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Model for stationary turn of an arbitrary vehicle

I P Troyanovskaya[1,2] **and S A Voinash**[3,4]

[1] Department of Automobiles and Crawler Vehicles, South Ural State University, Chelyabinsk, Russia
[2] Department of Applied Mechanics, South Ural State Agrarian University, Troitsk, Russia
[3] Department of Land Transport Systems, Rubtsovsk Industrial Institute (Branch) of Polzunov Altai State Technical University, Rubtsovsk, Russia

[4] E-mail:sergey_voi@mail.ru

Abstract. The authors of the article propose a block-modular approach to constructing a mathematical model of stationary rotation of an arbitrary vehicle. This new method allows us to describe the curvilinear motion of a machine with any number of wheels and the type of transmission. Unknown ground reactions (driving force, shear force and stabilization moment in the wheel-to-ground contact) are functions of unknown coordinates of the instantaneous slip centre. The connection between the problem of force and the kinematic problem allows us to solve these problems simultaneously. Additional equations of holonomic (geometric) constraints reflect the position of the wheel and its installation angle relative to the vehicle frame. Equations of non-holonomic (kinematic) constraints describe the type of transmission and the way the wheel is moved. The advantage of the proposed approach is the ability to predict the behaviour of the machine during the design phase.

1. Introduction

Currently, automated control systems for various vehicles are particularly important [1, 2]. The maximum interest is caused by the curvilinear motion of the vehicle with the entrance and exit from the turn [3, 4], maneuvering [5], stability from falling [6] or lateral displacement [7]. The development of the control algorithm directly depends on the adequacy of the vehicle's motion model [8, 9]. Therefore, the actual task is to develop a methodology for compiling a mathematical model for the curvilinear motion of a vehicle, taking into account the dimensions of the machine, the number of wheels and the type of transmission.

The first design stage is building a stationary turn model, because it excludes the driver's impact and allows for the evaluation of the potential of the vehicle itself. The mathematical model of stationary rotation is a set of equations of motion and a set of constraints. Some restrictions reflect the design parameters of the car. Other restrictions reflect the transmission control scheme. Changing the control scheme is accomplished by replacing one constraint equation with another.

2. Motion equations

A stationary turn is characterized by a constant turning radius ρ and the velocity modulus V. The motion equations are algebraic equations:

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$$\left.\begin{array}{c} ma_{nx} = \Sigma F_x \\ ma_{ny} = \Sigma F_y \\ 0 = \Sigma M \end{array}\right\}, \quad (1)$$

where m is vehicle weight; a_{nx},a_{ny} are projections of the normal acceleration on the longitudinal and transverse centerline of the vehicle; ΣF_x,ΣF_y, ΣM are sum total of the projections of all the forces on the longitudinal and transverse centerline of the vehicle and the moments relative to the center of mass.

Ground-based reactions enter the right-hand side of equations (1) and are described on the basis of the mathematical theory of friction in the presence of sliding wheels along the ground [10].

3. Ground reactions in the wheel-to-ground contact

Today, most models of curvilinear movement of wheeled vehicles are based on the theory of lateral deviation [11, 12]. This approach neglects the sliding of the wheel relative to the ground.

However, sliding is an integral part of the movement for many special purpose vehicles. We described the soil reactions on the basis of the mathematical theory of friction [13, 14]. They are functions of unknown coordinates of instantaneous gliding centers (figure 1):

$$\left.\begin{array}{l} P_x = \iint_{\eta\xi} \varphi q \frac{y-\eta}{\sqrt{(x-\xi)^2+(y-\eta)^2}} d\xi d\eta, \\ P_y = -\iint_{\eta\xi} \varphi q \frac{x-\xi}{\sqrt{(x-\xi)^2+(y-\eta)^2}} d\xi d\eta, \\ M = \iint_{\eta\xi} \varphi q \sqrt{(x-\xi)^2+(y-\eta)^2}\, d\xi d\eta, \end{array}\right\}, \quad (2)$$

where ξ,η are sizes of the wheel-to-ground contact;$P_x, P_y, M$are resulting force factors in the wheel-to-ground contact; q is normal pressure in the wheel-to-ground contact; x, y are coordinates of the instantaneous slip center of the wheel with the ground; φ is coefficient of friction of the wheel slip relative to the grund.

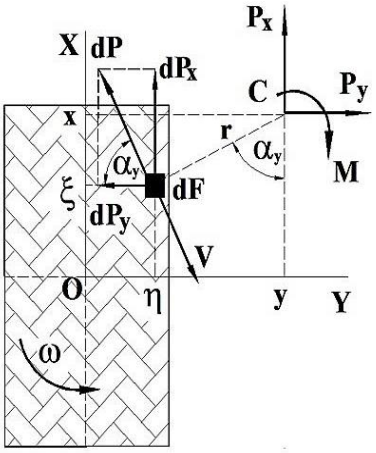


Figure 1. The frictional forces in the wheel-ground contact when it rotates glide.

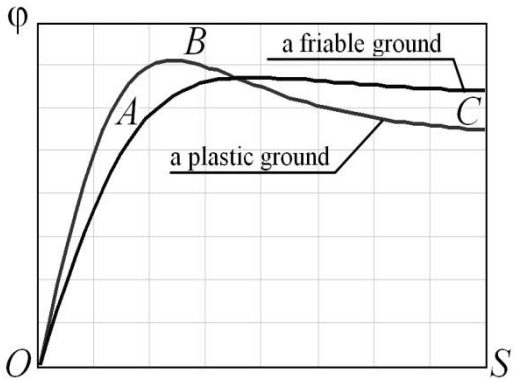


Figure 2. Dependence of friction coefficient φ on sliding S.

Coefficient of friction φ varies depending on the slip value *S* (figure 2). For small slips *S*, coefficient of friction φ takes into account the elastic properties of the tire [15].

The dependence in figure 2 is well described by formula [15]:

$$\varphi = \varphi_{max}\left(1 + \frac{\chi}{\mathrm{ch}(S/\lambda)}\right)\mathrm{th}\left(\frac{S}{\lambda}\right), \quad (3)$$

where *S* is slipping; λ, χ are the coefficients characterizing the elastic properties of the wheel and the soil; φ_{max} is factor of friction at full slipping.

Formulas (2) relate the problem of force and the kinematic problem. This allows you to solve them together.

4. Kinematic dependencies on did vehicle's turn

We examined in detail the kinematics of the wheel at the turn of the vehicle. This allowed us to write the coupling equations to find the unknown coordinates of the instantaneous slip centres *x* and *y* entering into equations (2).

The actual wheel speed differs from the theoretical speed modulo and direction due to its slip. The contact area of the wheel at the turn of the vehicle slides and instantly rotates. The instantaneous sliding center C of the contact pad of the wheel has coordinates *x* and *y* in the local wheel coordinate system (figure 1).

The sliding speed of the center of instantaneous sliding of *C* is zero. Then the velocity of the hull point, which lies above the instantaneous glide center, has the theoretical speed of the wheel and is directed along the plane of its rotation [16]. Consequently, the instantaneous center of sliding of the contact zone wheel-to-ground lies on the perpendicular, lowered from the center of rotation of the vehicle to its plane of rotation. This condition is written down by the equation ofholonomic (geometric) constraints:

$$y_c sin\alpha = x_c cos\alpha, \tag{4}$$

where α is the angle of the wheel's turn relative to the vehicle's frame;x_c,$y_c$are the coordinates of the center of instantaneous slip in the global coordinate system associated with the turning center of the vehicle. The relationship between the global coordinate system and local coordinate systems allows you to take into account the dimensions of the vehicle and the relative position of the wheels.

The number of geometric equations is equal to the number of wheels and characterizes the presence of steerable wheels (angle of turn) and their mutual arrangement.

Non-holonomic (kinematic) constraints are determined by the type of transmission or the mode of motion of the wheel. Their number is equal to the number of wheels of the vehicle too. They are written in the form of force relationships often [17]:

– *driven wheels* are characterized by the absence of pulling force (longitudinal component of the sliding friction force), which corresponds to the absence of transverse displacement of the instantaneous slip center

$$y = 0. \tag{5}$$

– *locked wheels* have no theoretical velocity, and their instantaneous slip center coincides with the turn center of the entire vehicle

$$\sqrt{x_c^2 + y_c^2} = 0. \tag{6}$$

– *the inter-wheel differential* provides equal torques between the drive wheels *i* and *j*, which corresponds to the equality of the pulling forces at equal wheel radiuses

$$P_{xi} = P_{xj}. \tag{7}$$

– *the interaxle differential* ensures the torque distribution between *i* and *j* axes with the proportionality factor μ

$$P_{xi} = \mu P_{xj}. \tag{8}$$

– *blocking of the differential* between wheels *i* and *j* ensures leveling of the theoretical velocities for them, which at the same angular velocity ω has the form

$$\sqrt{x_{ci}^2 + y_{ci}^2} = \sqrt{x_{cj}^2 + y_{cj}^2}. \quad (9)$$

– *rotation by the example of caterpillar machines* is provided by a given ratio k of theoretical speeds between *i* and *j* sides

$$\sqrt{x_{ci}^2 + y_{ci}^2} = k\sqrt{x_{cj}^2 + y_{cj}^2}. \quad (10)$$

5. Methodology of building a mathematical model

The number of unknowns for an arbitrary vehicle with n number of wheels is $2n + 2$:

- x_i, y_i are coordinates of the instantaneous slip centers of wheel *i* ($i = 1 \ldots n$);
- x_0, y_0 are coordinates of the turn center.

The number of equations is equal to the number of unknowns:

- three equations of motion (1) with allowance for (2);
- *n* equations of geometric constraint (4);
- *n* equations of kinematic constraint (5-10) depending on the transmission layout.

The result of solving the system of algebraic equations are all the force and kinematic parameters of the turning of the vehicle:

- tractive effort, transverse shear force, stabilizing torque in the contact of each wheel,
- the radius of the pivot and the speed of sliding: each wheel,
- the power required for the curvilinear motion of the vehicle [18].

6. Conclusion

A particular feature of the proposed approach is the block-modular construction of the mathematical model. The mathematical model consists of equations of motion and a block of constraints. The number of kinematic and geometric constraint equations is equal to the number of wheels of the vehicle.

The change in the transmission control system is made by replacing one equation of kinematic constraints with another. This allows you to compare the movement of the vehicle with different types of transmission sat the design stage.

The model includes all the main design parameters (overall dimensions, number of wheels, angles of their rotation relative to the body). This allows to evaluate the influence of each design parameter on the characteristics of curvilinear motion. The result is the ability to optimize vehicle parameters at the design stage.

The forces in the wheel-ground contact are written on the basis of the mathematical theory of friction. This allows you to take into account the sliding and elastic properties of the wheel and the ground.

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