IOP Conf. Series: Materials Science and Engineering **450** (2018) 062014 doi:10.1088/1757-899X/450/6/062014

Numerical modeling of surge wave in downstreams of the waterworks

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Abstract. Basic requirements for water measuring and water accounting devices developed to fulfil static control of water distribution have not changed virtually in recent decades. It is due primarily to the use of water accounting and measuring instruments developed and used in conditions of steady uniform water flow when unique dependence on depth and water discharge on the rise and decline of the waterline levels is observed. Under operation of watercourses in the unsteady water flow regimen, the existing means of water measurement are not applicable. Simultaneous direct measurement of hydraulic parameters in controlled cross sections of waterways is complicated even under steady water flow regimen. Nowadays there is the lack of data to predict extreme values of discharges and water depth in downstreams of the waterworks due to the absence of tested and feasible methods of water depth and discharge measurement in calculated cross sections of waterways. With the application of the complete integral theory, a hydraulic calculation of the changes in the flow rate and the depth of flow in fixed sections of the downstream of the waterworks was performed. It is based on the analytical method of linearized system solution of one-dimensional differential equations in partial derivatives of Saint-Venant hyperbolic type.

1. Introduction

In problems associated with the operation of the discharge waterworks in the sections of watercourses diverting water from the facilities, the most important problem is to promptly obtain reliable predictive information about changes in the parameters of water bodies in various modes of their operation. Currently, the most effective way to obtain such information is to use methods of mathematical modelling and simulation studies of transient processes.

It should be borne in mind that real water bodies characterized by complex morphometry and complex transient dynamic processes of water flow depend on a large number of hydraulic parameters. In this regard, for the correct description of real water bodies, in general, very complex multifactor mathematical models are required. At the same time, solving practical problems requires, first of all, such mathematical models that allow one to obtain concrete numerical estimates for a minimum estimated time. Therefore, obtaining simplified dependencies that adequately describe real hydraulic processes in the continuation of the minimum estimated time is an urgent problem.

In the method of characteristics, a difference scheme designed to solve the formulated problem is based on characteristic directions. A feature of the scheme is the possibility of relief selection of parts related to the movement of water. When choosing a finite difference scheme, factors such as numerical stability, accuracy, computation time, as well as ease of programming and inclusion of boundary

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conditions were taken into account. Algorithms for the functioning of transient hydraulic process control tools based on the method of characteristics are discussed in [25-32].

The law of regulation of the water supply is a boundary condition in the perturbation of the watercourse. It can be set in two versions: for the flow of water and for the depth of the watercourse [4]. To improve accuracy, the method of hydraulic calculation of flow rates and water depths in watercourses in the downstream of water-discharging waterworks was specified in the works. The boundary condition at the perturbation of the watercourse in this case was considered for the flow of water.

During surges through hydrosystems of large water flows, significant changes in water surface slopes occur, as well as high flow rates that can activate river and coastal deformations, which should be taken into account when predicting them [9-12, 14].

In conditions of reservoir overflow, an important problem is the prediction of changes in the water regime of watercourses. For this purpose, a simulation of unsteady flow in the downstreams of watercourses is performed, the operating mode of which is evaluated in terms of the implementation of a catastrophically high release [15-24]. The paper describes an approach to solve this problem based on numerical integration of Saint-Venant equations [3].

The purpose of this paper is to develop a method for numerical simulation of a surge wave in the downstream of spillway hydro-systems based on analytical solutions of the linearized partial differential equations of Saint-Venant hyperbolic type.

The paper formulates and solves a fundamentally new mathematical problem of calculating unsteady water flow using the boundary condition reflecting the physical characteristics of the hydraulic process under consideration with a sufficient degree of accuracy. The hydraulic calculations of extreme flows and water depths in critical sections of watercourses with unsteady water flow mode by solving the Saint-Venant equations using the proposed analytical method and characteristics method provide detailed information about the hydraulic mode of the spillway hydraulic systems and the nature of the flow movement in the lower pools.

2. Methods

We present an analytical method for solving the formulated problem. The process of propagation and transformation of long waves is described by a system of Saint-Venant hyperbolic-type one-dimensional partial differential equations [3]:

$$\frac{\partial Q}{\partial t} + 2U\frac{\partial Q}{\partial X} - (U^2 - C^2)B\frac{\partial H}{\partial X} - g\omega\left(i_0 - \frac{U^2}{C_{sh}^2 R}\right) = 0\ , \tag{1}$$

$$\frac{\partial Q}{\partial X} + B\frac{\partial H}{\partial t} = 0\ , \tag{2}$$

where: Q – water flow, m^3/s; C – initial perturbation propagation velocity, m/s; U - average flow rate of water in cross section, m/s; H – flow depth, m; X - spatial coordinate, m; t - time, s; ω - cross sectional area, m^2; B - waterway width along the water edge, m; g - gravity acceleration, m/s^2; i_0 - slope of watercourse; C_{sh} - Shezi coefficient, $m^{0.5}/s$; R - hydraulic radius, m.

Differential equations (1) and (2) are non-linear and, in general, they do not have an exact solution. To obtain approximate solutions, these equations are linearized [4-6].

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$$B_0\Delta H = -\frac{\zeta}{(U_0 \mp C_0)}\left[X-(U_0 \mp C_0)t\right] - \frac{K}{\overline{W}(U_0 \mp C_0)}\left[\exp\left(-\frac{\overline{W}X}{(U_0 \pm C_0)}\right)-1\right]$$
$$+\frac{M}{\overline{W}(U_0 \pm C_0)}\left[\exp\left(\frac{\overline{W}}{(U_0 \mp C_0)}(X-2U_0 t)\right) - \exp\left(\frac{2U_0\overline{W}}{(U_0 \mp C_0)^2}\left[X-(U_0 \mp C_0)t\right]\right)\right], \quad (3)$$
$$+\frac{M}{\overline{W}(U_0 \mp C_0)}\left[\exp\left(-\frac{\overline{W}X}{(U_0 \pm C_0)}\right)-1\right]$$

.

$$\Delta Q = -\zeta\left[X-(U_0 \mp C_0)t\right] - \frac{K}{\overline{\Pi}}(e^{-\overline{W}t}-1) + \frac{M}{\overline{W}}\left\{\frac{(U_0 \mp C_0)}{(U_0 \pm C_0)}\left[\exp\left(\frac{\overline{W}}{(U_0 \mp C_0)}(X-2U_0 t)\right)\right.\right.$$
$$\left.\left.-\exp\left(\frac{2U_0\overline{W}}{(U_0 \mp C_0)^2}\left[X-(U_0 \mp C_0)t\right]\right)\right] + \exp\left(\frac{\overline{W}}{(U_0 \mp C_0)}(X-2U_0 t)\right)-1\right\}. \quad (4)$$

The value of the constant parameter *K* is found from the boundary conditions:

$$\Delta Q = 0\,,\ \Delta H = 0 \ \text{when}\ t = T\,,\ X = (U_0 \pm C_0)T\,. \quad (5)$$

Substituting (27) into (25) and (26), we find:

$$K = \frac{\left\{\mp 2\overline{W}C_0 T + M\left[\frac{2U_0}{(U_0 \pm C_0)}e^{-\overline{W}T} - \frac{(U_0 \mp C_0)}{(U_0 \pm C_0)}\exp\left(\pm\frac{4\overline{W}U_0 C_0 T}{(U_0 \mp C_0)^2}\right)-1\right]\right\}}{(e^{-\overline{W}T}-1)}, \quad (6)$$

where *T* – the propagation time of the initial perturbation to the estimated target x, defined by the formula:

$$T = X\left(U_0 \pm C_0\right)^{-1} \quad (7)$$

In the derived relationships, the upper sign is taken if the direction of the disturbance movement coincides with the direction of the water flow in the watercourse. Otherwise, the bottom sign is taken.

The obtained analytical solutions (3) and (4) are applied in the work to calculate the transformation process of waves of one direction moving in an infinitely long prismatic channel of semi-limited length $(0 \le X < \infty)$. In this case, in relations (3), (4), the value of the parameter *M*=0 should be taken, that makes these relations much simpler.

Parameter $\overline{W}$ is defined by the equation (9), considered in the following form:

$$\overline{W} = \left(\beta\frac{(\overline{Q}-Q_0)}{B_0(\overline{H}-H_0)} - \gamma\right)\left[\frac{(\overline{Q}-Q_0)}{B_0(\overline{H}-H_0)} - (U_0 \mp C_0)\right]^{-1}. \quad (8)$$

Q_0 и H_0 are considered as initial values of flow and depth in the calculated area; $\overline{Q}$ и $\overline{H}$ - average values of flow and depth in the calculated area, determined from ratios:

$$\overline{H} = \frac{H_k + H_0}{2}\,, \quad (9)$$

$$\overline{Q} = \frac{Q_k + Q_0}{2}\,, \quad (10)$$

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where H_k - water depth determined from the boundary condition (13) at the value $t = t_k$; Q_k - water consumption corresponding to the depth determined from the boundary condition (3) with the value $t = t_k$.

3. Results

Initial calculation data: water flow Q_0 =520 m^3/s; average flow rate U_0=0.9 m/s; average waterway depth H_0=3.8 m; slope laying m=0; waterway width b=150 m; roughness coefficient n=0.02; waterway course i=0.00006.

The change in the initial position depth (x=0) of the hydroelectric complex downstream corresponds to the law (13):

$$H = H_0 + \xi t = 3.8 + 0.00002t; \quad 0 \le t \le 3600c.$$

where $\xi = \dfrac{\zeta}{B_0}$.

Let us calculate changes in depths and water flow in fixed sections X=0 m, X=5000 m, X=10000 m and X=20000 m, if at the initial moment in the prismatic channel of the river, in the calculated section, a flow pattern close to a uniform mode was observed .

1. We will calculate the following parameters C_0, β, D, γ, F, P, K и M using equations (6), (7), (8), (9) и (10):

$$C_0 = \sqrt{9.81 \times 3.8} = 6.106 \text{ м/с}, \ \beta = \frac{2 \times 9.81 \times 0.00006}{0.9} = 0.001,$$

$$F = \left(1 + 2\frac{1}{6}\right)\left[\frac{2}{(150 + 2 \times 3.8)} - \frac{150}{150 \times 3.8}\right] = -0.334,$$

$$\gamma = \left(2 \times 9.81 \times 0.00006 + 0.00006 \times 6.106^2 \times 0.334\right) = 0.0019,$$

$$P = 0.037, \quad K = -0.037 \ , \quad M = 0, \quad \overline{W} = 0.000003.$$

2. Formula (29) is used to determine the propagation time of the initial perturbation to the estimated alignment.

3. Changes in the depth and flow of water over time in fixed sections are calculated by means of formulas (5) and (6) using the software package. The calculation results are shown in figures 1 and 2.

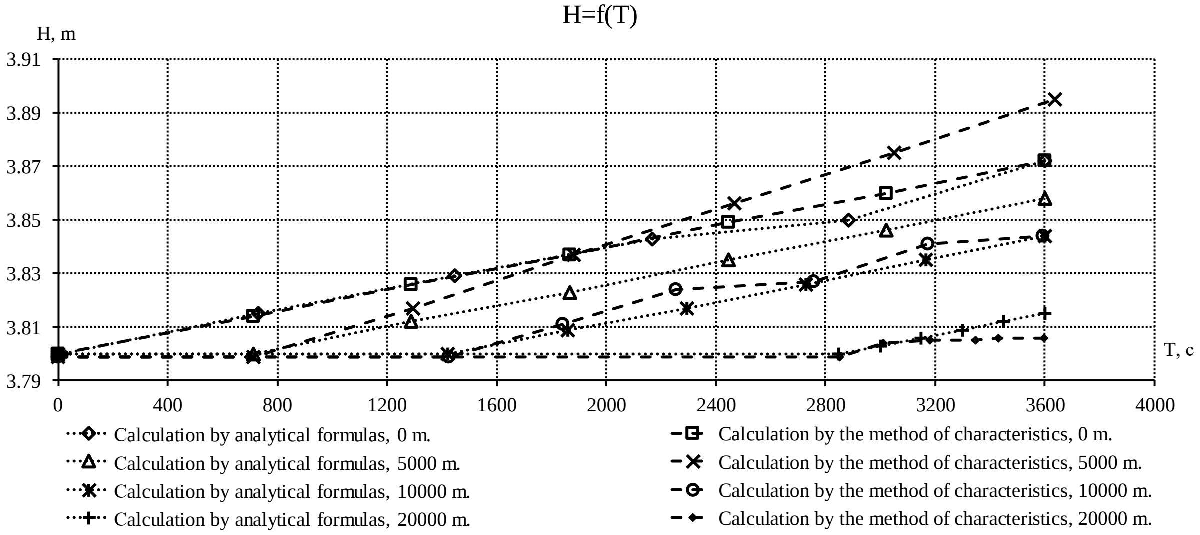

Figure 1. Graph of changes in water depths from time in fixed gauges.

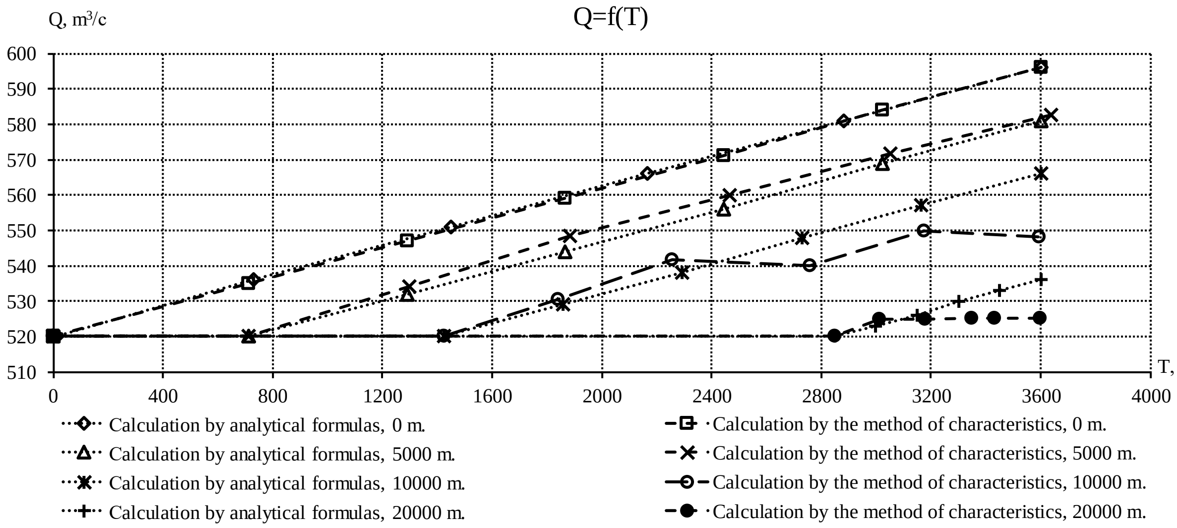

Figure 2. Schedule of changes in water consumption from time in fixed sections.

4. Discussion

Using the theory of a complete integral, analytical solutions (5), (6) were obtained. These solutions are used to calculate the depths and flow rates in fixed sections of the downstream of the hydroelectric complex, describing the process of transformation of waves of one direction, moving in an infinitely long prismatic channel of semi-limited length $\left(0 \leq x < \infty\right)$.

Analytical solutions (5) and (6) can be generalized for real release hydrographs. In this case, the curve of the hydrograph with any degree of accuracy is replaced by a linear approximating broken line. Knowing the solution of a non-stationary problem for one step, it is possible to obtain a total solution for all steps.

The calculations were performed using the analytical formulas (5), (6) derived in the work and the method of characteristics [3–9], which is considered as analogue.

5. Conclusions

The paper proposes an analytical method for solving a linearized system of partial differential equations of Saint-Venant hyperbolic type. The process of linearization of these equations was considered in [5, 6]. The mathematical algorithm for the analytical solution of the Cauchy problem using the classical theory of a complete integral, studied in [7, 8], was used to solve Saint-Venant equations.

Analytical solutions are obtained that describe the transformation of waves of one direction moving in an infinitely long prismatic channel of semi-limited length $\left(0 \leq x < \infty\right)$ with an initial uniform flow of water.

Examples of calculation by two methods are given: the developed analytical method for the hydraulic calculation of the discharge of water discharge from reservoirs and the adopted classical method of characteristics, which is considered as analogue.

Comparison of the calculation results for the two methods allowed us to determine the maximum relative error, which for the calculated depths and water flow does not exceed 3.5%.

The introduction of the considered method will allow to optimize the processes of water measurement and minimize idle and non-technological discharges of water from the reservoirs.

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