Where  $n_{i\max}$  is the maximum number of wind turbines in the  $i$ -th wind farm,  $P_{Gi\min}$  and  $P_{Gi\max}$  are the minimum and maximum active outputs of the  $i$ -th conventional generator unit,  $C_G$  is the set of conventional generator units.

This paper introduces the CCP theory to deal with inequality constraints [4-5]. State variables include the node voltage amplitude, the reactive output of conventional generator units, the line flow, the up and down rotation reserve of system, the ramp rate constraint of conventional units and so on, as follows:

$$\begin{cases} P_r \{U_{i\min} \leq U_i \leq U_{i\max}\} \geq \alpha_1 & i \in C_{PQ} \\ P_r \{Q_{Gi\min} \leq Q_{Gi} \leq Q_{Gi\max}\} \geq \alpha_2 & i \in C_G \\ P_r \{0 \leq P_{Li} \leq P_{Li\max}\} \geq \alpha_3 & i \in C_L \\ P_r \left\{ \sum_{i=1}^n (P_{Gi\max} - P_{Gi}) \geq P_{SRup} \right\} \geq \alpha_4 & i \in C_G \\ P_r \left\{ \sum_{i=1}^n (P_{Gi} - P_{Gi\min}) \geq P_{SRdn} \right\} \geq \alpha_4 & i \in C_G \\ P_r \left\{ \sum_{i=1}^n r_{Gi} \geq \sum_{j=1}^m (p_{ij} - p_{(i-1)j}) \right\} \geq \alpha_5 & i \in C_G, j \in C_W \end{cases} \quad (10)$$

Where  $U_{i\min}$ ,  $U_{i\max}$  are the minimum and maximum voltage amplitudes of node  $i$ .  $Q_{Gi\min}$ ,  $Q_{Gi\max}$  are the minimum and maximum reactive output of the  $i$ -th conventional generator unit.  $P_{Li}$  is the power flow on the  $i$ -th branch,  $P_{Li\max}$  is the maximum limit of the power flow on the  $i$ -th branch,  $C_L$  is the set of branches.  $P_{SRup}$ ,  $P_{SRdn}$  are the up and down rotation reserve of the system, generally 5% of the total system load.  $r_{Gi}$  is the maximum ramp rate of the  $i$ -th conventional unit.  $p_{ij}$ ,  $p_{(i-1)j}$  are the output sample sequence of the wind farm  $j$ ,  $\alpha_1 \sim \alpha_5$  are the corresponding confidence level of the inequality constraint.

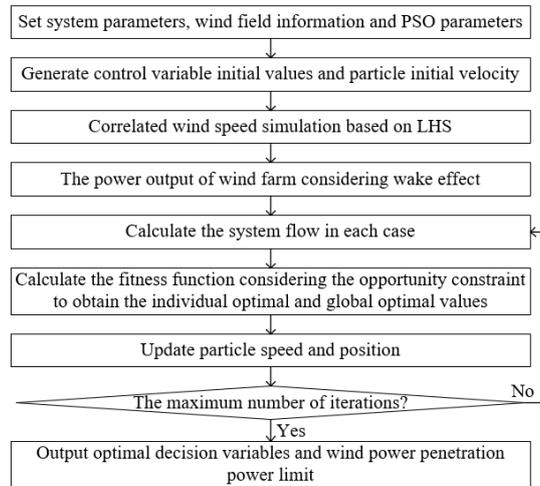
### 3.4. The particle swarm optimization algorithm

For the opportunity constraint, we can use the random simulation technique [1]. For chance constraints with a random variable  $\xi$ , as follows:

$$P_r \{g(x, \xi) \leq 0\} \geq \alpha \quad (11)$$

The stochastic simulation algorithm is as follows:

- a) Set counter  $N'=0$ .
- b) Generate  $\xi$  according to probability distribution  $\Phi(\xi)$ .
- c) If  $g(x, \xi) \leq 0$  is true,  $N' = N'+1$ .
- d) Repeat steps b) and c) for  $N$  times.



**Figure 3.** The process of PSO algorithm.

e) If  $N'/N \geq \alpha$ , the opportunity constraint is true, otherwise it is not true.

The basic idea of PSO algorithm is to randomly initialize a group of particles, and regard each particle as a feasible solution to the optimization problem, the quality of the particle is determined by a preset fitness function [3]. The process of PSO algorithm based on stochastic simulation is shown in Fig.3.

**4. Simulation case study**

We take the IEEE-30 node system [13] as an example. It is assumed that the wind farm operates with a constant power factor 1.00. The rated power of the wind turbine is 1 MW, the rated wind speed is 15 m/s, and the cut-in wind speed is 5 m/s, the cut-out wind speed is 25m/s. The wind speed signal obeys the Weibull distribution with a shape factor of 2.0 and a scale parameter of 7.5. The wind farm access nodes are 17, 24, and the confidence level is set to 0.95. When considering the wind speed correlation, the wind power penetration limit at different correlation levels is shown in TABLE I.

**Table 1.** The wind power penetration limit at different correlation levels

Correlation coefficient	Number of installed wind turbines		Wind power penetration limit
	Node 17	Node 24	
0	31	23	0.2854
0.2	23	29	0.2748
0.4	27	45	0.2643
0.6	25	23	0.2537
0.8	34	11	0.2378

It can be seen from TABLE I that when the wind speeds of the wind farms have different correlation coefficients, the installed capacity of each wind farm and the wind power penetration limit of the system are significantly different, and the latter decreases as the correlation coefficient increases. The reason is that the increase of the wind speed correlation increases the probability that the wind speed of the adjacent wind farms increase and decrease at the same time, and the cumulative effect will cause the wind speed fluctuation and the corresponding wind power output fluctuation to be more severe, thereby affecting the safe operation of the system. Therefore, an increase in wind speed correlation will reduce the ability of system to absorb wind power.

On the basis of considering the correlation of wind speed, we consider the influence of wake effect at the same time, the wind power penetration power limit under different correlation degrees is shown in TABLE II.

**Table 2.** The wind power penetration limit considering wake effect

Correlation coefficient	Number of installed wind turbines		Wind power penetration limit
	Node 17	Node 24	
0	43	24	0.3541
0.2	40	22	0.3277
0.4	38	22	0.3171
0.6	30	28	0.3066
0.8	24	32	0.2960

It can be seen from TABLE II that after considering the wake effect, the wind power access capability of the system is significantly increased. The reason is that the wind speeds captured by all the wind turbines of the actual wind farm are not the same. Due to the wake effect, the wind speed will be attenuated, and the resulting active output of the wind farm is also different than the ideal state. The active output of the wind farm is relatively reduced, and the disturbance to the system is reduced, so that the system can absorb the wind power to a greater extent. Therefore, in order to ensure that the system absorbs wind power to the maximum extent, the wake effect of the actual wind farm should be considered.

## 5. Conclusion

This paper proposes a wind power penetration limit analysis method that considers the wake effect and wind speed correlation. The method establishes the wind farm active output model considering the wake effect and wind speed correlation. At the same time, considering the AC power flow constraints and safety constraints, the wind power penetration power limit calculation model is established and solved by particle swarm optimization algorithm. When the wind speeds of multiple wind farms have different correlation degrees, the wind power access capability of the system is quite different, and gradually weakens as the correlation degree increases. Considering the wake effect, the system can absorb wind power to a greater extent. Therefore, considering the wake effect and wind speed correlation, the wind power penetration power limit can be obtained more accurately to ensure the safe operation of the system.

## 6. References

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