Efficient Evaluation of Mars Entry Terminal State Based on Gaussian Process Regression

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Abstract. Trajectory optimization technology used for Mars entry is one of the key technologies for planetary exploration. Evaluation of the performance of the entry trajectory under conditions of complex atmospheric dynamics, various vehicular design parameters, and multiple constraints in the process of entry, are important issues pertaining to the design of trajectories. In this study, an efficient evaluation approach of the terminal state for Mars entry is proposed based on Gaussian process regression to evaluate the maximum terminal altitude for different entry velocities, terminal Mach numbers, and vehicular parameters. Additionally, the influences of entry flight-path angle, lift-drag ratio, and ballistic coefficient, on the maximum terminal altitude are analyzed. A genetic algorithm is used in the optimization solver to avoid local minima and to guarantee the data quality of the training samples used for Gaussian process regression. The mean function, kernel function, and hyperparameters are selected as the optimization parameters for Gaussian process regression to describe the correlation between samples, and the maximum terminal altitude prediction model is then established. Numerical simulations demonstrate that the proposed method can achieve the evaluation of more than 3000 group scenarios within tens of seconds with a mean relative error that is less than 4%.

1. Introduction
Planetary exploration is one of the main fields of deep space exploration. Mars, the closest earth-like planet in the solar system to earth is usually the preferred target for human planetary exploration. However, unlike the earth's environment, Mars entry encounters many challenges given the planet's physical environment, with its thin atmosphere and increased gravity, thereby making it difficult to decelerate the vehicle [1,2]. When the vehicle is decelerated to meet the deployment constraints imposed for parachutes or other deceleration devices, the terminal altitude can affect the maneuver time of the landing process and the range of the target landing site that directly affects the success of the entire mission. Therefore, the approach used to evaluate the terminal altitude is an important issue in the design of Mars exploration missions.

However, when designing the entry trajectory the process involves large-scale work of trajectory design because different entry modes and control mechanisms lead to a variety of flight plans. Considerable amount of research work has been conducted on the entry trajectory design. Carman, Ives, and Geller et al. introduced the guidance algorithm of constant-bank angle profile that was adopted by the Apollo mission for the study of Mars entry vehicles [3,4]. Mendeck and Carman et al. emphasized the importance of the variable-bank profile for the optimization of Mars-entry trajectories [5]. Grant and Shuang et al. considered the terminal altitude as the performance index for the assessment of the maximum terminal altitude that could be reached by the variable-bank profile design in specific application scenarios [6,7]. Lafleur and Jacob et al. comprehensively studied the maximum terminal altitude that a Mars vehicle could reach with combinations of different vehicular parameters and entry process constraints [8,9]. However, all the existing research studies have focused on the complex modeling of the entry process with different vehicular parameters, depending on a large number of numerical calculations, or comprehensively studied the possible entry scenarios of different vehicular types, entry velocities, and deceleration devices that required considerable computation time in an inefficient manner, or simplified the constructed model to improve the solution's efficiency. It is difficult to design an approach that is comprehensive, optimal, and efficient.

Gaussian process regression can use part of the observed values to establish the mapping relationship of the input-output model, so that when the new input values are given, the corresponding output can be estimated. Since the prediction method based on GPR does not need a large number of complex modeling functions, its computational cost is considerably reduced. The idea of GPR was first proposed by Krige for the estimation of the distribution of gold in mines [10]. Thus far, the GPR method has been applied to the field of aerospace research, and includes prior work on the aircraft's airfoil design [11], aircraft's aerodynamic coefficient assessment [12], and for the assessment of the main belt that asteroids can reach [13]. Inspired by this thought, this study presents an efficient evaluation method based on Gaussian process regression to evaluate the terminal state of Mars entry.

This study is organized as follows. Section 2 introduces the Mars trajectory optimization algorithm proposed herein, uses it to generate a large number of simulation results, and provides sample data for the prediction model based on Gaussian process regression. In Section 3, a maximum terminal altitude prediction model of Mars entry is established based on Gaussian process regression, using the existing sample data. Section 4 discusses the effectiveness of the maximum terminal altitude prediction model of Mars entry, and uses the prediction model to analyze the characteristics of the trajectory, and provide the design references under a given mission scenarios.

2. The optimal trajectory for Mars entry
In order to provide sample data for the GPR-based prediction model, and to ensure that the sample data has sufficient empirical information, this section uses the terminal altitude as the performance index, and explores the maximum terminal altitude that the optimal trajectory of the Mars entry can reach under different combinations of entry velocities, terminal Mach numbers, lift-drag ratios, ballistic coefficients, and process constraints, in a comprehensive manner.

2.1. Dynamics model for Mars entry
Considering the rotation of Mars, the following dynamics model is applied.

$$\frac{dr}{dt} = V \sin \gamma$$
(1)

$$\frac{d\theta}{dt} = \frac{V \cos \gamma \sin \psi}{r \cos \varphi}$$
(2)

$$\frac{d\psi}{dt} = \frac{1}{V} \left[\begin{array}{l} \dfrac{L \sin \sigma}{\cos \gamma} + \dfrac{V^2}{r} \cos \gamma \sin \psi \tan \varphi + \dfrac{\Omega^2 r}{\cos \gamma} \sin \varphi \cos \varphi \sin \psi \\ - 2\Omega V (\tan \gamma \cos \psi \cos \varphi - \sin \varphi) \end{array} \right]$$
(3)

$$\frac{d\gamma}{dt} = \frac{1}{V}\left[\begin{array}{l} L\cos\sigma + \cos\gamma(\frac{V^2}{r} - g) + 2\Omega V\cos\varphi\sin\psi \\ + \Omega^2 r\cos\varphi(\cos\gamma\cos\varphi + \sin\gamma\cos\psi\sin\varphi) \end{array} \right] \tag{4}$$

$$\frac{d\varphi}{dt} = \frac{V\cos\gamma\cos\psi}{r} \tag{5}$$

$$\frac{dV}{dt} = -D - g\sin\gamma + \Omega^2 r\cos\varphi(\sin\gamma\cos\varphi - \cos\gamma\sin\varphi\cos\psi) \tag{6}$$

Herein, r is the distance between the vehicle and the center of Mars, and Ω is the rotation speed of Mars, $g = mu/r^2$ is the local gravitational acceleration, mu is Mars gravitational constant. Equivalently, $\theta, \varphi, \gamma, \psi, \sigma$ are longitude, latitude, flight-path angles, heading angles, and bank angles [14]. The variables L, D are the lift acceleration and drag acceleration. In accordance with the method in [8], 10 bank angles were provided in the velocity domain, and the bank angle profiles are then obtained by interpolation. Assume that the aerodynamic parameters can be fully expressed by the ballistic coefficient and lift-drag ratio, as stated in equations (7) and (8).

$$\beta = \frac{m}{C_D S} \tag{7}$$

$$L = kD \tag{8}$$

Among them, β is the ballistic coefficient, k is the lift-drag ratio, C_D is vehicle drag coefficient, and S is the reference area, and $D = \frac{1}{2\beta}\rho V^2$, ρ is the Mars atmospheric density. Based on the observation data of the Viking, a simplified model of the atmospheric index is adopted herein equation(9), whereby $\rho_0 = 1.474 \times 10^{-2}\,\text{kg/m}^3$, $hs = 8.8057 \times 10^3\,\text{m}$, and h is the vehicular altitude from the Mars surface [15].

$$\rho = \rho_0 e^{(-h/hs)} \tag{9}$$

2.2. Vehicular parameters and Mars entry scenarios
Entry vehicles are classified into manned and unmanned, and they have three different shapes, which respectively correspond to the ballistic, ballistic lift, and lift models.

At present, most of the vehicular lift-drag ratio values are less than unity, except for lift-body vehicles, such as the space shuttles. All known Mars missions have used unmanned vehicles with ballistic coefficients of approximately $100\,\text{kg/m}^2$. Considering the need of manned vehicle landing on Mars, and the increase of the required load, future vehicular ballistic coefficients will be correspondingly larger. Based on the use of manned vehicles in other missions, the ballistic coefficient mainly ranges between 200 and $1000\,\text{kg/m}^2$. At present, there are mainly three types of vehicular entry velocities: a) $3.3\,\text{km/s}$ for entries from a $500\,\text{km}$ circular orbit, b) $4.7\,\text{km/s}$ for entries from a 1-sol elliptical orbit, and c) direct entry velocities, which usually vary between 3-7 km/s [1,8].

Therefore, referring to the typical distribution of characteristic vehicular parameters, the ballistic coefficient used in this study was selected to range from $100\,\text{kg/m}^2$ to $1000\,\text{kg/m}^2$ in increments of 100 kg/m^2. The values of the lift-drag ratio were selected to be in the range of 0.1 to 0.9, at 0.1 increments. Moreover, the terminal Mach numbers were equal to 2.0, 3.5, and 5.0, corresponding to the Mach number of deployed parachutes, inflatable decelerators, and propulsive decelerators. The entry velocities were equated to $3.3\,\text{km/s}$, $4.7\,\text{km/s}$, and $5.5\,\text{km/s}$. The maximum acceleration limit of manned vehicles was $4.5\,g_0$ and the maximum acceleration limit of unmanned vehicles was $30\,g_0$, where $g_0 = 9.8066\,\text{m/s}^2$ and represents the Earth surface gravity acceleration. A total of 1620 groups were evaluated with different combinations of vehicular parameters and Mars entry scenarios. The

convective heat rate was constrained to 1000 W/cm^2 [8].

2.3. Optimization model

In this study, the entry flight-path angle and the bank angle are considered as the optimization variables. The entry flight-path angle and 10 bank angles evenly spaced within the velocity range are then optimized [9]. The optimal bank angle profile is obtained through interpolation, and the performance index is then calculated. Using a genetic algorithm-based optimization solver, the local minimum is avoided by virtue of its global goodness, so as to ensure the quality of the optimization results.

2.3.1. The objective function. Mars entries are expected to have a higher terminal altitude h_{term}. Correspondingly, this study selected the terminal altitude as the optimization goal and performance index to maximize terminal altitude. As shown in equation (10), r_{term} is the distance from the planet's center to vehicle at the instant when the deceleration device is deployed, and R_m is the radius of Mars.

$$h_{term} = r_{term} - R_m \tag{10}$$

2.3.2. Process constraints. The maximum acceleration limit reflects the deceleration acceleration F that the vehicle or astronaut can withstand. The convective heat rate constraint reflects the heat rate Q that the vehicle can withstand.

$$F = \sqrt{L^2 + D^2} \leq F_{\max} \tag{11}$$

$$Q = Q_z \left(\frac{V}{V_e}\right)^{3.08} \sqrt{\rho / \rho_0} \leq Q_{\max} \tag{12}$$

The minimum altitude during the process of entering can be classified in two situations. When restrictions are imposed, $h_{\min} \geq 10\text{km}$, and when no restrictions are imposed, $h_{\min} \geq -5\text{km}$ is the simulation condition.

2.3.3. Terminal constraints. Terminal Mach number indicates the velocity at which the deceleration devices can be deployed, and the minimum terminal altitude limits the altitude of the vehicle when the deceleration devices is deployed.

$$M_{term} = M_f \tag{13}$$

$$h_{term} \geq 0\text{km} \tag{14}$$

Therefore, 3240 groups of simulations were conducted with combinations of different Mars entry scenarios, vehicular parameters, and process constraints. A total of more than 1400 groups of optimal solutions were obtained.

3. Design of terminal altitude evaluation method based on Gaussian process regression

In this study, Gaussian process regression is used to design the prediction model of the optimal terminal altitude for Mars entry. Firstly, use the combination of entry velocity, terminal Mach number, ballistic coefficient, lift-drag ratio, acceleration limit, and the minimum altitude as the input x of the prediction model. Use the maximum terminal altitude as the output y of the prediction model.

$$x = \begin{bmatrix} V_0 & M_{term} & \beta & \dfrac{L}{D} & F_{\max} & h_{\min} \end{bmatrix}^T \tag{15}$$

$$y = h_{term} \tag{16}$$

Secondly, select the appropriate mean function and kernel function to design the framework of the prediction model. In this study, the zero mean function and the Gauss kernel function are selected, the corresponding expressions are in equations (17) and (18). Thus, the prediction data and the training

data are subject to a joint normal distribution distribution as in equation (19). Therefore, the mean and variance of the predicted data can be calculated by equations (20) and (21).

$$m(x) = 0 \tag{17}$$

$$K(x, x') = \sigma_f^2 \exp\left(-\frac{1}{2l^2}(x_p - x_q)^2\right) \tag{18}$$

$$\begin{bmatrix} y \\ y_* \end{bmatrix} \sim N(0, \begin{bmatrix} K(x,x) & K(x,x_*) \\ K(x_*,x) & K(x_*,x_*) \end{bmatrix}) \tag{19}$$

$$\overline{y_*} = K(x_*,x)[K(x,x) + \sigma_n^2 I]^{-1} y \tag{20}$$

$$\mathrm{cov}(y_*) = K(x_*,x_*) - K(x_*,x)[K(x,x) + \sigma_n^2 I]^{-1} K(x,x_*) \tag{21}$$

Subsequently, optimize the model hyperparameter ω using the training data. Suppose that the distribution of ω is Gaussian distribution. The Bayesian formula can estimate the likelihood function of the hyperparameter $p(\omega \mid y, x)$, as in equation (23). The joint probability distribution in accordance to the dataset is shown in equation (24) and the marginal probability is calculated in equation (25).

$$\omega = \begin{bmatrix} l & \sigma_f & \sigma_n \end{bmatrix} \tag{22}$$

$$p(\omega \mid y, x) = \frac{p(y \mid x, \omega) p(\omega)}{p(y \mid x)} \tag{23}$$

$$p(y \mid x, \omega) = \prod_{i=1}^{n} p(y_i \mid x_i, \omega) = \prod_{i=1}^{n} \frac{1}{(2\pi\sigma_n^2)^{\frac{n}{2}}} \exp(-\frac{(y_i - y_*)^2}{2\sigma_n^2}) \tag{24}$$

$$p(y \mid x) = \int p(y \mid x, \omega) p(\omega) d\omega \tag{25}$$

The optimal hyperparameters can be obtained by maximizing the logarithm of the edge likelihood of the training data, as shown in equation (26). By optimizing the hyperparameters, the probability of the model training output is maximized, and the prediction error is controlled within a reasonable range.

$$\log p(y \mid x, \omega) = -\frac{1}{2} y^T (K + \sigma_n^2 I)^{-1} - \frac{1}{2} \log \left| K + \sigma_n^2 I \right| - \frac{n}{2} \log 2\pi \tag{26}$$

Finally, using the hyperparameters solved by optimization, the prediction model of the maximum terminal altitude is determined. Thus, the use of specific parameters of the exploration mission allows the prediction model to be used to quickly and accurately evaluate the maximum terminal altitude that the vehicle can reach.

4. Numerical results and discussion
The simulations were conducted in MATLAB 2015b using an I5-2520 processor at 2.50 GHz, and a memory of 8 GB.

Based on the Gaussian process regression, the prediction model established in Section 3 was used to predict the maximum terminal altitude of Mars entry. Part of the sample data was selected as training data and the remaining data were used as test data to evaluate the maximum terminal altitude and validate the prediction model. In order to ensure that proper and adequate information can be extracted under different entry velocities, terminal Mach numbers, vehicle ballistic coefficients, and lift-drag ratios and process constraints, the training samples need to have data under different input parameters, and be distributed as evenly as possible. Therefore, in this study, the training samples were randomly and uniformly extracted under different input parameters to ensure that each acquired information type can be learned.

4.1. Performance verification of prediction model for Mars maximum terminal altitude
To validate the prediction model proposed herein, it was trained to evaluate the maximum terminal altitude and analyze the prediction errors of the test data. A total of 138 sets of data were extracted as

test data, and 213, 276, 345, 460, and 690 sets of data were evenly extracted from the rest of the data as training data to predict the terminal altitude and analyze the prediction errors. The error performance of the prediction results is shown in Table 1, and the relationship between the prediction error and the size of the training data is shown in Figure 1.

Figure 2 and Table 1 indicate that the use of the GPR-based prediction model to evaluate the terminal altitude yields a mean relative error of 3.7% for the predicted results, a relative error within 7% for nearly 95% of the test data, and a maximum relative error that does not exceed 10%.

Table 1. Performance evaluation of prediction model

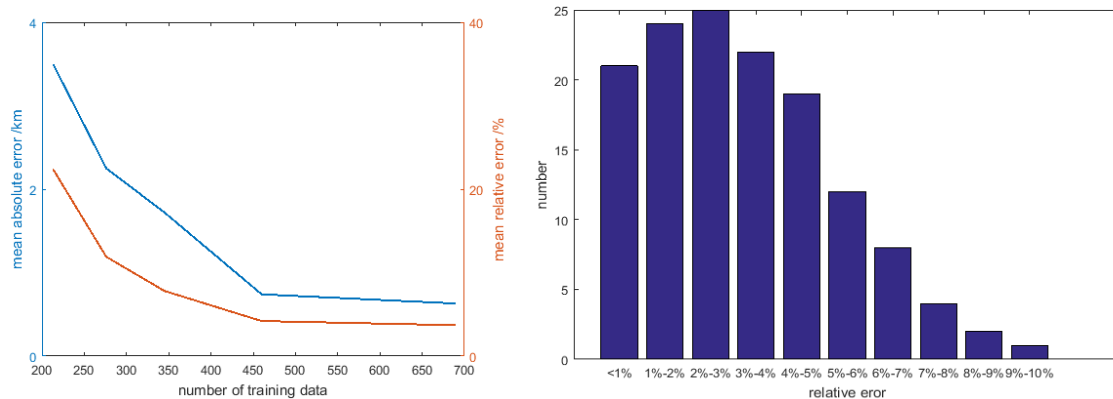| number of test data	number of training data	Mean absolute error/km	Mean relative error	Maximum relative error	Training,test time-consuming/s
138	213	3.50	22.4%	36.8%	0.81
138	276	2.25	11.9%	28.4%	1.50
138	345	1.72	7.8%	19.6%	3.79
138	460	0.74	4.2%	13.5%	18.4
138	690	0.63	3.7%	9.4%	25.7

Figure 1. Prediction error and the training data size.   **Figure 2.** Relative error distribution of test data.

As shown in Figure 1, when the training data becomes more than 500 groups, the performance of the prediction model exhibits a minor increase, and the mean relative error of the prediction results stabilizes near 4%. Figure 2 shows the relative error distribution with 138 groups test data and 690 groups training data.

4.2. Application to an elliptical orbit entry with a parachute decelerator
The prediction model based on the Gaussian process regression can evaluate the terminal altitude with different Mars entry scenarios, and analyze the characteristics of the entry trajectory with the entry flight-path angle obtained by the optimization algorithm. Due to the length of the space, this section takes the elliptical orbit entry with parachute decelerator as an example to show the numerical results.

Manned Mars exploration is the focus of future Mars explorations. Compared with unmanned vehicles, the atmospheric entry process of manned vehicles imposes strict constraints for the maximum acceleration limit, thereby affecting the maximum terminal altitude. Figure 3 show the maximum terminal altitude and optimal entry flight-path angle of unmanned vehicles. Figure 4 show the maximum terminal altitude and optimal entry flight-path angle of manned vehicles.
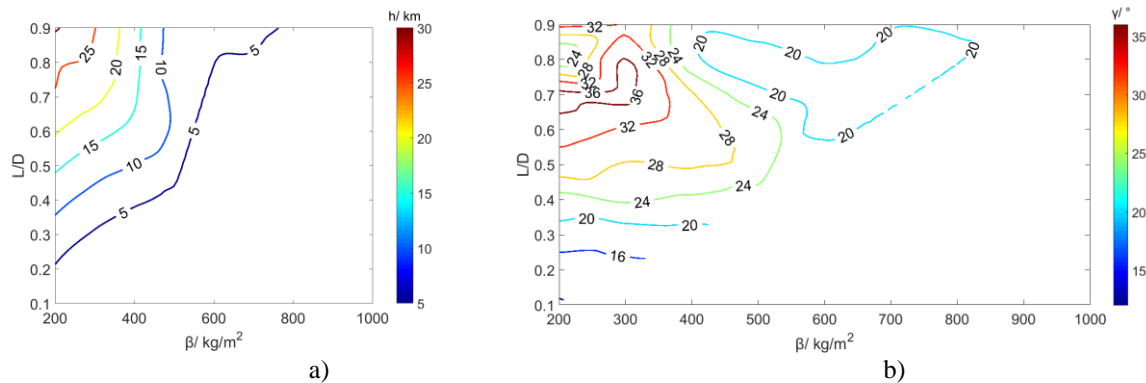
a)   b)

Figure 3. Maximum terminal altitude and optimal entry flight-path angle of unmanned vehicle.

a)   b)
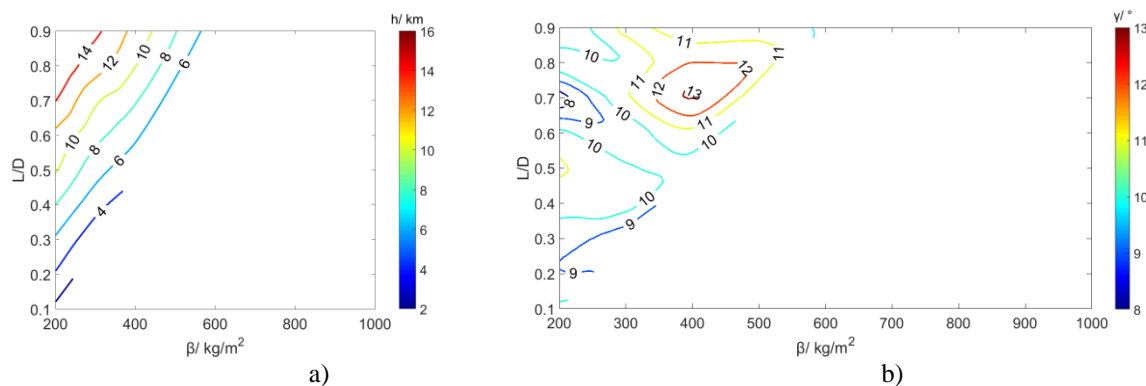
Figure 4. Maximum terminal altitude and optimal entry flight-path angle of manned vehicle.

As it can be observed from Figure 3 a) and Figure 4 a), in order to reach a larger terminal altitude, the vehicle is expected to have a higher lift-drag ratio and a lower ballistic coefficient, and the unmanned vehicle can reach a much higher terminal altitude than the manned vehicle. Moreover, the manned vehicle can hardly meet the deployment conditions of the parachute when the ballistic coefficient is more than $600 \, \text{kg/m}^2$. Figure 3 b) and Figure 4 b) show that for a manned vehicle with an elliptical orbit entry and uses the parachute as the decelerator, the optimal entry flight-path angle is approximately $10°$. Compared to the unmanned vehicle, the optimal entry flight-path angle is much smaller. At the same time, when the vehicle has a lower lift-drag ratio, a relatively small entry flight-path angle should be used to enter the Mars atmosphere. As the lift-drag ratio increases, the optimal entry flight-path angle become larger. However, when the vehicle has a higher lift-drag ratio, the optimal entry flight-path angle fluctuates as the ballistic coefficient and lift-drag ratio change. The reason for these outcomes may be that a larger entry flight-path angle provides the vehicle with more time to decelerate in the dense atmosphere, and the increase of lift-drag ratio can ensure that the vehicle decelerate without impacting the Mars surface. However, as the lift-drag ratio increases, a larger entry flight-path angle will cause an acceleration overload that the vehicle cannot withstand, so when the maximum constraints are reached, the optimal entry flight-path angle decrease instead.

5. Conclusions

In this study, an efficient evaluation approach of the Mars entry terminal state was proposed, based on Gaussian process regression, to evaluate the maximum terminal altitude for unmanned and manned vehicles of different entry scenarios, vehicular parameters, and process constraints. The maximum terminal altitude that the vehicle can reach under different combinations of parameters was optimized using a Mars entry trajectory optimization algorithm that provided the sample data for the GPR-based prediction model. The proposed method was used to evaluate the maximum terminal altitude data from more than 3000 groups. The evaluation process took less than 25.7 s. In contrast to traditional

trajectory optimization algorithms, which solve one group of data within tens of minutes to hours, the assessment efficiency was greatly improved. Compared with the optimal solution obtained using the optimization algorithm, the mean relative error of the evaluated results was 3.7%, and 7% for almost 95% of the predicted results, based on Gaussian process regression. Using the GPR-based method proposed herein, the Mars maximum terminal altitude prediction model can provide valuable insights to the design of the Mars exploration mission. Because of its high-efficiency calculation attributes, it is expected to provide support as an auxiliary means for online optimization of the Mars entry trajectories design.

References
[1]   Braun, R. D., and Manning, R. M., "Mars Exploration Entry, Descent and Landing Challenges," 2006 IEEE Aerospace Conference. Big Sky, MT, USA, Mar. 2006.
[2]   Wells, G., Lafleur, J., Verges, A., Manyapu, K., Christian, J., Lewis, C., and Braun, R., "Entry, Descent, and Landing Challenges of Human Mars Exploration," AAS Guidance and Control Conference, American Astronautical Society Paper 06-072, Feb. 2006.
[3]   Moseley, P. E., "The Apollo Entry Guidance: A review of the mathematical development and its operational characteristics," TRW Note No. 69-FMT-791, Dec. 1969.
[4]   Carman, G., Ives, D., and Geller, D., "Apollo-Derived Mars Precision Lander Guidance," AIAA 1998-4570.
[5]   Mendeck, G. F., and Carman, G. L., "Guidance Design for Mars Smart Landers Using the Entry Terminal Point Controller," AIAA Atmospheric Flight Mechanics Conference and Exhibit, AIAA Paper 2002-4502, Aug. 2002.
[6]   Grant, M. J., and Mendeck, G. F., "Mars Science Laboratory Entry Optimization Using Particle Swarm Methodology," AIAA Atmospheric Flight Mechanics Conference and Exhibit, AIAA Paper 2007-6393, Aug. 2007.
[7]   Shuang, L., and Yuming, P., "Mars Entry Trajectory Optimization Using DOC and DCNLP," Advances in Space Research, Vol. 47, No. 3, 2011.
[8]   Jarret, M. L. and Christopher, J. C., "Mars Entry Bank Profile Design for Terminal State Optimization," Journal of Spacecraft and Rockets, November. 2011.
[9]   Geethu, L. J., Geethu, N., and Ramanan, R. V., "Mars Entry Mission Bank Profile Optimization," Journal of Guidance, Control, and Dynamics, Vol. 37, No. 4, Jul-Aug. 2014.
[10]   Krige, D. G., "A Statistical Approach to Some Basic Mine Valuation Problems on the Witwatersrand," Journal of the Chemical, Metallurgical and Mining Society of South Africa, Vol. 52, No. 6, 1951, pp. 119-139.
[11]   Liu, X., Zhu, Q., and Lu, H., "Modeling Multiresponse Surfaces for Airfoil Design with Multiple-Output-Gaussian-Process Regression," Journal of Aircraft, Vol. 51, No. 3, 2014, pp. 740-747.
[12]   Robin, D., Muelenaere, J., and Elham, A., "Trajectory Driven Multidisciplinary Design Optimization of a Sub-Orbital Spaceplane Using Non-Stationary Gaussian Process," Structural and Multidisciplinary Optimization, Vol. 652, No. 4, 2015, pp. 755-771.
[13]   Haibin, S., Yuxin, L., "Assessing Accessibility of Main-Belt Asteroids Based on Gaussian Process Regression," Journal of Guidance Control and Dynamics, Vol.40, No.5, May 2017.
[14]   Pillai, J. R., John, A., and Ramanan, R. V., "Design and Analysis Tool for Mars Atmospheric Entry Missions," Journal of Aerospace Innovations, Vol. 5, No. 1, Mar. 2013, pp. 29-40.
[15]   Benito, J., Mease, K., "Characterizing the controllable entry states and reachable sites for planetary landing," International Planetary Probe Workshop, 2008, Atlanta, Georgia.