

# Evaluating Gravity-Assist Range Set Based on Supervised Machine Learning

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**Abstract.** The dynamics of gravity-assist (GA) trajectories contain strong nonlinearity, which makes the traditional methods for impulse transfer range set (RS) are intractable to deal with the gravity-assist RS. This paper develops a novel method to evaluate the gravity-assist RS based on regression methods in supervised machine learning (SML) field. The performances of three powerful regression methods with several common kernel functions are assessed. The Gaussian Processes Regression (GPR) method with Matérn 3/2 kernel is selected because of the minimum mean squared error ( $1.11 \times 10^{-3} \text{ km}^2/\text{s}^2$ ). The predicting model based on GPR is constructed to make prediction from the orbital elements of destination orbits to the total velocity increment of corresponding optimal GA trajectories. The percentage error of predicting model is no more than 2%. Millions pairs of sample points are generated by the trained predicting model. The points with specified value of total velocity increment are extracted, of which the envelope constitutes the gravity-assist RS. Both of Venus GA and Mars GA trajectories are considered in this paper.

## 1. Introduction

The reachable domain of spacecraft designates the set of positions that the spacecraft can reach with given initial orbit and specified fuel constraint. Reachable domain is a useful tool for mission planning and collisions risk assessment between two spacecraft [1]. Existing researches on reachable domain are based on the description of the positional parameters in Cartesian coordinate [2] [3] or the Keplerian orbital elements [4]. Reachable domain is also known as range set (RS) in the researches described by Keplerian orbital elements. These researches were usually based on impulse transfer, in which appropriate mathematics tools could be used to derive the relationship between the boundary accessible points and the velocity increment. However, it is difficult to calculate the gravity-assist RS because the GA models are more complex comparing with impulse transfer models [5] [6]. For GA problem, the traditional methods in the field of RS are intractable to obtain the relationship discussed above.

Regression methods in supervised machine learning (SML) aim to predict unsuspected data accurately and efficiently by detecting implicit relationships in training set [7]. These methods can substitute for the traditional methods, and dig out the relationship between the orbital elements of destination orbit and the total velocity increment  $\Delta V_{opt}$  of corresponding optimal GA trajectory. Inspired by this idea, a novel method based on SML is proposed to evaluate the gravity-assist RS numerically in this study. First, the feature and target of predicting model based on SML are determined. A hybrid optimization combining Differential Evolution (DE) and SNOPT is employed to solve the trajectory-optimization problem based on GA model with deep space maneuver (DSM).



Later, the performances of three powerful SML regression methods with several common choices of kernels are assessed. Finally, millions pairs of sample points are generated by the trained predicting model thanks to the high efficiency of SML methods. The points with specified value of  $\Delta V_{opt}$  are extracted, and the envelope of these extracted sample points constitutes the gravity-assist RS. Both of Venus GA and Mars GA trajectories are considered in this paper.

## 2. Problem Formulation

The regression methods in SML field are able to construct the predicting model from the features to target based on the training set that contains numbers of pairs of feature values and corresponding target values. In this paper, we aim to evaluate the gravity-assist RS with the description of Keplerian orbital elements. Considering ephemeris-free model, the mapping from orbital elements  $[a, e, i, \Omega, \omega]$  to the  $\Delta V_{opt}$  of corresponding optimal GA trajectory is in strong demand. Therefore  $\mathbf{x} = [a, e, i, \Omega, \omega]$  is selected as the feature of predicting model, and  $y = \Delta V_{opt}$  is regard as the target. The basic idea of regression is to find the mapping  $f: \mathbf{x} \rightarrow y$  based on the training set, expressed by  $\mathcal{D} = \{\mathbf{x}_i, y_i | i = 1, 2, \dots, n\}$ . The orbital elements in training set are randomly sampled in the region  $U$ , defined as

$$U = \{(a, e, i, \Omega, \omega) | 1.7 \leq a \leq 3.5 AU, 0 \leq e \leq 0.5, 0 \leq i \leq 30 \text{ deg}, 0 \leq \Omega \leq 360 \text{ deg}, 0 \leq \omega \leq 360 \text{ deg}\} \quad (1)$$

Actually, this region covers more than 90% the current known main-belt asteroids. The calculation of gravity-assist RS within this region is significant for the target selection in practical mission.

In this paper, the single GA trajectory is considered, and the candidate GA bodies contain Venus and Mars. The gravity-assist model with DSM is introduced to design the GA trajectories. The objective function of GA trajectory with DSM is given by

$$\Delta V_{GA} = \|\Delta \mathbf{V}_D\| + \|\Delta \mathbf{V}_{DSM1}\| + \|\Delta \mathbf{V}_{DSM2}\| + \|\Delta \mathbf{V}_I\| \quad (2)$$

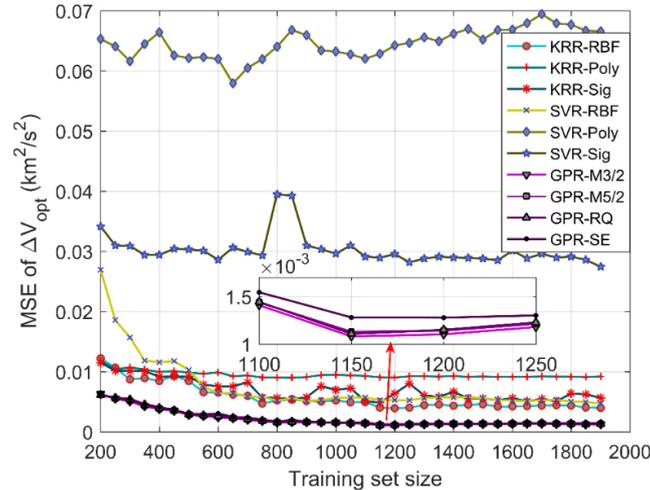
where  $\Delta \mathbf{V}_D$  and  $\Delta \mathbf{V}_I$  denote two impulses applied at departure from a circular Earth parking orbit (200km height) and the injection to the destination orbit.  $\Delta \mathbf{V}_{DSM1}$  and  $\Delta \mathbf{V}_{DSM2}$  represent two deep space maneuvers applied in the legs before and after GA flyby respectively. Hybrid of DE [8] and SNOPT is employed to solve the optimal GA trajectory with minimum  $\Delta V_{GA}$ . DE executes a global search and provides a high-quality initial guess for SNOPT. SNOPT performs precise local search based on the solution from DE. The minimum  $\Delta V_{GA}$  for different destination orbits are obtained, which are regarded as corresponding  $\Delta V_{opt}$  in subsequent sections.

## 3. Predicting Model Building and Range Set Calculating

### 3.1. Performance assessment for supervised machine learning regression methods

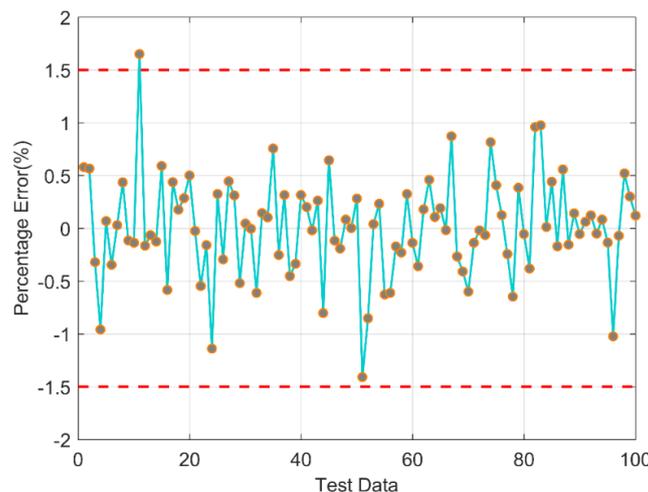
In this section, the performances of three powerful SML regression methods are assessed to determine the final predicting model, including Gaussian Processes Regression (GPR), Support Vector Regression (SVR) and Kernel Ridge Regression (KRR). Considering that these methods are kernel based, the selections of kernel functions are also considered in the performance assessment, including the Squared Exponential (SE), the Matérn class (M3/2 or M5/2), the Rational Quadratic (RQ), the Polynomial (Poly) kernel, the Radial Basis Function (RBF) and the Sigmoid (Sig). A training set  $\mathcal{D}$  with 2000 random orbital elements data in  $U$  and the corresponding  $\Delta V_{opt}$  is constructed to train the predicting models based on different SML regression methods. Define  $\mathcal{C} = \{(\mathbf{x}_i^*, y_i^*) | i = 1, 2, \dots, 100\}$  as the test set, which is generated in the same way as training set  $\mathcal{D}$ . The test set  $\mathcal{C}$  is used to measure the

accuracy of predicting models by calculating Mean Squared Error (MSE) or Percentage Error (PE) between the prediction output and the true value. Taking Mars gravity-assist case as an example, the MSE distribution is shown in figure 1.



**Figure 1.** MSE distribution of different SML regression methods with different kernels

In general, the MSEs of SML regression models decline as the training set size grows, and the MSE curves tend to converge after the training set contains about 1400 training pairs. Among all the SML regression models in figure 1, GPR models show the overwhelming superiority with faster convergence rate and lower stable value of MSE. In addition, the GPR models are relatively insensitive to the selection of kernel functions, while the performance of other models shows significant disparity with different kernels. As shown in the enlarged view of GPR models, the Matérn 3/2 kernel has a slight advantage with  $1.11 \times 10^{-3} \text{ km}^2/\text{s}^2$  minimal MSE at 1150 training set size. In conclusion, GPR model with Matérn 3/2 kernel function is selected to generate the gravity-assist RS, and the training set size is chosen as 1150 in subsequent sections of this study. The PE distribution of predicting model based GPR with Matérn 3/2 kernel is shown as figure 2. It can be seen that the maximum PE is no more than 2%, and the vast majority of PEs are less than  $\pm 1\%$ . According to the MSE and PE of predicting model, it can be concluded that the accuracy of the predicting model is satisfactory.



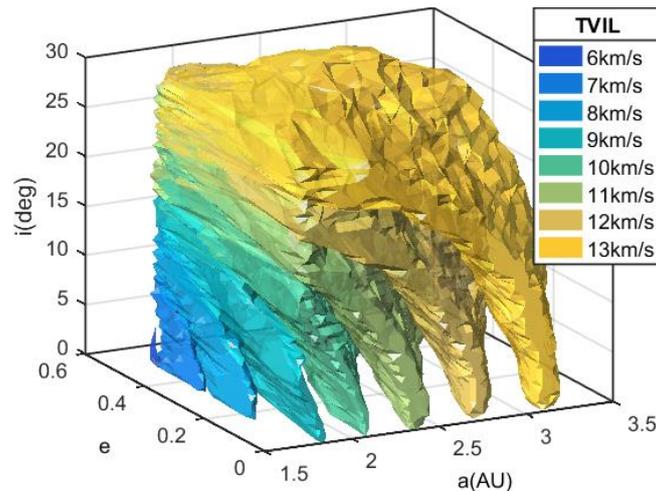
**Figure 2.** PE distribution of predicting model

### 3.2. Evaluating the gravity-assist range set

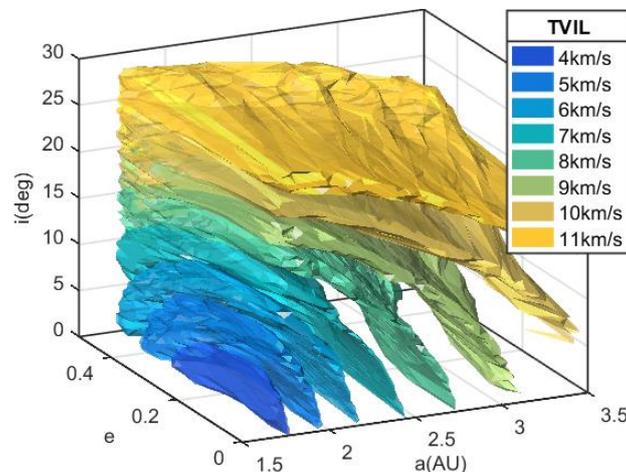
Thanks to the high-accuracy GPR predicting model constructed in this paper, the  $\Delta V_{opt}$  of the optimal

GA trajectories can be easily obtained given the orbital elements inputs. A large-scale dataset contains millions pairs of orbital elements and corresponding  $\Delta V_{opt}$  is generated using the predicting model. These orbital elements are sampled uniformly in the region  $U$  with the sampling frequency expressed by  $50 \times 50 \times 50 \times 25 \times 25$ , of which the multipliers correspond to each element of  $[a, e, i, \Omega, \omega]$ . These 78,125,000 sampling points are used to detect the gravity-assist RS by extracting the points with specified value of  $\Delta V_{opt}$ . The slack variable  $\Delta V_{slack} = \pm 0.01 \text{ km/s}$  is introduced because it is difficult to find the sample points with  $\Delta V_{opt}$  equal to a given value exactly without any fractional error. For example, it means that  $\Delta V_{opt}$  of 6.99~7.01 km/s will be regarded as 7 km/s in this study. The envelope of the extracted sampling points can be regarded as the corresponding RS via optimal gravity-assist trajectories with specified Total Velocity Increment Limit (TVIL) for missions. Finally, the RS via Venus GA trajectories and Mars GA trajectories are shown in figure 3 and 4 respectively.

As we can see in figure 3, only a small region in  $U$  is reachable when the TVIL for missions is set to 6 km/s, and it tends to be completely unreachable given lower TVIL via Venus GA trajectory. The RS extends out obviously with the increase of TVIL. The pattern for Mars GA trajectory is analogous, while the minimum TVIL that make the region  $U$  partly reachable is lower than 4 km/s (see figure 4). This reflects the advantage of Mars GA trajectory in reducing total velocity increment for missions. In general, the destination orbits with lower  $a$  and  $i$  are easier to reach via both Venus GA trajectories and Mars GA trajectories.



**Figure 3.** RS via Venus gravity-assist trajectory



**Figure 4.** RS via Mars gravity-assist trajectory

#### 4. Conclusions

In this paper, a novel method based on SML is proposed to evaluate the gravity-assist RS. The SML regression methods are introduced to dig out the implied relationship between the orbital elements  $[a, e, i, \Omega, \omega]$  and the corresponding  $\Delta V_{opt}$ . The property of several powerful SML methods with different kernel functions are assessed in this study, and the GPR model with Matérn 3/2 kernel is finally selected because it realizes minimum MSE ( $1.11 \times 10^{-3} \text{ km}^2/\text{s}^2$ ) with fewer training data (1150). The orbital elements in training set  $\mathcal{D}$  are randomly sampled in a significant region  $\mathcal{U}$ , which covers the majority of the main-belt asteroids. The corresponding  $\Delta V_{opt}$  are calculated by solving a NLP problem based on the gravity-assist model with DSM. The hybrid of DE and SNOPT is employed to solve the optimal  $\Delta V_{opt}$ . Using the trained predicting model, millions pairs of orbital elements and corresponding  $\Delta V_{opt}$  are generated. These sample points are used to detect the gravity-assist RS by extracting the points with specified value of  $\Delta V_{opt}$ . The envelope of the extracted sampling points with same  $\Delta V_{opt}$  constitutes the corresponding range set. The RSs of different TVILs via both Venus GA trajectories and Mars GA trajectories are evaluated using proposed method.

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