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Coupling bending-torsional vibration model of gear - rotor

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Abstract. A rotor - bearing model of 5 degrees of freedom was established by finite element method, and a gear pair meshing model of 5 degrees of freedom was established by Newton's second law. By introducing the azimuth, the gear rotor system could be designed with the gear pair arranging at will. Then through the transformation of coordinate system, the coordinate system of rotor-bearing model and gear pair model were unified, and the two were coupled together, and a complete gear-rotor model with uniform coordinate system was established.

1. Introduction

In a geared rotor system, due to the existence of a pair of gears, the motion of each rotor in the system was coupled and interacted with each other, and its dynamic characteristics were different from that of a single-rotor system. Therefore, it is necessary to study the whole geared rotor system. Many scholars had done a lot of work on the dynamic modeling and analysis of the geared rotor system. Mohiuddin considered the gyro effect and established the bending and torsional coupled vibration model of single rotor by using finite element method [1]. Rao et al. established the finite element model of the gear drive rotor system, analyzed the bending and torsional coupling vibration of the gear box and the geared rotor-bearing system, and discussed the influence of gear meshing stiffness on the natural frequency and mode vibration mode [2-3]. Lee et al. studied the bending vibration and torsional vibration characteristics of the gear rotor system with the increase of rotating speed, and discussed the variation of the bending and torsional coupled vibration strain energy [4].

However, in the selection of the coordinate system of the whole gear-rotor system, the alignment of gears couldn't be fully expressed by taking the center line of gears as the axial positive direction [2-4] or the meshing line of gears as the axial positive direction [5]. In addition, during the rotation process of the gear pair, the transverse vibration of the main and passive gears would inevitably lead to the change of the central displacement of the two gears, which will lead to the change of the contact degree of the gear pair and the change of the meshing stiffness of the gear pair. In this paper, the rotor - bearing model with 5 degrees of freedom was established by taking the above factors into full consideration, and the meshing model of gear pair with 5 degrees of freedom was established by Newton's second law. By introducing the azimuth, the gear rotor system could be designed so that the gear pairs could be arranged at will, and then the coordinate system of the rotor-bearing model and the gear pair model could be unified through the transformation of coordinate system, and then coupled together, the complete pg-rotor model with the uniform coordinate system was established. The model took into account the change of meshing stiffness and meshing damping of gears in motion, which was closer to the actual situation.

2. Motion differential equation of rotor

The geared rotor system can be divided into shafts, disks, gear pairs, bearings and other elements, and the kinematic differential equations of each element are established respectively. Then the generalized coordinate kinematic differential equations of the geared rotor system can be obtained by combining them.

2.1. Motion differential equation of shaft unit

The expression of the axial element's torsional kinetic energy and potential energy is:

$$\begin{aligned} T_e &= \frac{1}{2}\int_0^l \rho I_p \left(\frac{\partial \varphi(s,t)}{\partial t}\right)^2 \mathrm{d}s = \frac{1}{2}[\dot{\varphi}_A \quad \dot{\varphi}_B]\boldsymbol{M}_s[\dot{\varphi}_A \quad \dot{\varphi}_B]^T \\ U_e &= \frac{1}{2}\int_0^l G I_p \left(\frac{\partial \varphi(s,t)}{\partial s}\right)^2 \mathrm{d}s = \frac{1}{2}[\varphi_A \quad \varphi_B]\boldsymbol{K}_S[\varphi_A \quad \varphi_B]^T \end{aligned} \tag{1}$$

2.2. Motion differential equations of disk unit

The expression for the torsional kinetic energy of the disk is:

$$T_d = \frac{1}{2} J_p \dot{\varphi}_d^2 \tag{2}$$

3. Coupling bending-torsional vibration model of gear pair

3.1. Meshing stiffness of gear pair

The gear pair in this paper adopted standard involute cylindrical straight-tooth gear, so the average meshing stiffness of the gear could be expressed as:

$$k_m = b(0.75\varepsilon + 0.25)k' \times 10^6 \tag{3}$$

Where $k' = 1/(0.04723 + \dfrac{0.1551}{z_1} + \dfrac{0.25791}{z_2})$ was the meshing stiffness of single pair of teeth, z was the number of teeth, b was the gear width, ε was the gear coincidence degree.

In the process of gear pair movement, the change of center displacement of two gears was inevitable, and then the coincidence degree and meshing angle of gears also changed accordingly, which leaded to the change of meshing stiffness of gear pair. The gear coincidence degree and meshing Angle could be expressed as:

$$\varepsilon = \frac{1}{2\pi}\left[z_1(\tan\alpha_{a1} - \tan\alpha') + z_2(\tan\alpha_{a2} - \tan\alpha')\right] \tag{4}$$

$$\alpha' = \arccos\frac{R\cos\alpha}{R'} \tag{5}$$

Where, $R' = R + (x_1 - x_2)$ was the center displacement of two gears, $R = r_1 + r_2$ was the standard center distance of gear pair, x was the translational displacement of gear center, α was the dividing circular pressure Angle of gear, α_{a1} and α_{a2} were the circular pressure angle of the top of the teeth of passive gear respectively.

3.2. Motion differential equation of gear pair unit

To analyze the force and torque of each gear, the differential equation of motion of the gear pair was established according to Newton's second law:

$$\boldsymbol{M}_G^p \ddot{\boldsymbol{u}}_G^p + \boldsymbol{C}_G^p \dot{\boldsymbol{u}}_G^p + \boldsymbol{K}_G^p \boldsymbol{u}_G^p = \boldsymbol{Q}_G^p \tag{6}$$

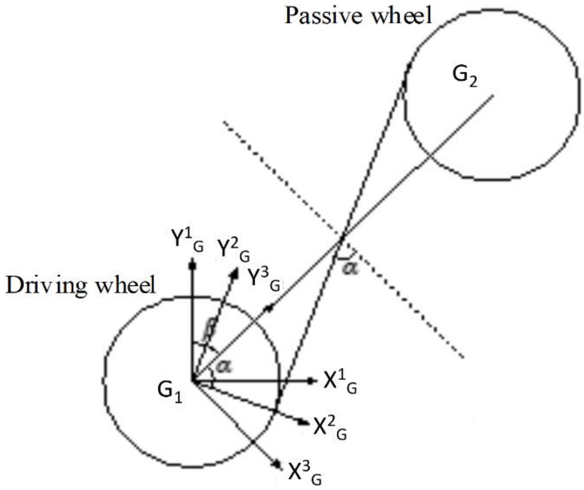

Fig.1 Coordinate of gear pair

Fig. 1 showed 3 coordinate system of the gear pair. The superscript c represents the direction of the Y-axis along the center line of the gear pair, and the superscript v represents the direction of the Y-axis along the vertical direction. α was the Angle of meshing equal to the dividing pressure angle of the gear, β was the angle between the center line of the gear pair and the vertical Y axis.

A rectangular coordinate system was established with the center line of gear as the Y-axis. Introducing the coordinate transformation matrix $\boldsymbol{T}$, where $s = \sin\alpha$, $c = \cos\alpha$.

Equation (6) could be obtained by coordinate transformation:

$$\boldsymbol{M}_G^p \ddot{\boldsymbol{u}}_G^p + \boldsymbol{C}_G^p \dot{\boldsymbol{u}}_G^p + \boldsymbol{K}_G^p \boldsymbol{u}_G^p = \boldsymbol{Q}_G^p \tag{7}$$

In the actual gear-rotor system, the center line of the gear pair does not necessarily recombine with the vertical Y axis, as shown in figure 2. The azimuth angle β was introduced to establish a new coordinate system $G_1 X_G^v Y_G^v$, which made the coordinate system of the gear pair coincide with the coordinate system of the rotor system. $S = \sin\beta$, $C = \cos\beta$.

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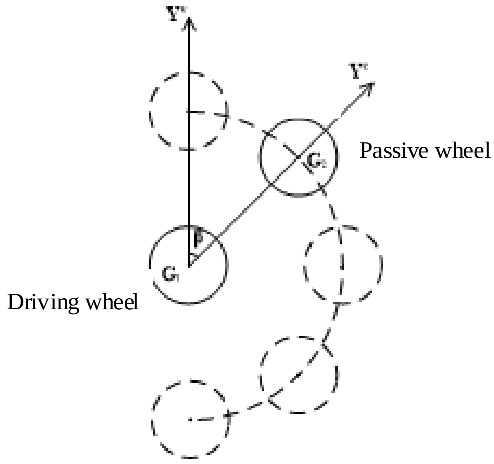

Fig.2 Style of gear pair arrangement

Equation (7) can be obtained by coordinate transformation:

$$\boldsymbol{DTM}_G^p\boldsymbol{T}^T\boldsymbol{D}^T\ddot{\boldsymbol{u}}_G^v+\boldsymbol{DTC}_G^p\boldsymbol{T}^T\boldsymbol{D}^T\dot{\boldsymbol{u}}_G^v+\boldsymbol{DTK}_G^p\boldsymbol{T}^T\boldsymbol{D}^T\boldsymbol{u}_G^v=\boldsymbol{DTQ}_G^p \tag{8}$$

4. Conclusion

Based on the finite element method and the Lagrangian equation, the kinematics differential equation of the rotor-bearing system with 5 degrees of bending-torsional coupling was obtained. Based on Newton's second law, the kinematics differential equation of 5 degrees of freedom with bending-torsional coupling of the gear pair was obtained by taking the changes of meshing stiffness and meshing damping into account. Then considering the arrangement mode of the actual gear pair, the coordinates of the rotor-bearing model and the gear pair model were unified by coordinate transformation.

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