Direction-finding method solution phase ambiguity in determining the spatial orientation of space vehicle

I N Kartsan, M O Petrosyan, S V Efremova and V V Khrapunova

Reshetnev Siberian State University of Science and Technology, 31, Krasnoyarsky Rabochy Av., Krasnoyarsk, 660037, Russia

E-mail: kartsan2003@mail.ru

Abstract. When creating multichannel GNSS receivers that implement interferometric methods for measuring spatial orientation using GLONASS and GPS signals, the main problem is the resolution of phase ambiguity when measuring the phase difference for spatially separated antennas. In order to improve the accuracy of determining the spatial orientation, interferometers with the distance between the antennas (base length) reaching several meters are used. The ambiguity in measuring the phase shift is due to the fact that the wavelength of the measured signals is sufficiently small (about 19 cm), which is much smaller than the length of the bases of the interferometer. The methods for resolving phase ambiguity can be divided into two classes: one-stage ones that operate on the basis of the results of each measurement and methods based on filtering that require measurement of phase shifts over a certain time interval.

1. Introduction

One of the promising directions for the development of GNSS receivers is the use of antenna arrays as an antenna system. This makes it possible to improve the noise immunity and accuracy of measurements of radio navigation parameters, and also provides a reliable resolution of phase ambiguity and an increase in the accuracy of measuring the spatial orientation of the object associated with the antenna system due to the redundancy of the measured parameters [1]. The use of antenna arrays can be multifunctional. When measuring the coordinates and the velocity vector of the object, it is possible to form a narrow radiation pattern for each GNSS space vehicle separately, which provides a significant increase in the signal-to-noise ratio and an increase in noise immunity [2]. When measuring the angles of the spatial orientation of the antenna array, a multi-base interferometer can be used.

Consider the algorithm for resolving phase ambiguity in the antenna array using the example of an 8-element antenna array which antenna elements are evenly distributed in a circle with a radius of 1 m (figure 1).

To resolve the phase ambiguity, the most often used method is the exhaustive method [3]. In the case of a multi-base interferometer, due to redundancy in the interferometer bases, it is possible to resolve the phase ambiguity for each navigational space vehicle separately. For this, in the basis for the phase ambiguity resolution algorithm should be used the direction-finding method for determining the spatial orientation, that is, in the coordinate system associated with the object [3, 4].
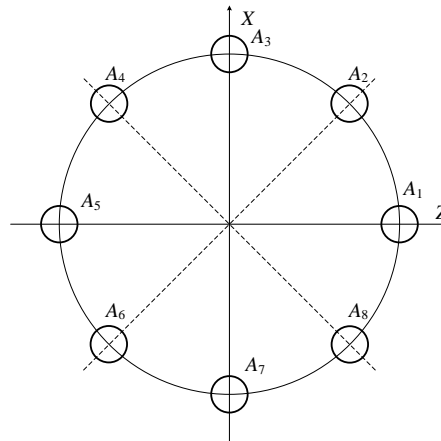
Figure 1. Scheme of 8-element antenna array.

Phase ambiguity occurs for each navigation space vehicle for each base, a complete search of all combinations is impossible due to the large number of options. To reduce the number of searches in the direction-finding algorithm, on the first step the phase resolution of phase ambiguity for each navigation space vehicle is made separately [5-7].

2. Proposed author method of resolution of phase ambiguity, based on direction finding

When using an m-antenna interferometer, one of the antennas is used as a reference antenna and, together with the remaining antennas, forms m-1 vector bases [6, 7-9].

Let's select a reference antenna, with respect to which phase shifts are measured, so that the first two bases are located along the axes of the antenna system. In this case, antenna A6 will be used as the reference.

The direction cosines of the vector-base can be determined from the equation on the basis of the scalar product of the vectors:

$$\frac{\lambda \varphi}{2\pi} = \Phi = k_x x + k_y y + k_z z, \tag{1}$$

where Φ is the phase shift, expressed in units of length, which is the difference in the travel of sign signals of navigation space vehicles between the antennas $A6$ and A_4; kx, ky, kz – direction cosines direction vector on navigational space vehicle; x, y, z – coordinates of base-vector.

The calculation of the direction cosines of the base-vectors is carried out on the basis of the equation. (1). When using a two-base interferometer, the initial system of equations includes 2N linear equations (2), where N – number of observable navigational vehicles.

$$\begin{cases} k_{xi}x_1 + k_{yi}y_1 + k_{zi}z_1 = \Phi_{1i}, \\ k_{xi}x_2 + k_{yi}y_2 + k_{zi}z_2 = \Phi_{2i}, \end{cases} \tag{2}$$

where i is the number of navigational space vehicles.

In the coordinate system associated with the object, the coordinates of the base-vectors are known, and the unknowns – are the direction cosines of directions on the NSV [10–12]. The system of equations (2) can be supplemented by the equations of coupling between the direction cosines of the directions on the navigational space vehicles (NSV).

$$\sqrt{k_{xi}^2 + k_{yi}^2 + k_{zi}^2} = 1 \tag{3}$$

and the linkage equations between the directions for different NSV.

$$k_{xi}k_{xk} + k_{yi}k_{yk} + k_{zi}k_{zk} = \cos\gamma_{ik} \tag{4}$$

The initial system of equations for determining the direction cosines of a base-vector for a multi-base interferometer includes Nb·N linear equations (1), where Nb – number of interferometer bases, N – number of navigational space vehicles; N linkage quadratic equations between the directional cosines of the directions on the NSV (3) and N·(N – 1)/2 quadratic equations between the directions on the NSV (4).

$$\begin{cases} k_{xi}x_j + k_{yi}y_j + k_{zi}z_j = \Phi_{ij}, \\ \sqrt{k_{xi}^2 + k_{yi}^2 + k_{zi}^2} = 1, \\ k_{xm} \cdot k_{xn} + k_{ym} \cdot k_{yn} + k_{zm} \cdot k_{zn} = \cos\gamma_{mn}. \end{cases} \tag{5}$$

The system of equations (5) is identical to the system of equations for a multi-base interferometer. The unknown coordinates of the base-vectors s in the system (5) are played by the unknown coordinates of the direction-vectors on the NSV, and the role of the coefficients is known coordinates of the base-vectors. Thus, the given system of equations is symmetric to parameter groups, one of which is the coordinates of the base-vectors, and the other is the direction cosines of directions on the NSV.

For the antenna array under consideration, the coordinates of the base-vectors are given in table 1.

Table 1. Coordinates of the base-vectors.

№	1	2	3	4	5	6	7
Z	- 0.293	0.0	0.707	1.414	1.707	1.414	0.707
X	0.707	1.414	1.707	1.414	0.707	0.0	- 0.293

When using the direction-finding algorithm, the direction cosines of the direction-vector on the NSV are unknown [5, 13-15]. To determine them, three equations are needed. Taking into account the linkage equation between the components of the unknown vector, two bases are required to compose the initial set of solutions [16]. The minimal system of equations has the form

$$\begin{cases} k_x x_1 + k_y y_1 + k_z z_1 = \Phi_1, \\ k_x x_2 + k_y y_2 + k_z z_2 = \Phi_2, \\ \sqrt{k_x^2 + k_y^2 + k_z^2} = 1. \end{cases} \tag{6}$$

We will choose a related system of equations so that both bases lie in the horizontal plane of the object, i.e. $y_1 = 0$, $y_2 = 0$.

Then the system of equations takes the form

$$\begin{cases} k_x x_1 + k_z z_1 = \Phi_1, \\ k_x x_2 + k_z z_2 = \Phi_2 \\ \sqrt{k_x^2 + k_y^2 + k_z^2} = 1. \end{cases} \tag{7}$$

The linear part of the system of equations (7) describes the projection of the direction vector onto the NSV in the horizontal plane of the object; the nonlinear equation can be used to determine the vertical component. In this case, taking into account that the NSV signals can only be received from the upper hemisphere (relative to the antenna array plane), the values of ky must take only positive values. With respect to the linear part of the system of equations (7), the locus of points of possible

positions of the vector-direction on the NSV lies inside the circle of unit radius. If the desired vector lies in the horizontal plane, then it is located on the border of this region.

To search through possible solutions, it is necessary to display the range of possible values in the basis of the base-vectors B1, B2. In the basis B1, B2, the border of the range of possible values is displayed in the form of an ellipse, while the maximum values of the path difference are equal to the length of the bases, and the eccentricity of the ellipse depends on the angle between the bases-the eccentricity of the ellipse decreases with a decrease in the angle between the bases. Figure 2 shows the range of possible values of path differences for the length of bases $B_1 = 2$ m, $B_2 = 1$ m, angle between the bases 30^0.
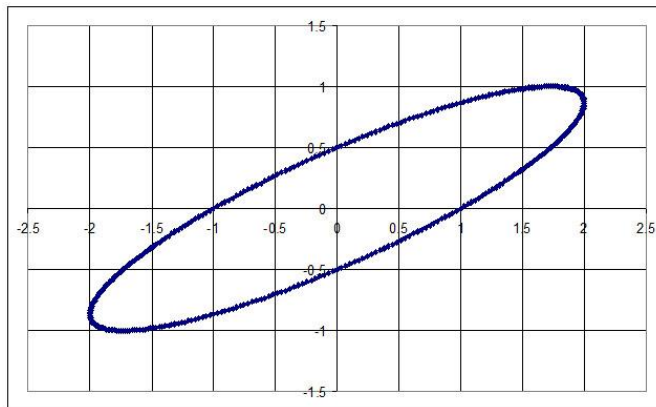

Figure 2. The range of possible values of path differences for the length of bases B1 = 2 m, B2 = 1 m, angle between the bases 30^0.

In case of decrease in angle between the directions on NSV, on the contrary, the range of possible values of phase ambiguity decreases, and within the limit, when the directions almost coincide, the ambiguities for these NSV are equal, and the search is almost unnecessary [17,18]. However, this increases the error in calculating the coordinates of the difference vector.

Thus, when pairs of NSV are selected, on the one hand, it is not necessary to take pairs with large angles between them, and on the other hand, not very close navigation space vehicles. In addition, the difference vector can be directed both to the upper half-plane, and to the lower (relative to the horizontal plane of the object).

Let us simulate the proposed algorithm.

Let us suppose that the azimuth of the NSV is 30°, spot angle is – 60°. The wavelength for the L_1 range is $\lambda = 0.1872$ m. We assume that the wave front coming from the NSV, is flat. Table 2 shows the phase shifts of the NSV signals for the considered antenna array configuration and the location of the NSV.

Table 2 depicts the phase shifts of the NSV signals due to the difference in the course of each of the bases.

Table 2. Phase shifts of the NSV signals.

Base	B_1	B_2	B_3	B_4	B_5	B_6	B_7
The difference in stroke, m.	0,086381	0,612262	1,269381	1,672762	1,366631	1,0605	0,403381
The difference in stroke, degree.	166,1173	1177,427	2441,117	3216,85	2628,137	2039,423	775,7327

Initial configuration of the base-vectors – B_2, B_6 (see figure 2) We substitute the coordinates of the base-vectors from table 2 into the system of equations (7).

$$\begin{cases} 1.414 \cdot k_x = \Phi_1, \\ 1.414 \cdot k_z z_2 = \Phi_2, \\ \sqrt{k_x^2 + k_y^2 + k_z^2} = 1. \end{cases}$$

or

$$\begin{cases} k_x = \Phi_1 / 1.414, \\ k_z = \Phi_2 / 1.414, \\ k_z = \sqrt{1 - \left(k_x^2 + k_z^2\right)}. \end{cases}$$

The initial rejection of solutions that do not fall within the range of possible values will be performed by the criterion:

$$\left(k_x^2 + k_z^2\right) < 1$$

The ambiguity range for bases is:

$$N_{max} = \pm B_1/\lambda = 1.414/0.1872 = 7.6$$

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Let's take $N_{max} = \pm 8$.

Table 3 shows the possible values of Φ_1 and Φ_2. The boldface indicates the true values. Note that the coefficient k_x depends only on Φ_1, and the coefficient k_z – depends only on Φ_2.

Table 3. Possible values of Φ_1 and Φ_2.

N	Φ_1	Φ_2	N	Φ_1	Φ_2
−8	−1,44694	−1,5603	1	0,237862	0,1245
−7	−1,25974	−1,3731	2	0,425062	0,3117
−6	−1,07254	−1,1859	3	**0,612262**	0,4989
−5	−0,88534	−0,9987	4	0,799462	0,6861
−4	−0,69814	−0,8115	5	0,986662	0,8733
−3	−0,51094	−0,6243	6	1,173862	**1,0605**
−2	−0,32374	−0,4371	7	1,361062	1,2477
−1	−0,13654	−0,2499	8	1,548262	1,4349
0	0,050662	−0,0627	−	−	−

Table 4 shows the possible values of k_x and k_z. The boldface indicates the true values.

Table 4. Possible values of k_x and k_z.

N	k_x	k_z	N	k_x	k_z
−8	−1,02329	−1,10347	1	0,168219	0,088048
−7	−0,8909	−0,97107	2	0,30061	0,220438
−6	−0,75851	−0,83868	3	**0,433**	0,352829
−5	−0,62612	−0,70629	4	0,56539	0,485219
−4	−0,49373	−0,5739	5	0,697781	0,61761
−3	−0,36134	−0,44151	6	0,830171	**0,75**
−2	−0,22895	−0,30912	7	0,962562	0,88239
−1	−0,09656	−0,17673	8	1,094952	1,014781
0	0,035829	−0,04434	−	−	−

The values $N = \pm\, 8$ are rejected immediately, since the result is greater than 1. Therefore, the maximum value should be rounded down, i.e. up to ± 7. Possible values can only lie in the upper hemisphere. If the search is carried out according to the difference of the NSV, then the possible positions of the vector can be in both the upper and lower hemisphere, and these positions differ only in the sign of the vertical component k_y.

The initial set of solutions with rejection $\left(k_x^2 + k_z^2\right) < 1$ consist of 180 possible solutions. Next, for each possible bearing, we calculate the path differences for the remaining bases, subtract the measured values of the phase shifts from them, and by subtracting the whole cycles, we bring the resulting differences to the range $\pm\lambda/2$.

The total discrepancy has a unique zero value corresponding to the correct solution. The smallest values of the discrepancy for false solutions are given in Table 5, in which the only correct solution is bold marked.

Table 5. Least values of discrepancies for false solutions.

N_1	N_2	Discrepancy, m (degree,)
3	**6**	**0**
3	- 4	0,013504 (25,97°)
1	1	0,023639 (45,46°)
5	5	0,023639 (45,46°)
3	2	0,032038 (61,61°)
1	-3	0,033951 (65,29°)
5	-3	0,033951 (65,29°)

We shall now estimate the noise immunity of the algorithm. Since residuals are the difference between the a priori and the measured values of the phase shift, the total residual, equal to the square root of the sum of the squares of the residuals, directly characterizes the error in measuring the phase shift at which the false solution differs from the true one [19, 20]. The total residual includes measurements according to 5 bases therefore the maximum error of measurement is 10...20°. At the same time, in the final set there is, apart from a true one, 1-2 false decisions. These results are comparable with the navigation algorithm, in which a search of ambiguities of 6 NSVs during a measurement on one base is performed.

Further rejection is performed by the angle between the direction cosines between the directions to different NSVs. At the same time, the number of NSV is at least 4, and more often 6-8 NSV for each of the GLONASS and GPS systems. As the experience of using the navigation algorithm with a two-base interferometer shows, the maximum error in measuring phase shifts is 20-40°, so we can expect that the direction-finding algorithm should at least have no worse immunity.

3. Conclusion

In this article, one of the problems in developing methods for accelerated resolution of phase ambiguity arising in multichannel GNSS receivers during the implementation of interferometric methods for measuring spatial orientation is solved. It should be noted that the developed method for resolving phase ambiguity in GNSS receivers equipped with multi-element antenna arrays based on the direction-finding method makes it possible to use search methods even at sufficiently long bases due to a small number of search options.

Acknowledgements

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