Assessment of work quality if there is lack of information

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Abstract. The article offers an approach to assess the work quality if there is a lack of information. A lack of information may be preconditioned by a small number of control objects, the features of controlled objects, lacking values of the features, as well as a low confidence of the features or values. The solution of the problem allows us, at a known a priori law of a random variable distribution, to make a decision using a small volume of the control sample without compromising the reliability of the solution. The approach uses the Lotfi Zadeh's possibility theory as a generalization of the fuzzy set theory. It suggests specifications for assigning the solution risk level, using the possibility theory. The solutions of two problems, using the specified theoretical base, are considered: the problem of assessing the quality system level and assessing the defectiveness of works. It is proved that at small sample volumes it is possible to obtain reliable solutions in determining the parameters of the normal distribution and the probability of defects.

1. Introduction

Quality indicators that characterize the features of an object or a process can be considered as information units. An information unit is defined by the four aspects: object, feature, value and confidence [1,2]. Therefore, a lack of information at quality control and assessment can be of four types: there are not enough control objects; insufficient features of the object; insufficient values of the feature; little confidence of the features or values.

All these situations arise when assessing the quality of construction. Inaccuracy (refers to the value) and uncertainty (refers to confidence), due to objective reasons, are inherent in the quality information, especially in the construction sector. Therefore, in the special literature attempts have been made to apply information methods for assessing the quality of construction works [3,4].

Considering production "noises" (technological variability) and control errors, the work quality is a probabilistic category. To control the quality of probabilistic objects or processes, it is necessary to create a mathematical model of the control object. The mathematical model in the form of a series of relationships based on the physical laws governing the operation of the control object enables to determine the signal at the output of the control object under the known input influences and initial states [5-7]. Mathematical recording of the laws of the technological process functioning often leads to a complex system of nonlinear differential equations that connect input and output variables and their derivatives.

The processes under consideration are subject to "noises", are weakly formalized systems, and are subject to statistical regularities. Stochastic processes and their random parameters are described by the laws of distribution. To establish the distribution law, it is necessary to perform at least 100–120 measurements, which is very laborious.

In some control situations, it is impossible to obtain a large data sample, for example:

- when testing structures or engineering systems;
- when controlling the parameters of small structural elements (for example, within one storey);
- when evaluating complex indicators averaged by a small sample of seizures, floors, objects;
- when monitoring the limit values of the safety function parameters of structural elements;
- when using an expert evaluation method and a small number of experts.

2. Preliminary remarks

In case of unstable distribution parameters within floors or seizures, the sample combined by the task, strictly speaking, cannot be considered homogeneous. At the same time, for small samples (at n<10 per floor) probabilistic methods are limitedly applicable. On the other hand, given the a priori distribution law, we can confine ourselves to a small sample volume, making conclusions on the general totality at minimal control costs.

The structural safety calculations operate with the areas on asymptotic "tails" of distributions, for which, on the one hand, a large sample is required, and, on the other hand, the extreme values of the parameters falling into the asymptotic parts of distributions have a decisive influence on the probability of a failure [8-11]. From this point of view, it is necessary to control, first of all, those elements that have the largest deviations from the average one. Since we are interested in the minimum safety or the worst quality, at the known distribution law, only average and minimum values can be controlled.

Proceeding from the aforesaid, quality assessment requires the use of methods that allow us to obtain reliable solutions when there is fuzzy source information or its lack. These methods are based on fuzzy sets and L. Zadeh's possibility theory, as well as on the theory of identification by the limited data [1,2,12-15].

3. Application of the possibility theory

When there is a lack of control or test data, the problem of quality or safety assessment can be solved using the possibility (fuzzy sets) theory. The works of Utkin V.S. and Kosheleva Zh.V. [16,17] consider the application of the possibility theory to evaluate the safety of structures in the conditions of limited information. In this case, the possibility distribution function (Figure 1) is considered as a "density" of the uncertainty measure and is represented in the form

$$\pi_x(x) = \exp\left\{-\left(\frac{x_n - a}{b}\right)^2\right\} \tag{1}$$

and the function parameters are taken equal:

$$a = \left(x_{\max} + x_{\min}\right)/2\,, \tag{2}$$

$$b = \left(x_{\max} - x_{\min}\right)/2\sqrt{-\ln\alpha}\,, \tag{3}$$

where $\alpha \in [0, 1]$ – risk level accepted depending on the number of measurements or the quality system level.

The higher the risk level α, the larger is the variance b and the lower is the function (1). Analogously, in statistics the wider the confidence interval of the average $x \pm t_{1-\alpha,n} S/\sqrt{n}$, the larger the confidence level $1-\alpha$.

The work [17] proposes to calculate the risk level by the formula

$$\alpha = 0{,}894 - 0{,}339 \cdot \ln n \tag{4}$$

where n – number of measurements.

The values α calculated by formula (4) are shown below.

n	2	3	4	5	6	7	8	9	10	11	12
α	0.66	0.52	0.42	0.35	0.29	0.23	0.19	0.15	0.11	0.08	0.05

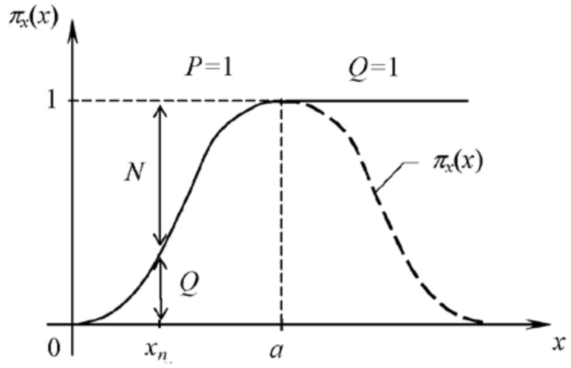

Figure 1. Possibilities distribution function.

Already at $n > 13$, the risk value becomes negative, which does not make sense. Consequently, formula (4) has a limited field of application. It is clear that at an increase of *n*, the risk asymptotically goes to zero, not reaching it. For calculations it is suggested to take $\alpha = 0.05$ at $n > 12$ and $\alpha = 0.01$ at $n > 100$, and also to relate this risk to the level of the construction quality system K_{CK} by the ratio $\alpha = 1 - K_{CK}$, as the higher the K_{CK}, the less is the risk of error.

At $a > x_n$, the possibility of zero defects when the parameter is limited from below is $P = 1$. In this case, the possibility of defectiveness is $Q = \pi_x(x)$. The necessity of zero defects is $N = 1 - \pi_x(x)$, the zero defect interval is $[N, 1]$. Let us consider the solution of two quality assessment problems using the specified theoretical base.

4. Problem of assessing the quality system level

Let us consider the problem of assessing the quality system level of a construction organization by three experts. We will compare the expert estimates: 0.74, 0.73 and 0.71 with the estimated values of 0.75 ... 0.85, dividing the quality area into three zones: high, satisfactory and unsatisfactory.

Let us find the parameters of the possibilities distribution function (PDF), which play the role of a mean and a standard deviation:

$$a = \left(x_{\max} + x_{\min}\right)/2 = (0,74 + 0,71)/2 = 0,725$$

$$b = \left(x_{\max} - x_{\min}\right)/2\sqrt{-\ln\alpha} = (0,74 - 0,71)/2\sqrt{-\ln 0,52} = 0,0185$$

Let us calculate the PDF value at $x_n = 0,75$

$$\pi_x(x) = \exp\left\{-\left(\frac{0,75 - 0,725}{0,0185}\right)^2\right\} = 0,163.$$

At $a < x_n = 0.75$ the possibility of an average level of the system quality is $P = 1$. The possibility of a satisfactory correspondence is $Q = \pi_x(x) = 0.163$. The need for an unsatisfactory correspondence is $N = 1 - \pi_x(x) = 0.827$. The interval $[N, 1]$ implies that with a confidence of 0.827 to 1 the quality system level is unsatisfactory. Accordingly, the quality system level is assessed as satisfactory with a confidence of 0 to 0.163.

When assessing by the lower border $a < x_n = 0.7$, the possibility of an unsatisfactory or satisfactory correspondence $P = 1$ (see Figure 1). The possibility of a high correspondence $Q = \pi_x(x) = 0$. Combining the extreme cases, we conclude that: the possibility of a high correspondence of the quality system level – 0, the possibility of a satisfactory correspondence is from 0 to 0.163, the possibility of an unsatisfactory correspondence is from 0.827 to 1.

5. Problem of assessing the work defectiveness level

Let us consider the task of assessing the level of defectiveness of construction works using the possibility method. For this purpose we will choose the most important parameters of material strength from [4]. Let us compare the results obtained using the possibility theory with the results of traditional statistical estimates.

On the average, the difference in the average probabilistic a and statistical $\bar{x}$ values is 3%, corresponding standard deviations of b and S_x – 11.9%, probabilities of defect-free operations $1–\pi_x(x)$ and P – 3.7%. Let us determine the influence of the number of measurements n on the parameters of the possibilities distribution function and its value. For this purpose, let us take the results of controlling the strength of a mortar of welds in a brick building [4]. Increasing the sample volume from 2 to n, we will find the values $x_{\max}$ and $x_{\min}$, average a, variability b and probability $N = 1– \pi_x(x)$. We will compare these values with statistical estimates of the mean $\bar{x}$, the standard deviation S_x, and the probability of zero defects P.

Table 1. Results of calculations using the probability theory and the statistical method.

Strength parameter, MPa	Probability theory method					Statistical method		
	a	α	b	$\pi_x(x)$	N	$\bar{x}$	S_x	P
Wall panel concrete	30.70	0.05	5.720	0.0302	0.9698	27.84	3.92	0.977
Wall panel concrete	27.80	0.05	3.813	0.0152	0.9848	27.35	3.38	0.985
Panel weld mortar	22.40	0.05	5.200	0.0034	0.9966	22.07	4.86	0.993
Panel weld mortar	13.20	0.19	2.018	0.0808	0.9192	13.26	1.92	0.955
Wall brick	18.35	0.05	4.247	0.5367	0.4633	17.42	4.26	0.715
Wall brick	13.75	0.05	2.167	0.0500	0.9500	12.14	2.92	0.768
Brickwork weld mortar	21.90	0.05	6.413	0.0320	0.9680	20.32	5.22	0.976
Brickwork weld mortar	11.70	0.05	1.329	0.1946	0.8054	10.97	1.50	0.741

The combined diagrams of the values of the compared characteristics for the strength parameter of the brickwork weld mortar are shown in Figure 2.

As we see, the best coincidence of variances and probabilities is observed at $n < 5$. The values of probabilities N and P are almost equal at $n < 10$. Thus, if there is a lack of control data, the methods of the possibility theory give acceptable results in terms of accuracy. In this case, a small sample $n < 5$ is sufficient to reliably determine the parameters of the normal distribution and the probability of a defect (the average error by the parameter values is 3–12%, by the defect probability – about 4%).

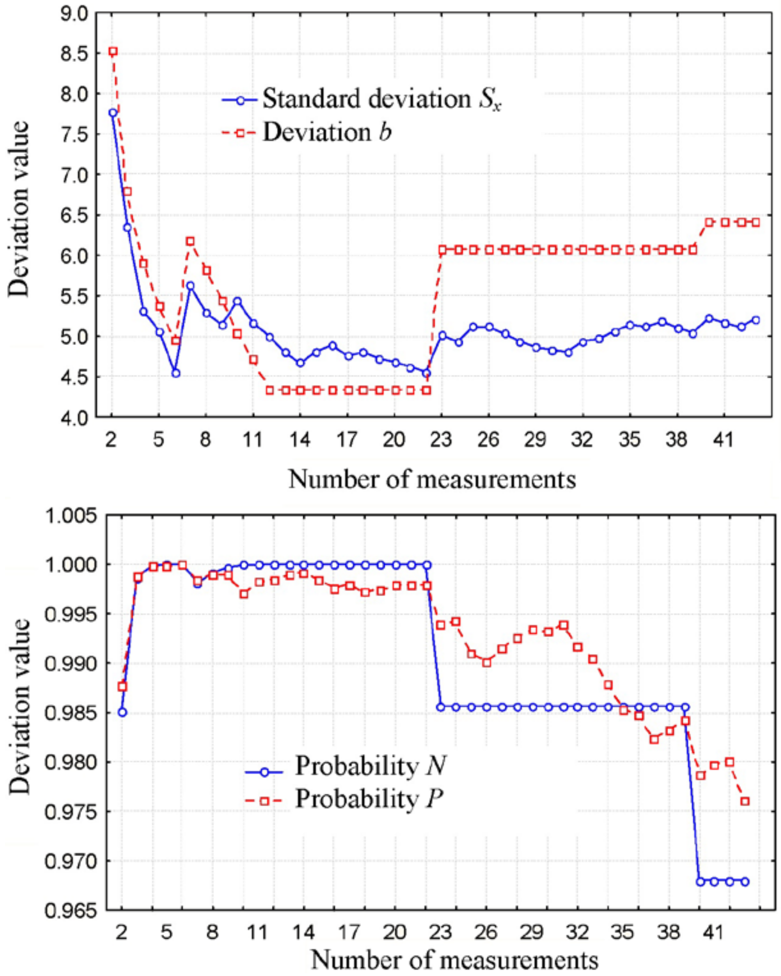

Figure 2. Diagrams of assessing the standard deviation and probability of zero defects calculated by the two methods.

6. Conclusion

To assess the accuracy of processes if there is a lack of information, it was proposed to use the methods of Lotfi Zadeh's possibility theory (fuzzy sets). We suggested specifications for assigning the solution risk level using the possibility theory. It is proved that at small sample volumes it is possible to obtain reliable solutions of fuzzy quality assessment problems. The comparative calculations using the statistical method and the possibility theory have shown that in case of limited information on the controlled parameters, a small sample is sufficient to reliably determine the parameters of the normal distribution and the probability of defects. In this case, the average error by the parameter values is 3–12%, and the defect probability is about 4%.

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