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A PSK Signal Symbol Rate Estimation Algorithm based on the Combination of Stochastic Resonance and Wavelet Transform

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Abstract. In non-cooperative communications, the deterioration of the channel makes the SNR of the received PSK signals at a low level, resulting in the failure to estimate the symbol rate. Stochastic resonance can use noise energy to strengthen the weak signals, and wavelet transform can effectively detect the transient phase. This paper proposes to use stochastic resonance combined with wavelet transform to estimate the symbol rate of the PSK signals. It not only overcomes the drawback that stochastic resonance disperses easily as a nonlinear system, but also reduces the influence of the optimal size of the wavelet. The simulation results show that this method can improve the output peak to a certain extent and reduce the threshold of SNR, which is of great significance for the symbol rate estimation for weak signals.

1. Introduction

In digital communication system, the symbol rate is related to a series of signal processing steps, such as demodulation, recovery of baseband sequence, estimation of time lapse, and so on [1]. In non-cooperative communications, it is often necessary to estimate the symbol rate blindly. However, if the signal-to-noise ratio (SNR) is extremely low, it is impossible to estimate the symbol rate accurately, and thus it is difficult to demodulate the signal, which may cause the loss of important information. Therefore, studying the symbol rate estimation problem under low SNR is of great value.

Stochastic resonance is a kind of nonlinear system in which periodic output is enhanced by the cooperation with periodic signals and random interference [2]. With the deepening of the research, scholars have found that stochastic resonance can also transfer the energy of noise to non-periodic signals with certain bandwidth, which enables stochastic resonance to be applied in many fields including signal processing. Meanwhile, if the stochastic resonance is only used for pre-processing the signals [3], the result is not stable due to the non-linear feature of the system, and it is usually not ideal. The wavelet transform can effectively detect and extract the signal phase when estimating the symbol rate [4], but the optimal scale is difficult to grasp, and its effect will deteriorate when the noise is strong.

To address the intermediate frequency PSK signals in low signal-to-noise ratio, a method of symbol rate estimation based on the combination of stochastic resonance and wavelet transform is proposed. First, the receiving signal is processed by the bistable stochastic resonance system, transferring some of the out-of-band noise energy to the in-band target signal. Then, the transient information can be detected by wavelet transform after energy enhancement. Finally, symbol rate is determined by calculating the spectrum of the squared envelope signal. This method can combine and strengthen the advantages of stochastic resonance and wavelet transform. After noise is greatly reduced and signal energy is enhanced, transient characteristics can be captured and extracted in time.

2. Classical PSK symbol rate estimation algorithm

2.1. Spectrum of the squared envelope signal

PSK can be written as

$$S(t)=\sum_{n}(\sqrt{S_n})g(t-nT_B-t_0)\exp[j(2\pi f_c t+\phi_n+\theta_c)]+\Gamma(t),\ \ 0\le t\le nT \tag{1}$$

where A denotes the amplitude, φ_n the phase with $\varphi_n\in\{(2m-1)\pi/M\mid m=1,2,...,M\}$, T_B the symbol duration, $R_B=1/T_B$ the symbol rate, t_0 a channel delay, θ_c the initial phase of the carrier, $g(t)$ a baseband pulse made of square-root raised cosine roll-off filter, and $\Gamma(t)$ the additive complex white Gaussian noise.

The classic method of estimating R_B depends on the spectrum of the squared envelope signal, because of the fact that $g(t)$ is band-limited, therefore discrete component can appear in the form of the single line spectrum which stands for the symbol rate after calculating the spectrum of the squared envelope signal [5]. $\hat{R}_B$ is estimated as follows:

$$\hat{R}_B=\arg\max_{f}|FFT[|s(t)|^2]| \tag{2}$$

The formula indicates that the symbol rate is the frequency point corresponding to the maximum spectral line in the field of frequency. This algorithm is not affected by the carrier and is easy to operate. Since it does not pre-process the signal, it cannot detect weak signal effectively, thus, only applicable to the signals with high SNR.

2.2. Wavelet transform

Wavelet transform is a kind of signal processing tool with great local characteristics, and it can detect and extract the phase of signals. Therefore, it can be applied to the detection of PSK symbol rate of signals. In addition, *Haar* wavelet function is widely used because of its simple form and prominent effect.

The discrete *Haar* wavelet is defined as

$$\frac{1}{\sqrt{k}}\psi(\frac{n}{k})=\begin{cases}1 & n=-\frac{k}{2},-\frac{k}{2}+1,...,-1\\ -1 & n=0,1,...,\frac{k}{2}-1\\ 0 & otherwise\end{cases} \tag{3}$$

where k is the scale.

For PSK signals, when the wavelet transform is in a symbol period, its value is fixed, and when the phase changes suddenly, the wavelet transform value will produce a corresponding sharp change [6].

In order to ensure the validity of wavelet transform, the scale k must be chosen under the condition of $k<f_s/R_B$ [7], in which f_s denotes the sampling rate. Since the sampling rate in stochastic resonance must be sufficiently large, the condition can easily be satisfied.

3. Combination method

3.1. Principle of stochastic resonance

The rectangular-shaped PSK signal can be considered as a periodic signal within one symbol interval with the effect of the carrier, and if the shaping filter is considered, it can be regarded as a similar periodic signal.

Bistable system is a typical nonlinear system, which can be represented by the Langevin equation

$$\frac{dx}{dt} = ax - bx^3 + A\cos\left(2\pi f_0 t + \varphi\right) + \Gamma\left(t\right) \tag{4}$$

where a and b are parameters of the system, $s(t) = A\cos\left(2\pi f_0 t + \varphi\right)$ the detected signal, A the amplitude, f_0 the frequency, and $\Gamma(t)$ the additive complex white Gaussian noise with mean value of 0 and the intensity of D.

The output's power spectrum of the bistable system is expressed as

$$S_t(D, f) = S_{f_0}(D, f) + S_\eta(D, f) \tag{5}$$

which consists of two parts: signal and noise, both of which are the function of the noise intensity D and frequency f. The output signal power spectrum is equation (6), and it is an impulse function for the single frequency signal. $r_k = \frac{a}{\sqrt{2\pi}} \exp(-\frac{a^2}{4bD})$, denotes the transition rate of the bistable system between potential wells. And the output noise power spectrum is equation (7), whose energy is distributed averagely in full band.

$$S_{f_0}(D, f) = \frac{2\pi A^2 a^2 r_k^{\ 2}}{D^2(4r_k^{\ 2} + 4\pi^2 f_0)} \delta(f - f_0) \tag{6}$$

$$S_\eta(D, f) = [1 - \frac{2A^2 a r_k^{\ 2}}{D^2(4r_k^{\ 2} + 4\pi^2 f_0^2)}] \frac{4ar_k}{4r_k^{\ 2} + 4\pi^2 f^2} \tag{7}$$

Considering narrow-band signal processing, the sampling rate determines the frequency band width. The input energy of the noise starts form 0 to f_s, so the signal to noise ratio of the output is

$$SNR_{out} = \frac{\frac{1}{\pi}\int_0^{+\infty} S_{f_c}(D, f)df}{\frac{1}{\pi}\int_0^{+\infty} S_\eta(D, f)df} = \frac{a^2 A^2 \pi \exp(-\frac{a^2}{4bD})}{\sqrt{2}D^2(\frac{a^2}{2\pi^2}\exp(-\frac{a^2}{2bD}) + \pi^2 f_c^2)} \bullet \frac{\arctan \frac{\pi f_s}{\frac{a}{\sqrt{2\pi}}\exp(-\frac{a^2}{4bD})}}{1 - \frac{A^2 a r_k^{\ 2}}{D^2(\frac{a^2}{\pi^2}\exp(-\frac{a^2}{2bD}) + 2\pi^2 f_c^{\ 2})}} \tag{8}$$

Stochastic resonance can converge the whole band energy and then transfer it to low frequency band [3], to enhance the specific signal energy accordingly. From the equation (8), we can conclude that the higher the sampling rate, the greater the output SNR gain, but the larger the computational complexity. Therefore, all these factors should be considered in the application.

Stochastic resonance not only has good performance which can greatly reduce the noise when extracting spectral lines, but also has some deficiencies: firstly, the target signal must be sampled at high sampling rate, which is usually greater than the bandwidth of more than 50 times; secondly, the system parameters are difficult to adjust[8][9], and when transferring the noise energy of the whole bandwidth to the low-frequency signal energy, the noise background at the bottom of the signal will be raised accordingly.

3.2. Combination with wavelet transform

Because processing the signal by stochastic resonance has certain limitations, the innovation in this paper focuses on the combination of stochastic resonance and wavelet transform, absorbing the advantages of them both and making up for their inadequacy, as long as the parameters of stochastic resonance be in a certain value range (in the same order of magnitude with carrier), and it is unnecessary to find the optimal parameters on purpose, which simplifies the steps. In the meanwhile, it also makes up the shortcoming of the wavelet transform whose optimal scale is difficult to confirm, because the ascension of the sampling rate makes it easy to meet the condition. Furthermore, the combination with the classic mode avoids moving the signal to baseband which require the precise estimation of carrier's frequency.

The system composition is given below, as shown in figure 1.

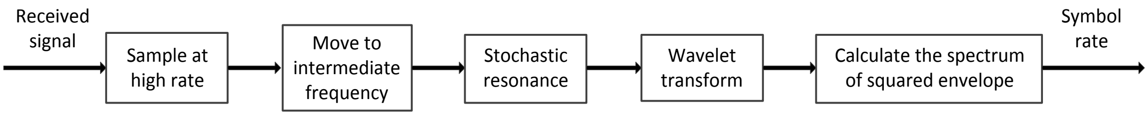

Figure 1. System block diagram of combining stochastic resonance with wavelet transform to estimate the symbol rate.

The algorithm is implemented as follows: Sample the PSK signals sent by message channel at high rate in order to satisfy the requirement of the stochastic resonance system; Move signals to the appropriate intermediate frequency to facilitate the determination of the system parameters of stochastic resonance; Pass the stochastic resonance system using iterative calculation; Use *Haar* wavelet function to transform the output of the last procedure with the minimum size of the wavelet scale as 2; Calculate the spectrum of squared envelope processed signal and observe the peak position, thus the symbol rate being obtained.

4. Simulation experiment and performance analysis

4.1. Effectiveness validation

Set the parameters of simulated signal: the symbol number N is 5000 and modulation way is QPSK, using ascending cosine shaping filter with roll-off factor of 0.6. The carrier frequency f_c is 10 Hz, the symbol rate R_B is 8 Hz, and the sampling rate f_s is 2000 Hz. Compare these three ways: the squared envelope method, the wavelet transform method, the stochastic resonance method with the new method: combining stochastic resonance with wavelet transform in the conditions which $Es/N0$ is 6dB. In addition, the scale k of the wavelet is 60, and the parameters of stochastic resonance system is $a = b = 10$. The results are shown in the figures below:

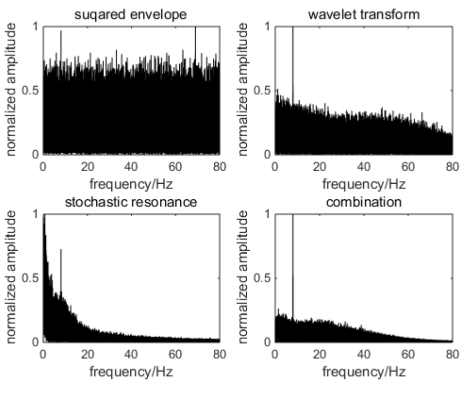

Figure 2. Output comparison diagram of the four methods.

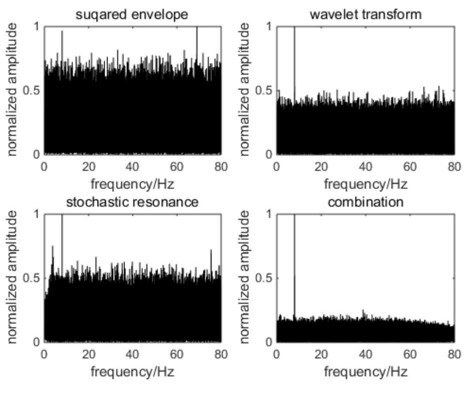

Figure 3. Output comparison diagram of the four methods after suppressing the background colour noise.

Figure 2 shows the output graphs of the four methods which have distinct differences. So as to automatically search for the spectral peak, a nonlinear filter is used to suppress the background colour noise [10], as shown in figure 3. It can be seen that, in the case of the failure of classical algorithm, the method of wavelet transform or stochastic resonance can be used to improve the performance, but at the same time, the output noise is strong either. The method of stochastic resonance combined with wavelet transform can greatly improve the spectral line and also remarkably reduce the background noise.

4.2. Analysis of the relative error

Make $Es/N0$ increase from -5 to 10, with the remaining parameters unchanged, and use the Monte Carlo simulation method in which the simulation number is 100. Because the symbol rate can be coarsely estimated according to the bandwidth, the peak value can be found within the range of $[0.5R_B, 1.5R_B]$. The relative error is defined as $\frac{|\hat{R}_B - R_B|}{M \times R_B}$ where M is the times of simulation. The graph of the relative error of the symbol rate estimation with the symbol signal-to-noise ratio is shown in figure 4.

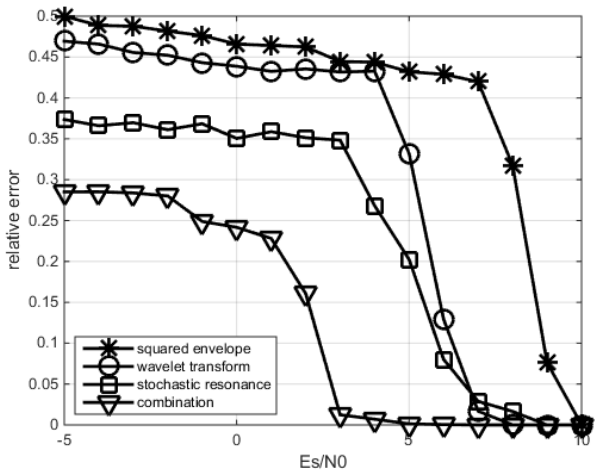

Figure 4. Relative error of QPSK signal varies with $Es/N0$.

It can be seen from the graph that with the increase of the $Es/N0$,the relative error of four methods are decreasing gradually, and there is an obvious steep descending process in each curve. The method of combining stochastic resonance with wavelet transform appears steep fall firstly, with the relative error lowering than other methods under the same noise condition.

4.3. The experimental summary

Through the simulation, compared the classic squared envelope way, the combination method, it can reduce the noise threshold at least by 6 dB for PSK (M = 2,4,8) signals. The anti-noise performance also has a higher degree of ascension than only using wavelet transform and stochastic resonance method. Table 1 shows the noise threshold of the four methods (the relative error is less than 0.05).

Table 1. Noise threshold of the four methods ($Es/N0$)

	BPSK	QPSK	8PSK
squared envelope	9	10	10
wavelet transform	6	7	7
stochastic resonance	6	7	7
combination	3	3	4

It is necessary to note that, from equation (9)(l is the oversampling rate), when $Es/N0$ is 5, SNR is about -7 dB under this experimental condition, which is due to the high sampling rate in the simulation, so our main concern is the degree of reduction of threshold under the same noise condition.

$$E_s / N_0(dB) = 10\log_{10}(l) + SNR(dB) \tag{9}$$

In addition, during the experiment, when signal-to-noise ratio is slightly lower, the stochastic resonance exists some failure situation, that is to say, when the energy is conversed from the high frequency to the low frequency, most of it moves to nearly zero frequency, which is caused by the nonlinearity of the system. The above problem is inevitable during the promotion and application of the stochastic resonance, and if only use the stochastic resonance method, it is unable to estimate the symbol rate (these singular values have been removed in the statistical noise threshold). But after the wavelet transform extracting the transient information, we can estimate the symbol rate accurately in the same resonance conditions, which shows that wavelet transform can excavate the transient information from the unideal results of the stochastic resonance.

5. Conclusion

In this paper, considering the characteristics of wavelet transform and stochastic resonance, a new method which combines them both to estimate the symbol rate is proposed and discussed, reducing the harsh conditions of using stochastic resonance method with the simple *Haar* wavelet. After the experimental verification, the combination method is still valid in a certain range of signal-to-noise ratio even though those two methods are invalid separately. Wavelet transform can improve the unideal results of the stochastic resonance, while stochastic resonance can make effective enhance pre-treatment for wavelet transform. The requirement of high sampling rate of stochastic resonance can also make the wavelet scale easy to meet the condition. In a word, this method is a new attempt to the symbol rate estimation, and it can reduce the nose threshold by about 6 dB compared with the squared envelope, wavelet transform and stochastic resonance those methods. The combination method is suitable for low SNR signals and can expand the train of thought for weak signal analysis, processing and parameter estimation.

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