

A Rapid Estimation for Interplanetary Low-Thrust Trajectories Using Support Vector Regression

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Abstract. During the preliminary phase of space mission planning and design, a large quantity of trajectory optimization problems have to be solved. Obtaining the optimal solutions of low-thrust trajectory is computationally challenging since the optimization problems usually involve an iterative numerical algorithm and the complicated numerical integration of the equations of motion. It is necessary to develop the rapid trajectory estimation methods for low-thrust transfer. In this paper, a new method based on machine learning has been proposed to estimate the optimal interplanetary low-thrust trajectory. The minimum-propellant low-thrust trajectory is optimized by using the hybrid optimization algorithm, which would provide the high-quality training samples for machine learning. Support vector regression is adopted to construct and train the estimation model. Numerical simulations demonstrate that the proposed estimation method and the percentage errors of random test samples are all lower than 5%. This application of machine learning method can accomplish very efficient low-thrust interplanetary trajectory evaluation and it is therefore suitable to extend the design flexibility in the practical exploration mission.

1. Introduction

Application of low-thrust propulsion system is effective and prospective for interplanetary exploration. Benefiting from its high specific impulse, low-thrust propulsion can decrease the fuel consumption significantly [1]. Some accomplished missions have profited from that, such as Hayabusa, Dawn, etc. The optimization of low-thrust trajectory has always been an intensive research area. Various techniques have been developed which are mainly categorized as indirect and direct methods [2]. Both methods have their strengths and shortcomings but they all require the time-consuming iteration. However, in the preliminary mission design, there are usually a lot of low-thrust trajectories need to be evaluated, which is important for the appropriate mission planning. For instance, in the selection of target asteroids, the fuel consumption is a key factor to determine the feasibility of missions. Therefore, the new methods for evaluating optimal low-thrust trajectory effectively are very desirable.

In this paper, a novel estimation method for the optimal interplanetary low-thrust trajectory is developed based on machine learning. Considering the minimum-propellant transfer, support vector regression (SVR) is chosen to train the prediction model of velocity increment [3]. Through the rational data processing, the SVR-based model can predict the continuous output values efficiently for the given inputs.

This paper is organized as follows. Section 2 describes the evaluation problem of optimal low-thrust trajectory. In Sec. 3, low-thrust optimization problem is solved firstly by using numerical algorithms, which provides the reliable database. Then the prediction model of trajectory parameters is



constructed and trained based on orbital dynamics and SVR. The numerical results are shown in Sec. 4 to verify and analyse the accuracy of proposed method.

2. Problem statement

Efficient estimation for optimal trajectories is of good importance in the preliminary mission design and tend to be solved by the trajectory optimization methods. Some of the well-known software, such as MALTO and VARITOP, are all medium- or high-fidelity [4]. These optimization methods solve low-thrust optimization problem by the numerical iteration, leading to a complicated and time-consuming computation.

The low-fidelity optimization tools to quickly obtain optimal trajectory are lacking. Only a kind of new methods based on curve-fitting is developed for interplanetary circular-coplanar low-thrust transfers. Kluever given the analytic functions for computing mass-optimized trajectory using solar electric propulsion [5]. This method is derived by fitting the database derived from the low-thrust coplanar and circular transfers between Earth orbit and a target orbit. It is assumed that the transfer starts with a powered arc, followed by a coast arc, and finally ends with another powered arc. Then, the analytic function of velocity increment for optimal low-thrust transfers is fitting as

$$\Delta V = A \bar{a}_t^B \text{ km/s} \quad (1)$$

where A and B are the fit coefficients. \bar{a}_t is the average thrust acceleration.

The numerical results proved that the analytic function of Eq. (1) is extremely efficient to predict velocity increment by comparing with the solutions from optimization algorithms. However, this method is only available for low-thrust circular-coplanar transfers. In terms of the general low-thrust transfer, the new technique must be developed to evaluate optimal trajectory.

3. Optimal trajectory evaluation based on support vector regression

In this section, an evaluation method for propellant-optimal interplanetary low-thrust trajectory is developed to compute trajectory parameters. The general low-thrust transfer from Earth is considered to generate the database and support vector regression is adopted to train the evaluation model.

3.1. Generation of optimal low-thrust trajectory database

Considering the minimum-propellant low-thrust transfer from Earth, the trajectory optimization problem is a nonlinear optimal control problem. In this paper, the Sims-Flanagan transcription is adopted to solve this problem [6]. This method discretizes the whole trajectory into N equal-size time steps. The continuous thrust of each step can be approximated by applying a bounded velocity impulse at the center of each time step. It assumes that the spacecraft moves in an unperturbed Keplerian orbit between applied thrust impulses. Figure 1 shows the diagram of a low-thrust trajectory using the Sims-Flanagan transcription. The maximal impulse ΔV_{\max} that the spacecraft can produce is determined as:

$$\Delta V_{\max} = \frac{DT_{\max} \Delta t}{m} \quad (2)$$

where T_{\max} is the maximal thrust magnitude that spacecraft can produce, D is the thruster duty cycle, Δt is the flight time of each time steps.

Taking the ephemeris-free low-thrust transfer into consideration, the decision variables of trajectory optimization problem are

$$\mathbf{X} = [E_0 \ E_f \ t_f \ m_f \ \Delta v_{ix} \ \Delta v_{iy} \ \Delta v_{iz}]^T \quad (1 \leq i \leq N) \quad (3)$$

where E_0 and E_f are the eccentric anomaly of the initial and final orbit, respectively. t_f is the transfer time. m_f is the final mass of the spacecraft. And $\Delta v_{ix}, \Delta v_{iy}, \Delta v_{iz}$ are the three components to express the i th discontinuous impulse at center of time step.

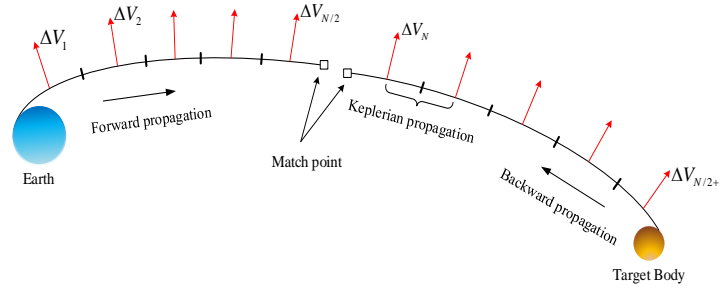


Figure 1. Schematic of low-thrust trajectory using the Sims-Flanagan transcription.

In order to reduce the sensitivity of the constraints to the decision variables, the whole trajectory is divided into two equal-time halves. The first half begins at the launch body and is propagated forward in time. So does the second half. Therefore, the state constraints of match point are:

$$\Psi(X) = z_F(X) - z_B(X) = [\Delta x \ \Delta y \ \Delta z \ \Delta v_x \ \Delta v_y \ \Delta v_z \ \Delta m] = 0 \quad (4)$$

where $\Delta x, \Delta y, \Delta z$ and $\Delta v_x, \Delta v_y, \Delta v_z$ are the three components of position and velocity vector difference in match points, Δm is the difference of spacecraft mass in two match points.

The hybrid optimization algorithm combining DE and SNOPT is adopted to solve optimization model. In detail, DE is used as a global optimizer to produce good potential solutions and then SQP as a local optimizer to fine tune the solution starting with the initial guess from DE. Particularly, the penalty function is used to handle the equality constraints.

3.2. Construction of evaluation model

In this section, the evaluation model to predict velocity increment for optimal low-thrust transfer is built and a popular data-driven machine learning method is adopted to train the prediction model.

To evaluate the minimum-propellant trajectory, the velocity increment ΔV for low-thrust transfer is always chosen as objective to be predicted. ΔV is determined theoretically and empirically by the propulsive parameters and the state vector of target body. Considering the ephemeris-free transfer, the target orbit can be described mainly by the orbital elements including semi-major axis a , eccentricity e and inclination i . The transfer time is also chosen as inputs, in other words, the optimal low-thrust transfer with the fixed time is evaluated. Then the prediction model is expressed as

$$f: R^7 \rightarrow R \Rightarrow [a, e, i, T_{\max}, I_{sp}, m_0, t_f]^T \mapsto \Delta V \quad (5)$$

Support Vector Regression (SVR) is used to train the prediction model [3]. It is assumed that the training samples set $U = \{(x_i, y_i) | i = 1, 2, \dots, N\}$ contains the inputs x_i and the corresponding output y_i where N is the number of samples. The basic idea of regression is to map the input x_i to y_i with a function $f(x)$. In the nonlinear problem, the mapping function $\Phi(x)$ is used to simplify the problem. Then the regression function form is presented by

$$f(x) = \langle \omega, \Phi(x) \rangle + b \quad (6)$$

where $\langle \cdot, \cdot \rangle$ denotes the dot product. ω, b represent the weight coefficient and constant respectively. Theoretically, the function $f(x)$ fit the training samples as flat as possible by seeking a small ω . One way to satisfy this is to minimize $\|\omega\|^2 = \langle \omega, \omega \rangle$. There are two slack variables ξ_i, ξ_i^* introduced to deal with other infeasible constraints. With a ε for the prediction accuracy, the problem is formulated as

$$\text{Minimize} \quad \frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^l (\xi_i + \xi_i^*) \quad (7)$$

$$\text{Subject to} \quad \begin{cases} y_i - \langle \omega, x_i \rangle - b \leq \varepsilon + \xi_i \\ \langle \omega, x_i \rangle + b - y_i \leq \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \geq 0 \end{cases} \quad (8)$$

where the constant C denotes the penalty factor. ξ_i, ξ_i^* represent the difference between the predicted and true values. This problem is solved more easily by the transformation of the dual formulation. The Lagrangian function is shown as follows

$$L := \frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^l (\xi_i + \xi_i^*) - \sum_{i=1}^l (\eta_i \xi_i + \eta_i^* \xi_i^*) - \sum_{i=1}^l \alpha_i (\varepsilon + \xi_i - y_i + \langle \omega, x_i \rangle + b) - \sum_{i=1}^l \alpha_i^* (\varepsilon + \xi_i^* + y_i - \langle \omega, x_i \rangle - b) \quad (9)$$

where $\eta_i, \eta_i^*, \alpha_i, \alpha_i^*$ are the Lagrange multipliers. This function in Eq. (9) has a saddle point with respect to the dual variables at the solution which is substituted by inferring the partial derivatives of L with respect to the primal variables. Then the dual optimization problem is written as

$$\text{Minimize} \quad \frac{1}{2} \sum_{i,j=1}^l (\alpha_i^* - \alpha_i)(\alpha_j^* - \alpha_j) K(x_i, y_j) + \varepsilon \sum_{i=1}^l (\alpha_i^* + \alpha_i) - \sum_{i=1}^l y_i (\alpha_i^* - \alpha_i) \quad (10)$$

$$\text{Subject to} \quad \begin{cases} \sum_{i=1}^l (\alpha_i^* - \alpha_i) = 0 \\ 0 \leq \alpha_i, \alpha_i^* \leq C \end{cases} \quad (11)$$

where $K(x_i, y_i)$ is the kernel function. The non-zero α_i and α_i^* are also the support vectors. By solving the dual optimization problem, the weight coefficient ω is computed

$$\omega = \sum_{i=1}^l (\alpha_i^* - \alpha_i) x_i \quad (12)$$

Besides, b can be determined by exploiting the Karush-Kuhn-Tucker (KKT) conditions.

$$\begin{aligned} b &= y_j - \langle \omega, \Phi(x) \rangle - \varepsilon \quad \text{for } 0 \leq \alpha_i \leq C \\ b &= y_j - \langle \omega, \Phi(x) \rangle + \varepsilon \quad \text{for } 0 \leq \alpha_i^* \leq C \end{aligned} \quad (13)$$

Then the mapping function can be presented in the support vector expansion as

$$f(x) = \sum_{i=1}^l (\alpha_i - \alpha_i^*) K(x_i, x) + b \quad (14)$$

According to Eq. (14), the determination of kernel function is the key component in SVR method. The commonly used kernel functions include linear kernels, polynomial kernels and Radial Basis Function (RBF). Their corresponding formulations are

$$\begin{aligned} K_{\text{linear}}(x_1, x_2) &= x_1' x_2 \\ K_{\text{polynomial}}(x_1, x_2) &= (1 + x_1' x_2)^p \\ K_{\text{RBF}}(x_1, x_2) &= \exp(-\|x_1 - x_2\|^2) \end{aligned} \quad (15)$$

Each kernel function has distinct characteristic because of its special and inherent formulation for computing the predictors which should be compared as the problem. Finally, to determine the suitable training sample set, the Mean Square Error (MSE) is chosen as objective to present the prediction accuracy. The lower MSE means the more accurate predicted values and the number of training samples can be determined by limiting an acceptable MSE.

4. Numerical results

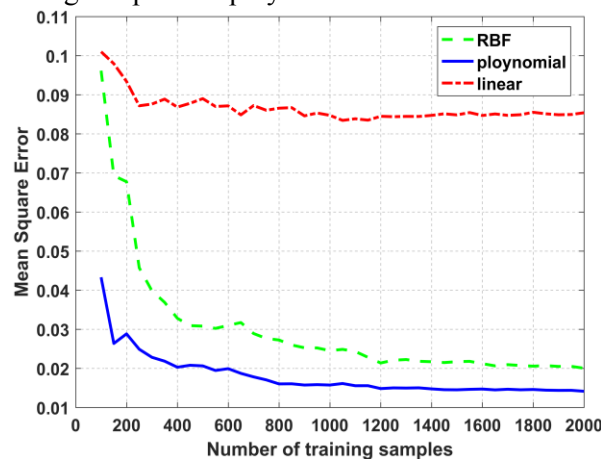
To verify the efficiency of proposed method, the evaluation model is constructed to compute trajectory parameters. Considering low-thrust transfer from Earth's orbit with zero excess velocity, the model in Eq. (5) is trained based on SVR to predict the total velocity increment. To guarantee the performance of prediction, the seven input parameters are generated following a uniform distribution in the input domain. Without loss of generality, the range for each parameter is set as table 1.

Table 1. Ranges of seven input parameters.

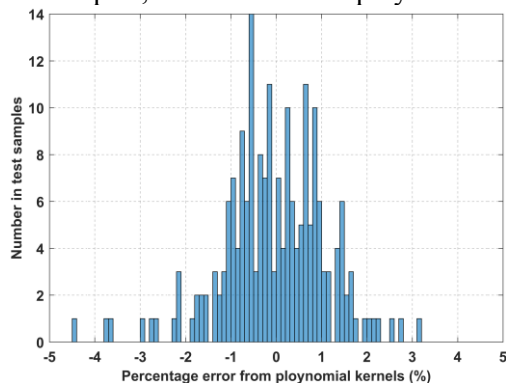
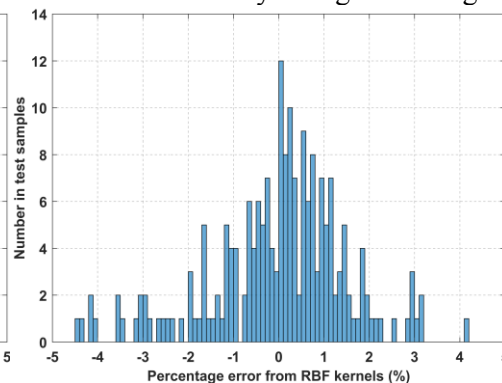
Input parameters	ranges
Semi-major axis a (AU)	2.1~2.5
Orbital eccentricity e	0~0.3
Orbital inclination i (deg)	0~20
Maximal thrust magnitude T_{\max} (mN)	300~500
Thrust specific impulse I_{sp} (s)	2500~3500
Initial mass of spacecraft m_0 (kg)	1000~2000
Flight time t_f (days)	1000~1500

For the inputs from table 1, the optimal low-thrust trajectory database used for SVR is obtained by using the Sims-Flanagan transcription and DE-SNOPT. Then, SVR is adopted to train the prediction model. The prediction model mainly depends on the determination of kernel function and training samples set. Choosing 200 test samples randomly, figure 2 shows the relationship between MSE and training samples set size for three kernels in Eq. (15). The degree of polynomial kernel is set to 3.

As shown in figure 2, it is obvious that polynomial kernel function performs better for this prediction problem. In addition, with the increase of training samples set size, MSE of test samples decreases and tend to be stable after fluctuation for all three kernels. According to this selection criterion, the number of training samples for polynomial kernel function are chosen as 800.

**Figure 2.** Relationship between MSE and number of training samples for three kernels.

To show the prediction deviation more clearly, the percentage error (PE) between the predicted outputs and optimal outputs of test samples is chosen for the prediction of SVR-based model. For 200 random test samples, the PE results of polynomial kernel are exhibited by histograms in figure 3.

**Figure 3.** PE of polynomial kernels.**Figure 4.** PE of RBF kernels.

It can be seen that the maximal PE is less than $\pm 5\%$. The SVR-based model is capable of predicting the velocity increment for interplanetary low-thrust transfer with sufficient precision. Comparatively, figure 4 shows that of RBF kernel function. The distribution of PE in figure 4 is more dispersive and the number of PE over $\pm 3\%$ is more than that in figure 3. Note that the generation of one optimal low-thrust trajectory by using hybrid optimization algorithm requires about 2 minutes, which means obtaining the 200 test samples takes about 6.67 hours. While the SVR-based prediction model can accomplish the computation like the analytical function.

By using the prediction model from polynomial kernel for low-thrust optimal transfer, we set the asteroid mission to Vesta for evaluating optimal velocity increment ΔV . Table 2 lists the predicted ΔV to Vesta for various inputs. Furthermore, it also shows the global optimal ΔV calculated by numerical optimization algorithm. As shown in table 2, the PE is varied from -0.59% ~ 1.54% . The corresponding deviation of ΔV is less than 157 m/s. Therefore, the evaluation method based on SVR is really efficient in the preliminary low-thrust mission design.

Table 2. Prediction accuracy of propulsion systems for low-thrust transfers to Vesta.

T_{\max} , mN	I_{sp} , s	m_0 , kg	t_f , days	Optimal ΔV , km/s	predicted ΔV , km/s	PE of ΔV , %
300	2500	2000	1450	10.574	10.702	1.21
350	3500	1000	1400	10.187	10.127	-0.59
400	3000	1000	1050	10.245	10.234	-0.11
450	2500	1500	1200	10.182	10.226	0.43
500	3500	1000	1400	10.183	10.340	1.54
500	2500	1500	1200	10.182	10.156	-0.26

5. Conclusion

In this paper, a novel approach is developed for evaluating interplanetary optimal low-thrust trajectory quickly and efficiently. Considering the minimum-propellant low-thrust transfer, the SVR-based model can predict optimal velocity increment accurately. Comparing to the numerical optimal solution, the percentage errors of 200 random test samples are all less than 5%. Thus, the proposed SVR-based models are powerful tools for evaluation of optimal low-thrust trajectory, especially in the case that a large number of candidate trajectories are considered. The introduction of machine learning also provides an advantage to reveal the relationship of trajectory parameters for low-thrust transfer, which makes it well suited for performing preliminary interplanetary low-thrust mission design.

References

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