

# Calculating the geometric parameters of the distribution of electron beam energy density on its section in EBW

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**Abstract.** The article considers various ways of calculating the diameter of an electron beam. Analysis of the energy density distribution curves in the cross section of the electron beam shows that they differ from the normal law. In view of this, a method is proposed for calculating the width of the distribution of energy density in the cross section of an electron beam, considering the shape of the distribution curve. The method is based on calculating the entropy of the distribution. The method of calculating the width of the distribution according to the probe characteristics is considered.

## 1. Introduction

Electron-beam welding (EBW) has become widespread in various branches of engineering. The electron beam is the main instrument in EBW. The energy parameters of the electron beam, namely the accelerating voltage  $U$ , the beam current  $I$  and the width of the distribution of energy density in its cross-section – the diameter of beam  $d$ , are the main energy characteristics in the process of electron-beam processing. Without the control and stabilization of the above parameters, it is impossible to provide the required quality of the technological process. This is especially important in the study of EBW technology, as well as for the repeatability of the technological process on various equipment and various power equipment in industrial conditions.

The power of the electron beam is equal to  $q = U \cdot I$ . If the diameter of the electron beam  $d$  is known, then it is possible to determine the power density  $q_2$  (W/cm<sup>2</sup>), which is one of the determining parameters of the process

$$q_2 = \frac{UI}{\pi d^2 / 4}.$$

Depending on the value of the power density  $q_2$  for the same energy input, a different configuration of the processing zone can be obtained. The most important and at the same time the most difficult parameter of an electron beam to determine is its diameter.

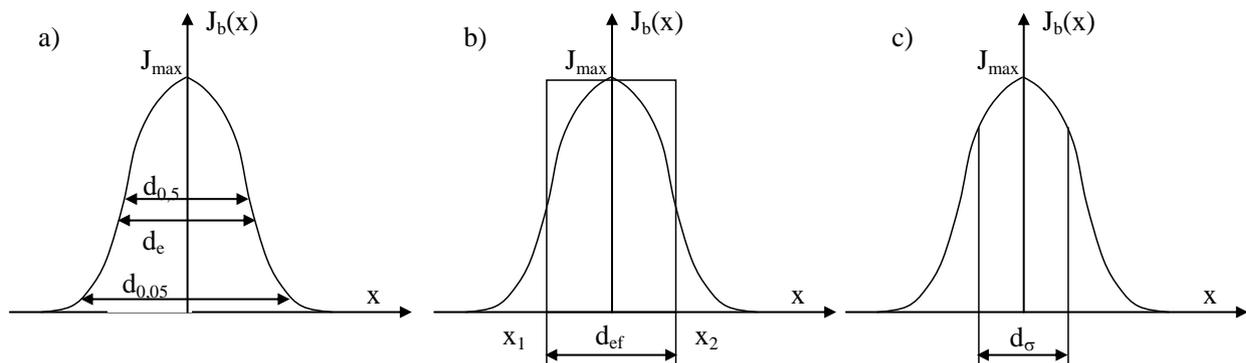
There are various ways to experimentally determine the diameter of an electron beam. The simplest method is the rotating probe [1, 2]. The essence of the method is that a thin rotating probe, made in the form of a tungsten wire with a diameter of 0.1 mm, crosses the electron beam perpendicular to its axis and selects a part of the current. The curve of the probe current (probe characteristic) is used to build the distribution of current density for the electron beam on its cross section and calculate the diameter. More accurate results are obtained when the beam is deflected to a slit diaphragm or a hole of small



diameter, under which the Faraday cup is placed [3 - 5]. To control the width of the energy density of the beam on the surface of the workpieces, an X-ray sensor with a collimated nozzle is used [6]. The sensor is oriented on the surface of the parts to be welded, and to determine the geometric parameters, the electron beam intersects the zone of view of the collimated sensor.

The diameter of the electron beam is calculated from the value of the width of the current distribution density.

As parameters characterizing the width of the electron beam current density distribution, the following are typically used (figure 1):



**Figure 1.** The methods for determining the geometric parameters of the electron beam.

The width of the distribution curve (Figure 1a), calculated at the characteristic levels of its maximum value  $J_{\max}$  [7] is:

- $d_{0,05}$  at 5% of the maximum amplitude  $J_{\max}$ ;
- $d_{0,5}$  at 50% of the maximum amplitude  $J_{\max}$ ;
- $d_e$  at  $1/e$  of the maximum amplitude  $J_{\max}$ .

The effective diameter of the electron beam  $d_{ef}$ , or the zero central moment of the distribution normalized by the maximum value of the distribution  $J_{\max}$  (Figure 1b) [6]

$$d_{ef} = \frac{1}{J_{\max}} \int_{-\infty}^{\infty} j_b(x) dx$$

In this case, the current distribution function  $F(x) = \int_{-\infty}^x j_b(x) dx$  having the maximum value of the distribution density  $j_b(0) = J_{\max}$  is replaced by an equal distribution function  $F^*(x) = \int_{x_1}^{x_2} j_b^*(x) dx$  that reflects the uniform law of the distribution density, for which the distribution density  $j_b^*(x) = J_{\max}$  is on the region  $d_{ef} = (x_2 - x_1)$ .

The effective diameter for a normalized normal distribution is

$$d_{ef} = \frac{1}{J_{\max}} = \sigma_b \sqrt{2\pi};$$

where  $\sigma_b$  is the mean square deviation of electrons from the axis of the electron beam, which is determined through the second central moment of the distribution function

$$\mu_2(x) = \sigma_b^2(x) = \int_{-\infty}^{\infty} x^2 j_b(x) dx.$$

The mean square diameter  $d_\sigma = 2\sigma_b$  indicates the interval at which 68% of the energy of the electron beam hits, provided that the energy density is distributed on the surface of the workpiece according to the normal law (Figure 1c).

It should be noted that the existing estimates do not take into account the shape of the distribution curve or are calculated on the basis that the density is distributed according to the normal law. Existing distributions may differ from the normal law [8]. It was shown in [9] that the current density distribution in different sections of the same beam can have different shapes (uniform distribution with a minimum in the centre and normal distribution). It is established that the geometry of the beam is affected by changing the accelerating voltage and beam current. The shape of the distribution is also affected by various cathode defects and changes in the pressure in the vacuum chamber [3, 5, 9]. The presence of various factors affecting the beam energy density have led to the fact that there is no single-valued relationship between the mean square deviation  $\sigma_b$  and the diameter of the electron beam  $d$  as an energy estimate for the introduced power density  $q_2$ .

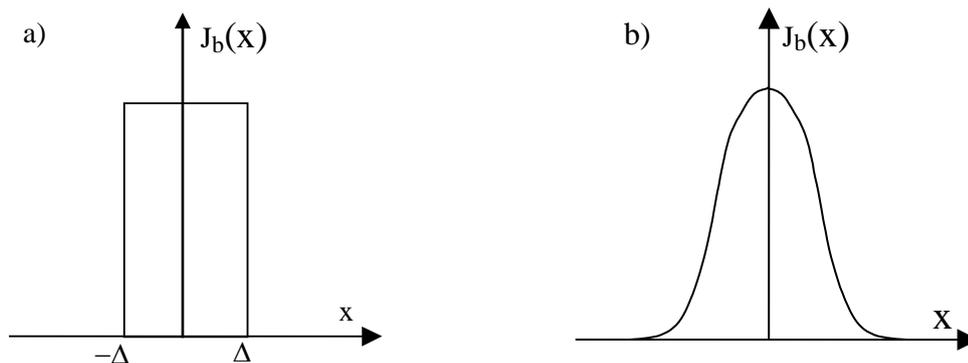
Considering the fact that the distributions are quite diverse and are described by different laws, they can be systematized by combining them into several classes. Inside a class, the distribution is described by a single analytic model, the parameters of which uniquely determine the form of the distribution. Such a systematization will simplify the process of identifying the form of distribution and will allow more accurate calculations of the electron beam diameter [10].

## 2. Calculation method of electron beam diameter

As a parameter that takes into account the law of distribution of energy density, entropy can be used to estimate the width of the distribution [10]

$$H(x) = - \int_{-\infty}^{\infty} j_b(x) \ln(j_b(x)) dx .$$

Entropy is a functional of the energy density distribution law and takes into account the features of this law. We will consider the numerical value of entropy and its relation to the mean square deviation for the uniform and normal distribution laws (figure 2).



**Figure 2.** The laws of distribution: a) - uniform; b) - normal.

Uniform energy distribution (figure 2a)

$$j_b(x) = \begin{cases} \frac{1}{2\Delta}, & npu|x| \leq \Delta \\ 0, & npu|x| > \Delta \end{cases} .$$

The entropy of the uniform distribution will be equal to

$$H_u(x) = - \int_{-\Delta}^{\Delta} \frac{1}{2\Delta} \ln \frac{1}{2\Delta} dx = \ln 2\Delta \quad (1)$$

The dispersion of the uniform distribution is

$$\sigma_b^2 = \int_{-\infty}^{\infty} x^2 j_b(x) dx = \int_{-\Delta}^{\Delta} x^2 \frac{1}{2\Delta} dx = \frac{\Delta^2}{3}.$$

From here, the entropy can be expressed in terms of the mean square deviation as

$$H_u(x) = \ln(2\sqrt{3}\sigma_b).$$

The normal distribution (Figure 2b)

$$j_b(x) = \frac{1}{\sqrt{2\pi}\sigma_b} \exp\left(-\frac{x^2}{2\sigma_b^2}\right).$$

The entropy value for the normal law is

$$H_n(x) = \int_{-\infty}^{\infty} j_b(x) \left( \ln(\sqrt{2\pi}\sigma_b) + \frac{x^2}{2\sigma_b^2} \right) dx = \ln(\sqrt{2\pi}\sigma_b) \int_{-\infty}^{\infty} j_b(x) dx + \frac{1}{2\sigma_b^2} \int_{-\infty}^{\infty} x^2 j_b(x) dx.$$

Since  $\int_{-\infty}^{\infty} j_b(x) dx = 1$  and  $\int_{-\infty}^{\infty} x^2 j_b(x) dx = \sigma_b^2$ , then

$$H_n(x) = \ln(\sqrt{2\pi}\sigma_b) + \frac{1}{2} = \ln(\sqrt{2\pi}\sigma_b) + \ln(\sqrt{e}) = \ln(\sqrt{2\pi e}\sigma_b) \quad (2)$$

When the above approach is used in practice for estimating the geometric parameters of the electron beam, it is more customary to operate not with the values of entropy, but with the width of the distribution density, giving it the name entropy diameter  $d_e$ .

As the entropy diameter, we choose the width of the uniform distribution density, the entropy of which is equal to the entropy of the density of the measured distribution.

We will calculate the entropy diameter for the case of the normal distribution of the electron beam current. For this, we equate the expressions (1) and (2).

$$d_e = 2\Delta = \sqrt{2\pi e}\sigma_b = 4,13\sigma_b.$$

In the case of an arbitrary shape of the distribution curve  $j_b(x) = \varphi(x)$ , the value of the entropy diameter is determined as follows:

We find the entropy of the distribution

$$H(x) = - \int_{-\infty}^{\infty} \varphi(x) \ln \varphi(x) dx.$$

We equate it to the expression (1) and get:

$$\ln 2\Delta = H(x) \quad d_e = 2\Delta = \exp(H(x)).$$

The entropy value of the diameter is obtained in the same units in which the width of the uniform distribution is measured, according to the relation

$$d_e = \exp(H_u(x)) = \exp(\ln(2\Delta)) = 2\Delta.$$

For an entropy estimate of the effective diameter, we choose an interval equal to two standard deviations of the normal distribution law, the entropy of which is equal to the entropy of the density of the measured distribution. This estimate is called the effective entropy diameter of the electron beam and is denoted by  $d_{ef.e}$ .

To calculate it, we find the entropy of the distribution of an arbitrary form  $H(x)$  and equate it to the expression (2)

$$\ln(\sqrt{2\pi e}\sigma_b) = H(x),$$

it means that

$$d_{ef.e} = 2\sigma_b = \frac{2}{\sqrt{2\pi e}} \exp H(x).$$

Considering that the value of the entropy diameter is  $d_e = \exp H(x)$ , the relationship between the effective entropy diameter and the entropy diameter has the form

$$d_{ef.e} = \frac{2}{\sqrt{2\pi e}} d_e = 0,484 \cdot d_e.$$

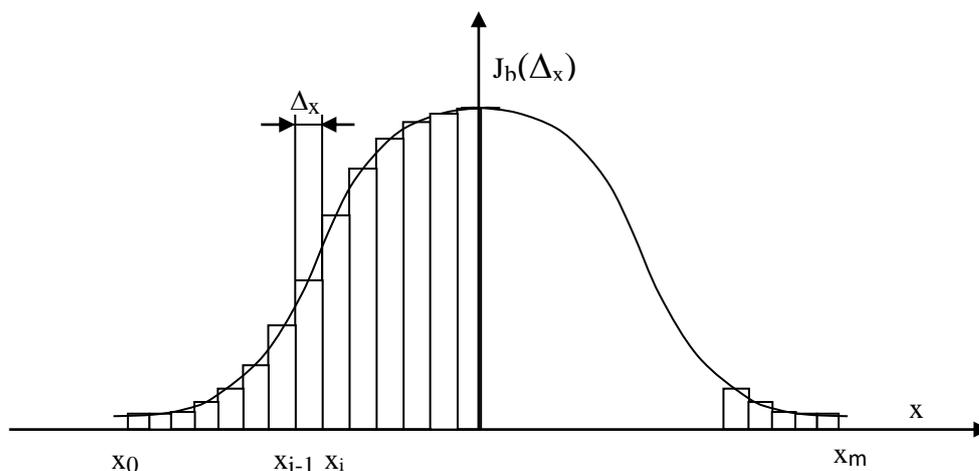
### 3. Experimental study

To calculate the entropy diameter by the probe characteristics, it should be taken into account that when estimating, the width of the distribution density of the electron beam is not set by the distribution law itself, but by a set of discrete values obeying this law. This is due to the fact that during the measurement of the density, the electron beam is deflected on a probe, a slit diaphragm, or by a projection of a collimator having certain dimensions. On the basis of this limited number of measurements, a stepped histogram can be constructed, approaching the actual distribution.

Let the step distribution (figure 3) consist of  $m$  columns with boundaries  $x_0, x_1, x_2, \dots, x_i, \dots, x_m$ . Each column has a width  $\Delta_x = x_i - x_{i-1}$  and includes  $n_i$  discrete measurement results. The distribution density throughout each of the columns remains constant and equal

$$J_b(\Delta_x) = \frac{n_i}{n \cdot \Delta_x},$$

where  $n = \sum_{i=1}^m n_i$ .



**Figure 3.** Step histogram of the beam current distribution.

The entropy of such a step distribution is equal to.

$$\begin{aligned}
 H &= - \int_{-\infty}^{\infty} j_b(\Delta_x) \ln(j_b(\Delta_x)) dx = - \sum_{i=1}^m \int_{x_{i-1}}^{x_i} \frac{n_i}{n \cdot \Delta_x} \ln \frac{n_i}{n \cdot \Delta_x} dx = - \sum_{i=1}^m \frac{n_i}{n \cdot \Delta_x} \ln \frac{n_i}{n \cdot \Delta_x} \int dx = \\
 &= \sum_{i=1}^m \frac{n_i}{n} \cdot \ln \frac{n \cdot \Delta_x}{n_i} = \sum_{i=1}^m \frac{n_i}{n} \ln \frac{n}{n_i} + \ln \Delta_x
 \end{aligned}$$

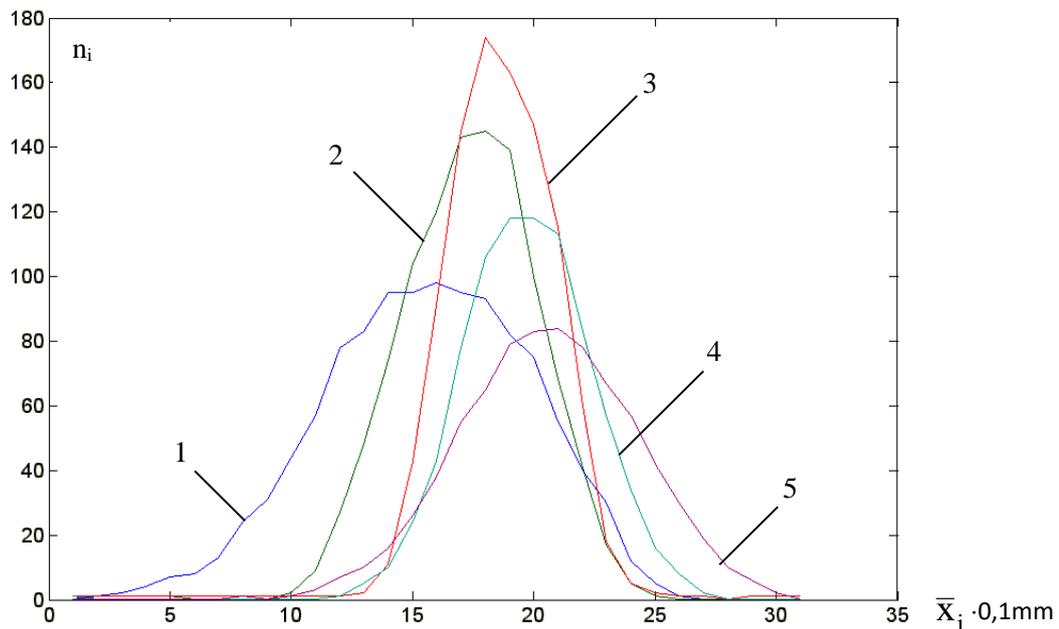
This expression can be converted into

$$H(\Delta_x) = \ln \Delta_x + \sum_{i=1}^m \ln \left( \frac{n}{n_i} \right)^{\frac{n_i}{n}} = \ln \left[ \Delta_x \prod_{i=1}^m \left( \frac{n}{n_i} \right)^{\frac{n_i}{n}} \right].$$

Then the entropy diameter will be equal to

$$d_e = \exp(H(\Delta_x)) = \Delta_x \prod_{i=1}^m \left( \frac{n}{n_i} \right)^{\frac{n_i}{n}} = \Delta_x \frac{n}{\sqrt[n]{\prod_{i=1}^m (n_i)^{n_i}}} = \Delta_x \cdot n \cdot 10^{-\frac{1}{n} \sum_{i=1}^m n_i \lg n_i}.$$

Figure 4 shows the probe characteristics obtained experimentally with the help of an automated system for monitoring the distribution of the density energy of the electron beam over its cross section [11, 12].



**Figure 4.** Probe characteristics of the density of the electron beam current distribution corresponding to different values of the current of the focusing system: 1 –  $I_f = 127$  mA; 2 –  $I_f = 129$  mA; 3 –  $I_f = 132$  mA; 4 –  $I_f = 134$  mA; 5 –  $I_f = 136$  mA.

A collimated x-ray sensor was used to control the distribution density. The sensor was located in such a way that the projection of the collimator, representing a 0.1 mm wide strip, was located in the immediate vicinity of the welding point.

To measure the geometric parameters of the current distribution, the electron beam periodically intersects the projection of the collimator. The calculation of the values of the standard deviation  $\sigma_b$  and entropy diameter  $d_e$  was made on the basis of statistical processing of a series of observations. Each series of observations consisted of eight implementations.

To estimate the accuracy of the measurements for each series of measurements, the entropy diameter of individual implementations was calculated. Then the obtained value was subjected to statistical processing. The standard deviation was calculated. The interval of the confidence error, which determines the interval for the confidence probability, was calculated from the expression [12].

$$\Delta_{0,9}(d_e) = 1,6 \cdot \sigma(d_e^*).$$

The relative error was calculated by the expression

$$\Delta_{rr}(d_e) = \frac{\Delta_{0,9}(d_e)}{d_e} \cdot 100\%.$$

The calculation results are shown in table 1.

**Table 1.** Beam density distribution parameters.

№ Set	Current of focusing system, mA	$\sigma_b$ , mm	$d_e$ , mm	$\Delta_{rr}(d_e)$ , %
1	127	0,424	1,701	1,8
2	129	0,293	1,162	6
3	132	0,28	0,953	8,9
4	134	0,282	1,082	8,8
5	136	0,72	1,499	4

#### 4. Conclusion

The width of the density distribution of the electron beam current must be estimated by a parameter that takes into account the shape of the distribution curve.

As a parameter that takes into account the current distribution law, the entropy  $H(x)$  and its nonlinear functional  $d_e = \exp(H(x))$  can be used to estimate the width of the distribution.

To reduce the influence of interference, the calculation of geometrical parameters should be made using the averaged probe characteristics, and the distribution parameters obtained for individual implementations should be used to assess the accuracy of the measurements.

#### Acknowledgements

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