Event-Triggered Output Regulation for A Class of Switched Linear Systems with Average Dwell Time Approach*

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Abstract. This paper studies the output regulation problem for a class of switched linear systems via designing an event-triggered state feedback controller. Compared with the time-triggered mechanism, event-triggered mechanism effectively reduces execution times of a control task. The designing of the controller for switched systems is difficult due to the mixture of event-triggered instants and switching instants. This paper gets sufficient conditions for the output regulation problem to be solvable by giving a co-design scheme of an event-triggered mechanism, state feedback controllers and switching rules satisfying the average dwell time (ADT) condition. The different coordinate transformations break the restrictions of a common solution for regulator equations of all subsystems. Moreover, the minimum inter-event interval is given to avoid Zeno behavior. Finally, a simulation example of the switched RLC circuit systems shows the validity of the results.

1. Introduction

The output regulation problem is a hot issue in the field of control theory. The purpose is to design a feedback controller with undesired disturbances in the system, while making the output asymptotically tracks a set of preset orbital sets under keeping the closed-loop system stable. Two important concepts of the regulator equation and the internal model principle of system effectively solved the output regulation problem of non-switched systems [1-2]. The switched system contains a series of continuous-time and discrete-time subsystems and the switching rules acting on it, which make the design of output regulation of switched systems more complicated. The method of common Lyapunov function is used in [3] to solve the output regulation problem for switched linear systems under arbitrary switching signals with a common solution of regulator equations for all subsystems. In order to further reduce the conservativeness, [4] employs the ADT approach and the different coordinate transformations and gives sufficient conditions of the solvability of the output regulation problem for switched linear systems without the common solution restrictions of the regulator equations. On this basis, when the states of the switched systems and the switched exosystems are unknown, [5] designs the switching rules and the controllers based on the measurable output tracking error, and thus solves the output regulation problem for the switched discrete-time systems. Inspired by the incremental passivity theory, [6] introduces the concept of incremental passivity for switched linear discrete-time systems and solves the output regulation problem by constructing the feedback incremental passive interconnection between the controlled plants and the switched internal models. Based on [6], the

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error-based feedback controller and error-based switching rules are designed to study the output regulation problem of switched nonlinear systems using the incremental passivity in [7].

Compared with the time-triggered control, event-triggered can sample according to the current system performance. In recent years, the event-triggered control achievements for switched systems have emerged rapidly. In [8], a state-based event-triggered mechanism is proposed, the event-triggered stability for switched linear systems is given using the ADT method without discussing the relationships between the switching instants and the trigger instants. [9] eliminates the Zeno phenomenon by giving the minimum inter-event interval, and the event-triggered stability problem of switched linear systems is proposed via considering the output-based event-triggered controller. Due to the mixture of the event-triggered instants and the switching instants, [10] considers the occurrences of multiple switchings within two consecutive triggering times, and gives the stability with event-triggered control for switched systems based on the observer states. As is known to all, it is so restrictive to make all subsystems of a switched system use a common event-triggered mechanism. Multiple event-triggering mechanisms are utilized in [11] which make the controller design more flexible, and the event-triggered robust control for uncertain switched linear systems with time-varying delays is studied using the method of multi-Lyapunov function. [12] studies the stability of observer-based output feedback control for switched linear neutral systems with mixed time-varying delays with event-triggered mechanism under the asynchronous switching. The above research works motivate us to conduct the event-triggered output regulation for switched linear systems in this paper.

In this paper, the event-triggered output regulation problem for the switched systems is investigated based on ADT and the different coordinate transformations. Meanwhile, the minimum inter-event interval is given to avoid Zeno behavior. Finally, a simulation example for a switched RLC circuit systems shows the validity of the results.

2. Problem description

Consider a continuous-time switched linear system:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + Z_{\sigma(t)}w(t) \\ e(t) &= D_{\sigma(t)}x(t) + G_{\sigma(t)}w(t) \end{aligned} \tag{1}$$

Where $x \in R^n$ is the system state, $u \in R^m$ is the control input, $\sigma : [0,\ \infty) \to \underline{N} = \{1,\ 2, \cdots q\}$ is the switching signal, $e \in R^p$ is the regulated error, $A_{\sigma(t)}, B_{\sigma(t)}, Z_{\sigma(t)}, D_{\sigma(t)}, G_{\sigma(t)}$ are known constant matrices of appropriate dimensions, $w \in R^r$ is the external input variable, which is produced by an exosystem

$$\dot{w}(t) = Sw(t) \tag{2}$$

Where S has only simple eigenvalues on the imaginary axis.

The switching sequence is represented by

$$\left\{ x(t_0) : (i_0, t_0), \cdots (i_j, t_j), \cdots \middle| i \in \underline{N},\ j = 0, 1, 2, \cdots \right\}$$

Which means that the i_jth subsystem is active when $t \in [t_j, t_{j+1})$, where $t_0 < t_1 \cdots t_j < \cdots$ are the switching instants. Using $t_0^e < t_1^e \cdots t_r^e < t_{r+1}^e < \cdots$ ($t_0^e = t_0$) to denote the triggered instants. Where t_r^e represents the rth triggered instants.

In this paper, we give the form of state feedback controller by discussing the relationship between the event-triggered instants and the switching instants.

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Case 1: There is no event-triggered when $t \in [t_j, t_{j+1})$, i.e. $t_r^e \le t_j < t_{j+1} < t_{r+1}^e$, the event-triggered state feedback controller is $u(t) = K_{i_j} x(t_r^e) + L_{i_j} w(t_r^e)$.

Case 2: There are l times event-triggered when $t \in [t_j, t_{j+1})$, i.e. $t_r^e < t_j < t_{r+1}^e < \cdots < t_{r+l}^e \le t_{j+1}$, the event-triggered state feedback controller is

$$u(t) = \begin{cases} K_{i_j} x(t_r^e) + L_{i_j} w(t_r^e), & t \in [t_j, t_{r+1}^e) \\ K_{i_j} x(t_{r+1}^e) + L_{i_j} w(t_{r+1}^e), & t \in [t_{r+1}^e, t_{r+2}^e) \\ \cdots & \\ K_{i_j} x(t_{r+l}^e) + L_{i_j} w(t_{r+l}^e), & t \in [t_{r+l}^e, t_{j+1}) \end{cases}$$

Where K_{i_j} and L_{i_j} are controller gains.

In summary, we can get the unified event-triggered state feedback controller of system (1) as follows

$$u(t) = K_{\sigma(t)} x(t_{r+h}^e) + L_{\sigma(t)} w(t_{r+h}^e), \quad t \in [t_j, t_{j+1}), \; h = 0, 1, \cdots l \tag{3}$$

The event-triggered mechanism based on state $x(t)$, external input variable $w(t)$ and the switching rules $\sigma(t)$ is considered in this paper, which is given by

$$t_{r+1}^e = \inf\{t > t_r^e \,\big|\, \| K_{\sigma(t)} \bar{e}(t) \|^2 \ge \eta \| K_{\sigma(t)} \bar{x}(t) \|^2 \} \tag{4}$$

Where $\bar{x}(t) = x(t) - \Pi_{\sigma(t)} w(t)$, $\bar{e}(t) = \bar{x}(t_r^e) - \bar{x}(t)$ is the difference between the current state $\bar{x}(t)$ and the recent event-triggered instants state $\bar{x}(t_r^e)$, $\eta > 0$ is a threshold.

Assumption 1: There exist matrix Π_{i_j} solving the linear regulator equations $\Pi_{i_j} S = A_{i_j} \Pi_{i_j} + Z_{i_j}$, $0 = D_{i_j} \Pi_{i_j} + G_{i_j}$.

When the set of measurable variables includes state $x(t)$ and external input variables $w(t)$, consider the coordinate transformation $\bar{x}(t) = x(t) - \Pi_{\sigma(t)} w(t)$. The switched systems (1) and the exosystem (2) compose the following closed-loop system under the controller (3)

$$\begin{aligned} \dot{\bar{x}}(t) &= (A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)}) \bar{x}(t) + B_{\sigma(t)} K_{\sigma(t)} \bar{e}(t) \\ e(t) &= D_{\sigma(t)} \bar{x}(t) \end{aligned} \tag{5}$$

The corresponding event-triggered state feedback output regulation problem is defined as follows

Definition 1: Under Assumption 1, for $\forall i_j \in \underline{N}$, given matrices A_{i_j}, B_{i_j}, Z_{i_j}, D_{i_j}, G_{i_j}, S and matrix Π_{i_j}, looking for matrices K_{i_j}, L_{i_j} and switching rules $\sigma(t)$ with event-triggered mechanism (4), we have (i) When $w(t) = 0$, the closed-loop system (5) is exponentially stable. (ii) When $w(t) \ne 0$, for any (x^0, w^0), the solution of the closed-loop system (5) under initial condition satisfied $\lim_{t\to\infty}(D_{\sigma(t)} \bar{x}(t)) = 0$.

Definition 2: Consider the closed-loop system (5), if there is a positive $c > 0$, $\alpha > 0$, for any initial conditions $x(t_0) \in R^n$, $w(t_0) \in R^r$ under event-triggered mechanism (4), the solution of closed-loop system (5) satisfy $\|\bar{x}(t)\| \le c e^{-\alpha(t-t_0)} \|\bar{x}(t_0)\|$, $\forall t > t_0$. Then the closed-loop system (5) is exponentially stable, which α is exponential decay rate.

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3. Main results

In this section, we give the sufficient conditions for the solvability of the output regulation problem for the switched linear systems (1) under the event-triggered mechanism (4) and the state feedback controller (3)

Theorem 1: Consider the switched systems (1) and the exosystem (2). Under Assumption 1, (a) for a given appropriate dimensioned matrix T_{i_{j-1},i_j} and the system (1) with the different coordinate transformations

$$\bar{x}(t_j^-) = x(t_j^-) - \Pi_{i_{j-1}} w(t_j^-)\,,\ \bar{x}(t_j^+) = x(t_j^-) - \Pi_{i_j} w(t_j^-)$$

$$x(t_j^-) - \Pi_{i_j} w(t_j^-) = T_{i_{j-1},i_j}(x(t_j^-) - \Pi_{i_{j-1}} x(t_j^-)) \tag{6}$$

(b) For given scalars $\eta > 0, \lambda > 0, \mu_0 \geq 1$, if exist appropriately dimensional matrices R_{i_j}, $P_{i_j} > 0$, such that the following inequalities hold

$$\begin{bmatrix} \Omega_{i_j}^T + \Omega_{i_j} + B_{i_j} B_{i_j}^T + 2\lambda P_{i_j} & R_{i_j} \\ * & -\eta^{-1} I \end{bmatrix} < 0 \tag{7}$$

$$P_{i_{j-1}} \leq \mu_0 P_{i_j} \tag{8}$$

Then the event-triggered output regulation problem for the switched systems (1) is solvable under arbitrary switching rules which satisfies ADT defined by $\tau_a \geq \tau_a^* = \dfrac{\ln \mu}{2\lambda}$ .where $L_{i_j} = -K_{i_j} \Pi_{i_j}$, $K_{i_j} = R_{i_j} P_{i_j}^{-1}$, $\Omega_{i_j} = A_{i_j} P_{i_j} + B_{i_j} R_{i_j}$, $\mu = \mu_0 \mu_1$, $\mu_1 = \max\limits_{i_{j-1}, i_j \in \underline{N}} \{\| P_{i_{j-1}}^{-1/2} T_{i_{j-1},i_j} P_{i_{j-1}}^{1/2} \|^2\}$, $i_{j-1}, i_j \in \underline{N}$.

Proof Define Lyapunov function candidate as follows

$$V(t) = V_{\sigma(t)}(\bar{x}(t)) = \bar{x}^T(t) \hat{P}_{\sigma(t)} \bar{x}(t) = \bar{x}^T(t) P_{\sigma(t)}^{-1} \bar{x}(t)$$

When $t \in [t_j, t_{j+1})$, $\sigma(t) = i_j$, the derivative of $V(t)$ along the trajectory of the closed-loop system (5), combining the event-triggered mechanism (4) and [13, Lemma 2.2], using Schur complement lemma [14], we have

$$\Omega_{i_j}^T + \Omega_{i_j} + B_{i_j} B_{i_j}^T + 2\lambda P_{i_j} + \eta R_{i_j}^T R_{i_j} < 0 \tag{9}$$

Pre-and post-multiplying matrix in (9) by $\hat{P}_{i_j} = P_{i_j}^{-1} > 0$, we obtain $\dot{V}_{i_j}(t) + 2\lambda V_{i_j}(t) < 0$. Thus, for $\forall t \in [t_j, t_{j+1})$, we have

$$V_{\sigma(t_j)}(t) < \mathrm{e}^{-2\lambda(t - t_j^+)} V_{\sigma(t_j)}(t_j) \tag{10}$$

Because the coordinate transformations are different, at the switching instants t_j, $V_{\sigma(t_j)}(t_j^+) \neq V_{\sigma(t_{j-1})}(t_j^-)$. According to the properties of the positive definite matrices, applying (6) and (8), we have

$$\| P_{i_j}^{-1/2} \bar{x}(t_j^+) \|^2 \leq \mu_0 \| P_{i_{j-1}}^{-1/2} T_{i_{j-1},\, i_j} P_{i_{j-1}}^{1/2} \|^2 \| P_{i_{j-1}}^{-1/2} \bar{x}(t_j^-) \|^2 . \tag{11}$$

By utilizing (10), (11) and the ADT [15] method, for any $t \in [t_j, t_{j+1})$, system (1) is exponentially stable when $w(t) = 0$. Thus, we obtain $\lim\limits_{t \to \infty}(D_{\sigma(t)} x(t) + G_{\sigma(t)} w(t)) = \lim\limits_{t \to \infty} D_{\sigma(t)} \bar{x}(t) = 0$.

Therefore, we can conclude that the output regulation for the switched linear systems (1) is solved.

Theorem 2: Consider the system (5) with event-triggered mechanism (4). For given any event-triggered instant t_{r+1}^e and all $t \in [t_{r+1}^e, t_{r+2}^e)$, the minimum inter-event interval is lower bounded by a positive scalar that is determined by

$$T = \frac{1}{\Theta_1} \ln[\frac{\eta^{1/2}\Theta_1}{(1+\eta^{1/2})(\Theta_1+\Theta_2)}+1] \tag{12}$$

Where $\Theta_1 = \max_{i_j \in \underline{N}}\{\| K_{i_j} A_{i_j} K_{i_j}^{-1} \|\}$, $\Theta_2 = \max_{i_j ?N}\{\|K_{i_j} B_{i_j}\|\}$.

Proof Combining the closed-loop system (5), we know

$$\frac{d}{dt}\| K_{i_j}\overline{e}(t) \| \le \Theta_1 \| K_{i_j}\overline{x}(t_{r+1}^e) + K_{i_j}\overline{e}(t) \| + \Theta_2 \| K_{i_j}\overline{x}(t_{r+1}^e) \|$$

Therefore, when $t \in [t_{r+1}^e, t_{r+2}^e)$, from above inequality, we have $\|K_{i_j}\overline{e}(t)\| \le \int_{t_{r+1}^e}^t e^{\Theta_1(t-s)}(\Theta_1+\Theta_2)\| K_{i_j}\overline{x}(t_{r+1}^e)\|ds$. From event-triggered mechanism (4), $\|K_{i_j}\overline{e}(t)\| < \frac{\eta^{1/2}}{1+\eta^{1/2}}\| K_{i_j}\overline{x}(t_{r+1}^e)\|$. Thus, the lower bound of the minimum inter-event interval can be obtained as (12).

4. Illustrative examples

Consider a switched RLC circuit as shown in Fig.1 [16], the circuit consists of an input power source, a resistance, an inductance and q capacitors that could be switched between each other. We consider the case $q = 2$, the model of switched RLC circuit is as follows

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{Lc_i} & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u + \begin{bmatrix} -M_i \\ -\frac{N_i}{L} \end{bmatrix} w \\ e_i = [0 \quad 1]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - M_i w, i = 1, 2 \end{cases} \tag{13}$$

Where $x = [Q \quad I]^T$,the input u is the voltage, c_i denotes the ith capacitor, v_a , v_b are exogenous signal. There exist matrices M_i , N_i ,such that $v_a = M_i w$, $v_b = N_i w$. $w = (w_1, w_2)^T$ is assumed to be generated by the exosystem (2)

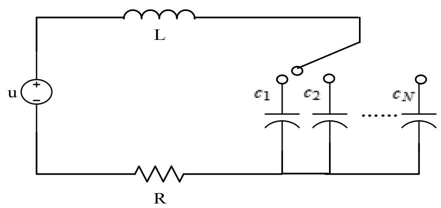

Figure 1. A switched RLC circuit

In the simulation, set $L = 0.1H$, $R = 0.2\Omega$, $c_1 = 0.5\mu F$, $c_2 = 3.6\mu F$. The system matrices are given by

$$A_1 = \begin{bmatrix} 0 & 1 \\ -20 & -2 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0 & 1 \\ -2.7778 & -2 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 10 \end{bmatrix},$$

$$Z_1=\begin{bmatrix}1.3 & 0.2\\ -2.4 & -1.7\end{bmatrix},\ Z_2=\begin{bmatrix}1 & 2\\ 0 & -5\end{bmatrix},\qquad D_1=D_2=[0\ \ 1]\ ,$$

$$G_1=\begin{bmatrix}1.3 & 0.2\end{bmatrix},\ G_2=[1\ \ 2],\ S=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

There exist the following matrices satisfying assumption 1

$$\Pi_1=\begin{bmatrix}0 & 0\\ -1.3 & -0.2\end{bmatrix},\ \Pi_2=\begin{bmatrix}0 & 0\\ -1 & -2\end{bmatrix}.$$

By choosing $\lambda=2$, $\eta=0.8$, $\mu_0=1.48$ and

$$T_1=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\ T_2=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}.$$

According to the theorem 1, we obtain $\tau_a^*=\dfrac{\ln\mu}{2\lambda}=0.9924$,

$$P_1=\begin{bmatrix}0.1568 & -0.3633\\ -0.3633 & 26.3712\end{bmatrix},\ P_2=\begin{bmatrix}0.1686 & -0.6597\\ -0.6597 & 30.3787\end{bmatrix},$$

$$K_1=\begin{bmatrix}0.8 & -0.3\\ 1.8 & 1.3\end{bmatrix},\ K_2=\begin{bmatrix}1.2 & 1.7\\ -0.6 & -0.8\end{bmatrix},$$

$$L_1=\begin{bmatrix}-0.39 & -0.06\\ 1.69 & 0.26\end{bmatrix},\ L_2=\begin{bmatrix}1.7 & 3.4\\ -0.8 & -1.6\end{bmatrix}$$

Given the initial state $\overline{x}(0)=[3\ \ -3]^T$ and ADT $\tau_a=1$, the switching signal is in Fig.2. The simulation results of Fig.3 shows that the output regulation problem for the RLC circuit system (13) is solvable under the switching signal. Fig.4 shows the errors of the closed-loop systems, the convergence curves in Fig. 4 mean that $e(t)\to 0$, as $t\to\infty$. Fig.5 shows inter-event interval, which verifies that the minimum inter-event interval in Theorem 2 has a positive lower bound.

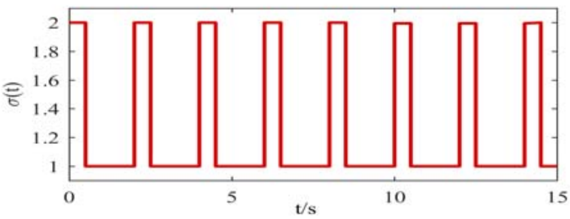

Figure 2. The switching signal

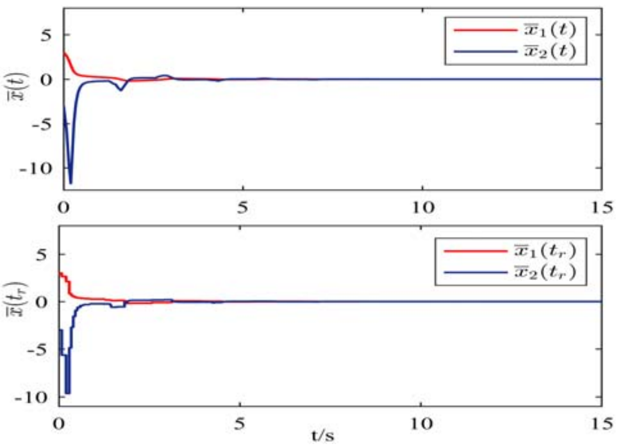

Figure 3. The states responses and the event-triggered state responses of the closed-loop systems

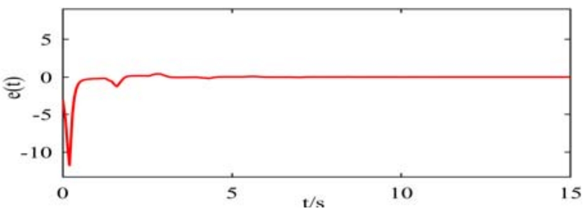

Figure 4. The errors of the closed-loop systems

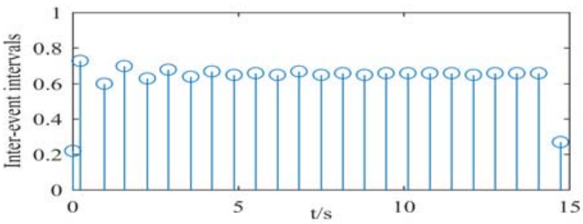

Figure 5. Inter-event intervals

5. Conclusion

In this paper, we have investigated the problem of output regulation for a class of switched linear systems with the event-triggered state feedback controller. Firstly, due to mixture of event-triggered instants and switching instants, we give the state feedback controller and state-based event-triggered mechanism. Secondly, the sufficient conditions for the output regulation problem is solvable by ADT and the different coordinate transformations. In order to avoid the Zeno behavior, the positive lower bound of the minimum inter-event interval is given. Finally, the application of a class of switched RLC circuit system verifies the validity of the conclusion.

Acknowledgments

This project is supported by the National Natural Science Foundation of China under Grant 61304056, the Supporting Program for High-level Talents Innovation of Dalian City of China under Grant 2016RQ049 and the Fundamental Research Funds for the Central Universities under Grants 3132018127 and 3132018125.

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