

Methods of regulating thrust in design of arch bridges

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Abstract. The paper deals with the identification of the optimal forms of arch span structures of bridges in terms of minimizing the expansion in their foundations. The calculations of thrust at different ratio boom lift for stairs in various arch design schemes (fully articulated, two-hinged, and hingeless arches). A graph of the dependence of the thrust on the ratio of the rise to the span for the bridge reinforced concrete arch, on the basis of which one can choose the most effective ratio of the boom to span for a particular case. A comparison of the distribution of the spread value for multi-span arch bridges with a constant and variable span width is performed. It is concluded that such ratio of the magnitudes of spans is the most effective for the equalization of forces, transmitted to the foundations of the arches of multispan bridges.

1. Introduction

Bridges of arch and combined systems are the most architecturally expressive bridge structures. First such bridges were built in Ancient Rome [1-3]. Arches or vaults in that era designed circular, built of stone blocks and overlapping small spans. Blocks in the masonry were held by friction force due to the compression of the arch [4], which caused the danger of destruction in case of underestimation in the calculations of a weak base (the mechanism of destruction of the stone arch is shown in [5]).

In ancient times, any building, in General, was corresponded the famous triad of Vitruvius "Firmitas, Utilitas, Venistas" [6]. One of the requirements of modern construction was economically advantageous. The emergence of new structural materials – steel and reinforced concrete allowed to create new structural forms of arch bridges. In such bridges two parts are distinguished: the main supporting structures – arches and beam roadway, based on arches with the help of racks [7].

In the design of modern arch bridges can be distinguished extensive and progressive ways to ensure the strength and stability of bridge structures. The first is based on the selection of the corresponding section, which is able to perceive the existing internal forces in the structure. The second is based on the search for such sizes, ratios and forms, in which the existing efforts will be minimal. In addition to vertical loads on the arched system wind pressure [8-10]. In cases where high arches are used in bridge structures, the effect of wind can create some types of aerodynamic instability. For arches typical of such aeroelastic phenomena as wind resonance and buffeting [11,12].

In professional literature there are no clear criteria for choosing the ratio of the boom of the arch to its flight. In [13] it is said: "in the existing arch bridges the flatness of the arches f/l varies in a very wide range (from $1/2$ to $1/18$). When riding on top of the arch is often used with a flatness of $1/7$ - $1/8$, and when driving lower $1/5$ – $1/6$ ". There is no answer to when to use $f/l=1/2$ - $1/5$ and $1/8$ - $1/18$ and in General what ratio to prefer in a given situation.



Being spacer systems, arch bridges create a large horizontal pressure on the base, but their foundations should not shift [10], which requires an increase in the volume and mass of the support, working due to the forces of gravity [14,15]. Displacement of the Foundation due to its insufficient mass will lead to the collapse of the arch. In case of design errors, it is necessary to restore the operational quality of the bridge construction over time, spending financial resources [16,17].

To reduce the horizontal impact of the arch on the Foundation used tightening, which are ropes of high-strength wire or reinforced concrete beams fixed to the arches. If the tightening perceives all the tension, then this design applies to combined systems. Arch bridges on the location of the roadway structure are divided into three types: with the movement on the bottom (Figure 1.a), with movement in the middle (Figure 1.b) and with movement on top (Figure 1.c.).

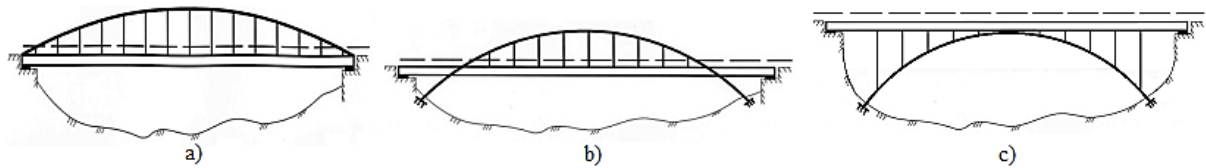


Figure 1. Movement on the bridge a) on the bottom, b) in the middle, c) on top.

When moving along the bottom or in the middle, it is possible to place a tightening inside or on the surface of the roadway structure of the bridge [18]. When driving on top of the tightening place in the roadway is impossible, and therefore should otherwise affect the amount of horizontal pressure on the Foundation.

2. Results of research on optimization of arch bridge shapes

As one of the ways to control the amount of the spread, you can take the selection of the correct ratio of the boom of the rise to the arch span f/l .

We investigate the effect of the ratio f/l on the expansion of the arch system. Since the bridge arches by the number of hinges can be three -, two-hinged and open-pit, consider each of the systems separately.

The three-hinged arch with span l and boom f is loaded with uniformly distributed load q [19,20]. The axis of the arch is outlined by a circle or parabola (Figure 2.).

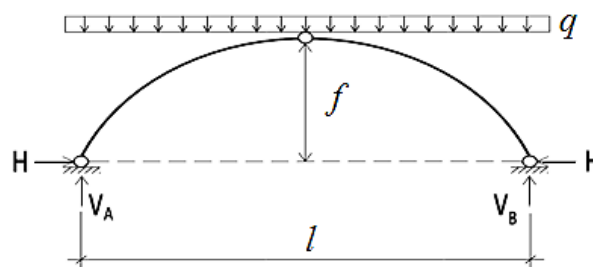


Figure 2. Three-hinged arch.

The vertical reference reaction $V_A = ql/2$, then the pressure will be $H = ql/8f$.

Taking for arch bridges of all kinds the ratio f/l from 1/18 to 1/2 we perform the calculation of the three-hinged arch spread for all combinations. The results are presented in table 1.

Table 1. The dependence of the thrust of the arch on the ratio f/l .

f/l	1/18	1/14	1/10	1/8	1/6	1/4	1/2
H/ql	2,25	1,75	1,25	1	0,75	0,5	0,25

Figure 3 shows a plot of the dependence of the pressure on the ratio f/l , which can be used to determine the pressure at any span and load on the three-hinged arch.

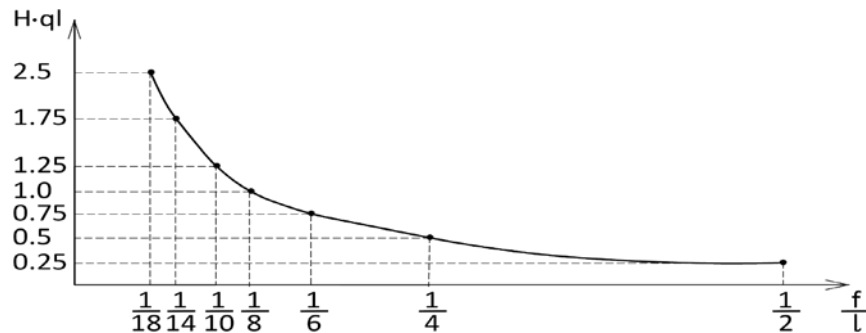


Figure 3. The graph of dependence of the arc from the ratio f/l .

We investigate the thrust arising in a two-hinged arch with similar parameters l , f and q . Two-hinged arch with parabolic axis (Fig. 4), once statically indeterminate. Cross-section form constantly on all the length of arch

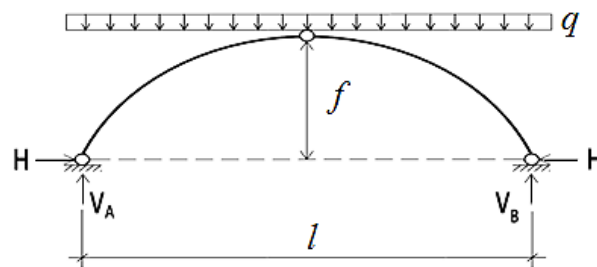


Figure 4. Two-hinged arch.

To determine the arch's thrust, static uncertainty is revealed in accordance with the expression (1).

$$H = - \frac{\int \frac{M_p y}{EI \cos \phi} dx + \int \frac{Q_p Q_1}{GA \cos \phi} dx + \int \frac{N_p N_1}{EA \cos \phi} dx}{\int \frac{y^2}{EI \cos \phi} dx + \int \frac{Q_1 Q_1}{GA \cos \phi} dx + \int \frac{N_1 N_1}{EA \cos \phi} dx} \quad (1)$$

In accordance with the design scheme (Fig.4) single and cargo forces in the cross section of the arch will be: $M_1 = y^2$; $Q_1 = -x \sin \phi = -\sin \phi$; $N_1 = l \cos \phi = \cos \phi$; $M_p = q/2(l-x)x$; $Q_p = q/(l/2-x) \cos \phi = Q_0 \cos \phi$; $N_p = q/(l/2-x) \sin \phi = Q_0 \sin \phi$.

In order to Express the stiffness of the bending, shear and tensile sections through a single value, let's define a rectangular cross-section with dimensions b and h , then: the sectional area will be: $A = bh$; axial moment of inertia: $I = bh^3/12 = Ah^2/12$. Take for the arch $h = 0,02l$, $b = h/1,6$. Then the cross-sectional area of the arch is expressed in terms of l : $A = 0,00025l^2$, axial moment of inertia $I = 0,000033Al^2$. For reinforced concrete $G = 0,42E$.

Taking into account the expressions for internal force factors, as well as the geometric characteristics of the cross section and replacing the integration by summation over sites of equal length, we obtain the following expression, which allows to calculate the thrust:

$$H = - \frac{\sum \frac{M_p y \Delta x}{EI \cos \phi} + \sum \frac{Q_0 \sin \phi \Delta x}{0,42 EA} + \sum \frac{Q_0 \sin \phi \Delta x}{EA}}{\sum \frac{y^2}{EA k \cos \phi} + \sum \frac{\sin^2 \phi \Delta x}{0,42 EA \cos \phi} + \sum \frac{\cos \phi \Delta x}{EA}} \quad (2)$$

where $k = 0,000033l^2$, $\Delta x = 0,1l$.

The calculation of the thrust is performed for all previously described boom-to-span ratios, and for $f/l = 0,1$ is presented in table 2.

Table 2. Results of the calculation of the thrust for double-hinged arch at $f/l = 0,1$

No.	y	tg ϕ	cos ϕ	sin ϕ	$\frac{y^2}{k \cos \phi}$	$\frac{\sin^2 \phi}{\cos \phi 0,42}$	M_p	Q_0	$\frac{M_p y}{k \cos \phi}$	$\frac{Q_0 \sin \phi}{0,42}$	$Q_0 \sin \phi$
	0,1l	-	-	-	-	-	ql^2	ql	ql	ql	ql
1	0	0,4	0,928	0,371	0	0,353	0	0,5	0	0,442	0,186
2	0,36	0,32	0,952	0,305	41,3	0,23	0,045	0,4	-51,6	0,29	0,122
3	0,64	0,24	0,972	0,233	127,7	0,13	0,08	0,3	-159	0,167	0,07
4	0,84	0,16	0,987	0,158	216,6	0,06	0,105	0,2	-270	0,075	0,032
5	0,96	0,08	0,997	0,08	280,1	0,015	0,12	0,1	-350	0,02	0,008
6	1	0	1	0	303	0	0,125	0	-379	0	0
7	0,96	-0,08	0,997	-0,08	280,1	0,015	0,12	-0,1	-350	0,02	0,008
8	0,84	-0,16	0,987	-0,158	216,6	0,06	0,105	-0,2	-270	0,075	0,032
9	0,64	-0,24	0,972	-0,233	127,7	0,13	0,08	-0,3	-159	0,167	0,07
10	0,36	-0,32	0,952	-0,305	41,3	0,23	0,045	-0,4	-51,6	0,29	0,122
11	0	-0,4	0,928	-0,371	0	0,353	0	-0,5	0	0,442	0,186
			10,67	$\Sigma =$	1634	1,576		$\Sigma =$	-2040	1,988	0,835

Substituting the values in the expression (2) we obtain the value of the thrust

$$H = -(-2040 - 1,988 + 0,835)ql / (1634 + 1,576 + 10,67) = 1,24ql$$

In determining the arc, taking into account only the bending, it is found that it differs by 0,8 percent from the calculated by the above method, that is, taking into account all three force factors, which is not essential.

Analyze the effect of the ratio f/l on the spacers hingeless arches are of parabolic shape (Fig.5), which is three times statically indeterminate. Choose the same basic system as for the two-hinged arch.

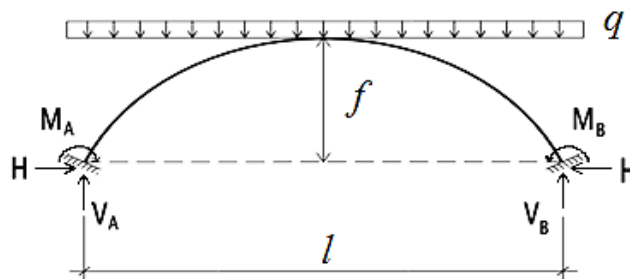


Figure 5. The stepless arch.

We group the unknown support moments as symmetric and skew-symmetric. In this case, the skewed reference moments will be zero, since the load on the arch is symmetrical. To reveal the static uncertainty and find two unknowns, we write a system of two equations.

$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + \Delta_1P = 0 \\ \delta_{21}X_1 + \delta_{22}X_2 + \Delta_2P = 0 \end{cases} \quad (3)$$

In accordance with the design scheme (Fig.5) single and cargo forces in the cross section of the arch will be: $M_1 = y^2$; $M_2 = 1$; $M_p = q/2(l-x)x$, then: $\delta_{11} = \sum \frac{y^2 \Delta x}{EI \cos \phi}$; $\delta_{12} = \sum \frac{y \cdot 1 \cdot \Delta x}{EI \cos \phi}$; $\Delta_{1p} = \sum \frac{M_p \cdot y \cdot \Delta x}{EI \cos \phi}$; $\delta_{21} = \sum \frac{y \cdot 1 \cdot \Delta x}{EI \cos \phi}$; $\delta_{22} = \sum \frac{1 \cdot 1 \cdot \Delta x}{EI \cos \phi}$; $\Delta_{2p} = \sum \frac{M_p \cdot 1 \cdot \Delta x}{EI \cos \phi}$.

The calculation of the stepless arch is carried out for all the ratios of the boom rise to the span; for $f/l = 0,1$ is presented in table 3.

Table 3. Results of the calculation of the thrust for headless arch at $f/l = 0,1$.

No.	x	y	tg ϕ	cos ϕ	$\frac{y^2}{\cos \phi}$	$\frac{y}{\cos \phi}$	$\frac{1}{\cos \phi}$	M_p	$\frac{M_p \cdot y}{\cos \phi}$	$\frac{M_p}{\cos \phi}$
	l	$0,1l$	-	-	$0,01l^2$	$0,1l$	-	ql^2	$0,1ql^2$	ql^2
1	0	0	0,4	0,928	0	0	1,078	0	0	0
2	0,1	0,36	0,32	0,952	0,147	0,407	1,05	0,045	-0,017	0,047
3	0,2	0,64	0,24	0,972	0,421	0,658	1,029	0,105	-0,0526	0,082
4	0,3	0,84	0,16	0,987	0,715	0,851	1,013	0,105	-0,0894	0,106
5	0,4	0,96	0,08	0,997	0,924	0,963	1,003	0,12	-0,1155	0,12
6	0,5	1	0	1	1	1	1	0,125	-0,125	0,125
7	0,6	0,96	-0,08	0,997	0,924	0,963	1,003	0,12	-0,1155	0,12
8	0,7	0,84	-0,16	0,987	0,715	0,851	1,013	0,105	-0,0894	0,106
9	0,8	0,64	-0,24	0,972	0,421	0,658	1,029	0,105	-0,0526	0,082
10	0,9	0,36	-0,32	0,952	0,147	0,407	1,05	0,045	-0,017	0,047
11	1	0	-0,4	0,928	0	0	1,078	0	0	0
					5,414	6,758	11,34		-0,674	0,835

The system of equations (3) after substitution of the displacements with a factor of 0.1 has the form

$$\begin{cases} 0,5414X_1 - 6,758X_2 - 0,674 = 0 \\ -6,758 + 113,46X_2 + 8,35 = 0 \end{cases}$$

Therefore $X_1 = H = 1,27ql$

From the performed calculations it follows that the magnitude of the thrust in the arch bridges of the systems depends on the current load of the span ratio f/l , but not depends on the consideration of longitudinal and lateral forces to be considered when disclosing the redundancies and the method of attaching the arch to the Foundation.

As another way to control the amount of spread, the design of different-sized spans for multi-span arch bridges is proposed, gradually reducing the size of spans from the middle to the abutments.

Most often, the length of the bridge is divided into several equal-sized spans (Figure 6). In this case, the extreme arch support will transfer to the extreme foundations of a large horizontal pressure H .

Central foundations are experiencing a lot of vertical pressure from adjacent spans, but is free from shear stresses due to the compensation of horizontal effort. In this case, reliable foundations are material-intensive and costly: intermediate due to large vertical loads, extreme because of the huge thrust.

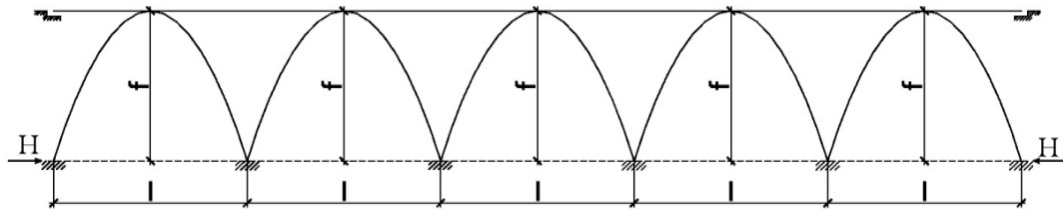


Figure 6. The scheme of the arch bridge with equal spans.

When dividing the length of the bridge into different-sized spans, the thrusts of adjacent spans are not fully compensated - in a smaller span, the spacer is less due to a decrease in the span on the one hand and an increase in the f/l ratio on the other. In this case, the intermediate foundations designed to take large vertical loads will take and small horizontal forces. The extreme foundations can be seriously lightened due to the significant reduction of shear forces. On the example of the bridge, the scheme of which is shown in figure 7, it is shown that in the case of successive reduction of spans, the horizontal pressure is distributed between all foundations.

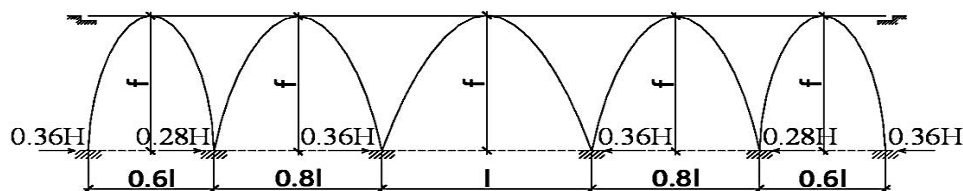


Figure 7. The scheme of the bridge with decreasing spans.

Main span at $f/l = 0,1$, $H = 1,25ql$.

Adjacent span $0,8l$, then $f/l = 0,125$ and the thrust in this case is $1,0ql$ related to the size of the main will be $0,8ql$. The difference between the horizontal pressure of the main and adjacent flight will be $0,45ql$ or $0,36H$.

3. Conclusions

Summarizing all the calculations and studies, we can draw the following conclusions:

1. The ratio of the height of the rise of the arch to the span has a significant impact on the size of the spread and as a result, on the stress-strain state of the foundations of bridge structures;
2. To align the efforts transferred to the foundations of the arches of multi-span bridges, it is most appropriate to design bridges with different size spans with a gradual decrease in spans to the abutments.

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