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Hob mill for trilobed rotor – Graphical method in CATIA

Nicuşor Baroiu, Virgil Gabriel Teodor, Florin Susac, Nicolae Oancea

111 Domnească Street, Galaţi, România, 800201

florin.susac@ugal.ro

Abstract. The machining of ordered curl of surfaces, associated to a centrode, is completed in very good yield and precision conditions with hob mill tool. In this case, the issue is to determine the enwrapping worm of the ordered curl of profiles, as a fundament for hob mill tool's construction. Based on the "generating trajectories" method, an algorithm for profiling the generating hob mill for machining the trilobed rotor of a peristaltic pump is presented. A graphical example, developed in CATIA design environment, shows the profiling technique.

1. Introduction

Some methods can be used for profiling the enwrapping worm for an ordered curl of profiles: fundamental method of Gohman for surfaces enveloping [1, 2]; complementary method of "minimum distance" [3] or the method of "substitutive circles" [3].

Advances in graphical design environment allow elaborating graphical methods for profiling a helical surface reciprocally enwrapping with an ordered curl of surfaces, associated to a circular centrode.

The hob mill enveloping an ordered curl of profiles is a classical type of enwrapping generation with a single point of contact. The solution of this problem, determining the primary peripheral surface of hob mill tool, is identified based on the second theorem of Olivier [1], as so as, using the "intermediary surface" method, based on the Gohman theorem [1, 2].

Others complementary methods were proposed, as the "minimum distance" method [3, 8]. This method allows approaching problems for a single point contact between enwrapping surfaces.

Issue of profiling worm tools, as so as, quality of these tools, was studied by Radzevich [9], referring to impact of the major design parameters of an involute hob on deviations of the hob tooth profile, by developing an analytical method for computation of the deviation, for optimal configuration of worm tool.

Hsieh et al. [10] propose a novel hub cutter design of straight side hob cutter with multiple pressure angles, in order to simplify the manufacturing process.

Jiang [11] use a CNC hob machine in order to minimize the tooth flank's deviation, demonstrates that the hob flank can be approximated to the given tooth flank by adjusting the coefficients of the polynomials based on their sensitivity.

In this paper, a problem concerning the hob mill profiling, for manufacturing a non involute profile, based on a complementary theorem of "generating trajectories family" [6] is presented. It is also presented an analytical development and a graphical application in CATIA.

The results for an axial profile of hob mill, determined using both methods, analytical and graphical, are compared.

2. Surface to be generated and generating rack gear tool

The rotor of a peristaltic pump is a composed cylindrical surface. The frontal profile is presented in figure 1, where a presented the reference systems too.

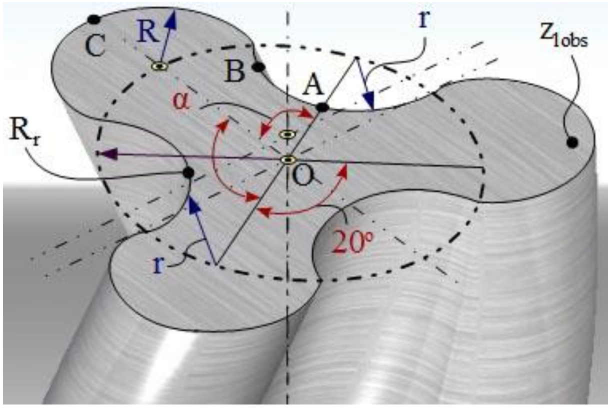

Figure 1. Frontal profile of the peristaltic pump's rotor

The rotor's profile is composed by an assembly of circle's arcs, AB, BD, DE, with radii r and R, fillet to each other.

The reference systems are defined, see Figure 2:

xyz is the world reference system, with z axis of rotor;

XYZ – mobile reference system, joined with the frontal profile of the rotor and with the associated centrode C_1;

$\xi\eta\zeta$ – mobile reference system, joined with the C_2 centrode of the future rack gear tool.

The analytical models of the frontal profiles of rotor are defined: AB, arc with radius r:

$$AB \left| \begin{array}{l} X = -R\cos\alpha - r\cos\theta; \\ Y = -R\sin\alpha + r\sin\theta; \\ Z = 0, \end{array} \right. \tag{1}$$

with θ variable parameter in limits $\theta_A = \alpha$; $\theta_B = \beta$, where:

$$\beta = \arcsin\left(\frac{R_r}{r+R}\cdot\sin\alpha\right), \tag{2}$$

and, similarly, the arc BC, with radius R:

$$BC \left| \begin{array}{l} X = -R_r + R\cos\eta; \\ Y = -R\sin\eta; \\ Z = 0, \end{array} \right. \tag{3}$$

with η as a variable parameter:

$$\eta_{\mathrm{C}} = 0;\ \eta_{\mathrm{B}} = \beta. \tag{4}$$

2.1. Generating movements of the rack gear – rack gear form
The relative motion between *ξηζ* and *XYZ* reference systems is writing:

$$\xi = \omega_3^T(\varphi)X - a;\ a = \begin{Vmatrix} -R_r \\ -R_r \cdot \varphi \\ 0 \end{Vmatrix}, \tag{5}$$

and the profiles' family:

$$\left(\Sigma_{AB}\right)_\varphi : \begin{vmatrix} \xi = \left(-R\cos\alpha - r\cos\theta\right)\cos\varphi - \left(-R\sin\alpha + r\sin\theta\right)\sin\varphi + R_r; \\ \eta = \left(-R\cos\alpha - r\cos\theta\right)\sin\varphi + \left(-R_r\sin\alpha + r\sin\theta\right)\cos\varphi + R_r\varphi; \\ \zeta = t. \end{vmatrix} \tag{6}$$

Similarly, for arc:

$$\left(\Sigma_{BC}\right)_\varphi : \begin{vmatrix} \xi = \left(-R_r + R\cos\eta\right)\cos\varphi - R\sin\eta\sin\varphi + R_r; \\ \eta = \left(-R_r + R\cos\eta\right)\sin\varphi + R\sin\eta\cos\varphi + R_r\varphi; \\ \zeta = t. \end{vmatrix} \tag{7}$$

For both (6) and (7) equations, the *t* parameter is defined along the cylindrical surface's generatrix – the rotor's flanks.

The enveloping of (6) and (7) profiles family, in the rolling movement of the two centrodes C_1 and C_2, with the rolling condition:

$$\lambda = R_r \cdot \varphi, \tag{8}$$

representing the generating rack-gear's profile.

2.2. The enveloping condition at contact between cylindrical surface and generating rack-gear
The establishing of enwrapping condition based on the "generating trajectories family" method [3] it is proposed. The method assumes to know the normals trajectories families. For the arc, from equations (1):

$$\vec{n}_{\Sigma_{AB}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r\sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos\theta \cdot \vec{i} - \sin\theta \cdot \vec{j}, \tag{9}$$

which represents the directrix parameters for the normal to the profile from which, in the current point onto profile, the normal form results:

$$\overrightarrow{N}_{\Sigma_{AB}} = \left[\left(-R\cos\alpha - r\cos\theta\right) + \lambda\cos\theta\right]\vec{i} + \left[\left(-R\sin\alpha - r\sin\theta\right) - \lambda\sin\theta\right]\vec{j}. \tag{10}$$

Similarly, for BC profile:

$$\vec{N}_{\Sigma_{BC}} = \left[\left(-R_r + R\cos\eta\right) - \lambda\cos\eta\right]\vec{i} + \left(-R\sin\eta + \lambda\sin\eta\right)\vec{j}, \tag{11}$$

with λ variable scalar parameter.

According to the "generating trajectories" theorem, [3], the characteristic curve onto Σ_{AB} and Σ_{BC} surfaces, is the geometric locus of points belonging to these surfaces, where the normals' families, in the generating motion (5), pass through the generatrix with direction $\vec{t}$ drawn from the gearing pole:

$$P\left|\begin{aligned}&\xi_p = 0;\\&\eta_p = R_r\varphi.\end{aligned}\right. \tag{12}$$

From (5) and (10), the normals family for the AB arc results:

$$\left(N_{\Sigma_{AB}}\right)_\varphi\left|\begin{aligned}&\xi = \left(-R\cos\alpha - r\cos\theta + \lambda\cos\theta\right)\cos\varphi - \left(-R\sin\alpha - r\sin\theta - \lambda\sin\theta\right)\sin\varphi + R_r;\\&\eta = \left(-R\cos\alpha - r\cos\theta - \lambda\cos\theta\right)\sin\varphi + \left(-R\sin\alpha - r\sin\theta - \lambda\sin\theta\right)\cos\varphi + R_r\varphi.\end{aligned}\right. \tag{13}$$

From condition that the $\left(N_{\Sigma_{AB}}\right)_\varphi$ normals family (13) pass through the gearing pole (12) results condition:

$$\varphi = -\alpha. \tag{14}$$

In this way, the (6) and (14) equations assembly represents the enwrapping of profiles family $\left(\Sigma_{AB}\right)_\varphi$, the S_{AB} profile of rack-gear tool.

Similarly, the normals trajectories family is established for profile Σ_{BC}, see form (3):

$$\left(N_{\Sigma_{BC}}\right)_\varphi\left|\begin{aligned}&\xi = \left(-R_r + R\cos\eta + \lambda\cos\eta\right)\cos\varphi - \left(-R\sin\eta - \lambda\sin\eta\right)\sin\varphi + R_r;\\&\eta = \left(-R_r + R\cos\eta + \lambda\cos\eta\right)\sin\varphi + \left(-R\sin\eta - \lambda\sin\eta\right)\cos\varphi + R_r\cdot\varphi.\end{aligned}\right. \tag{15}$$

From condition that the gearing pole coordinates check the equations for (15) family, see (12), the specific enwrapping condition results:

$$\varphi = 0. \tag{16}$$

In this way, for this case, the crossing profile equations for generating rack-gear, see relations (7), become:

$$C_{BC}\left|\begin{aligned}&\xi = R\cos\eta;\\&\eta = R\sin\eta.\end{aligned}\right. \tag{17}$$

3. Peripheral primary surface of hob mill

In figure 2, the reference systems are presented, as so as the positions of the future primary peripheral surface of hob mill, $\vec{V}$ axis.

In order to determine the peripheral primary surface of hob mill, reciprocally enwrapping with the cylindrical surface of generating rack-gear, the principles of helical motion decomposition are used [7].

We can consider that the helical motion with $\vec{V}$ axis and p helical parameter, when the peripheral primary surface of hob mill is generated, is decomposed in two motions:

- translation along the generatrix of the cylindrical surface of rack-gear flank, the direction $\vec{t}$, see figure 2;

- rotation around the $\vec{A}$ axis, parallel with axis $\vec{V}$ of the helical motion and at the a distance from $\vec{V}$,

$$a = p \cdot tg\theta. \tag{18}$$

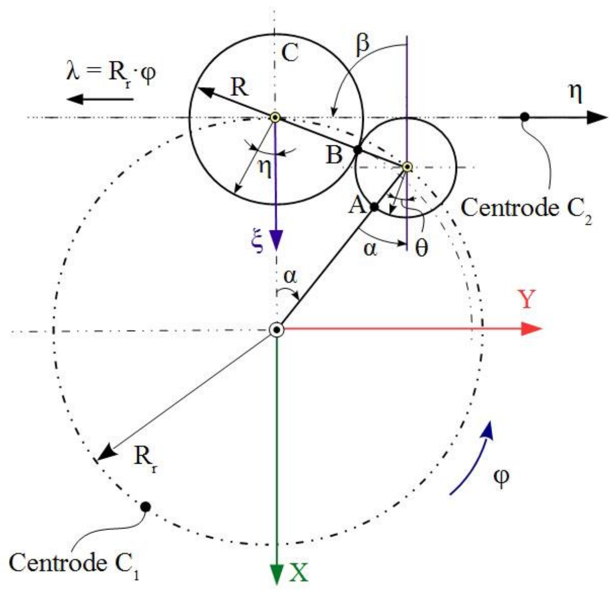

Figure 2. The positions of the future primary peripheral surface of hob mill; reference systems

In relation (18), with p was denoted the helical motion's parameter and with θ the angle between cylindrical surface's generatrix and the axis of hob mill.

The decomposition of the helical motion $\left(\vec{V}, p\right)$ may be symbolized:

$$\left(\vec{V}, p\right) \approx T\left(v, \vec{t}\right) + A\left(\vec{A}, \varphi\right). \tag{19}$$

The v and φ values are the moving parameters, with the component motions: v is scalar parameter along $\vec{t}$ direction and φ is angular parameter in rotation motion around the $\vec{A}$ axis, see figure 2.

In this way, the determining of rack-gear surface's characteristic $\left(S_{\Sigma_{AB}}\right)$ – equations (13) and (14) and $\left(S_{\Sigma_{BC}}\right)$ – equations (16) and (17) do not depend from the translation component $\left(T\left(v,\bar{t}\right)\right)$, when the $\left(S_{\Sigma_{AB}}\right)$ and $\left(S_{\Sigma_{BC}}\right)$ surfaces are self-generated.

So, the rack-gear surface's characteristic curve will depends on rotation around the $\vec{A}$ axis, only.

Regarding the *a* value, we can state: If the circular pitch of peristaltic pump rotor's teeth is defined as:

$$P_c = \frac{2\pi R_r}{z_{lobs}}, \tag{20}$$

the normal pitch of generating rack-gear is defined as, see figure 3:

$$P_{n_{cr}} = \frac{2\pi R_r}{z_{lobs}} \cdot \cos\beta_d, \tag{21}$$

where β_d is the angle between the cylindrical surface's generatrix and rotor's axis.

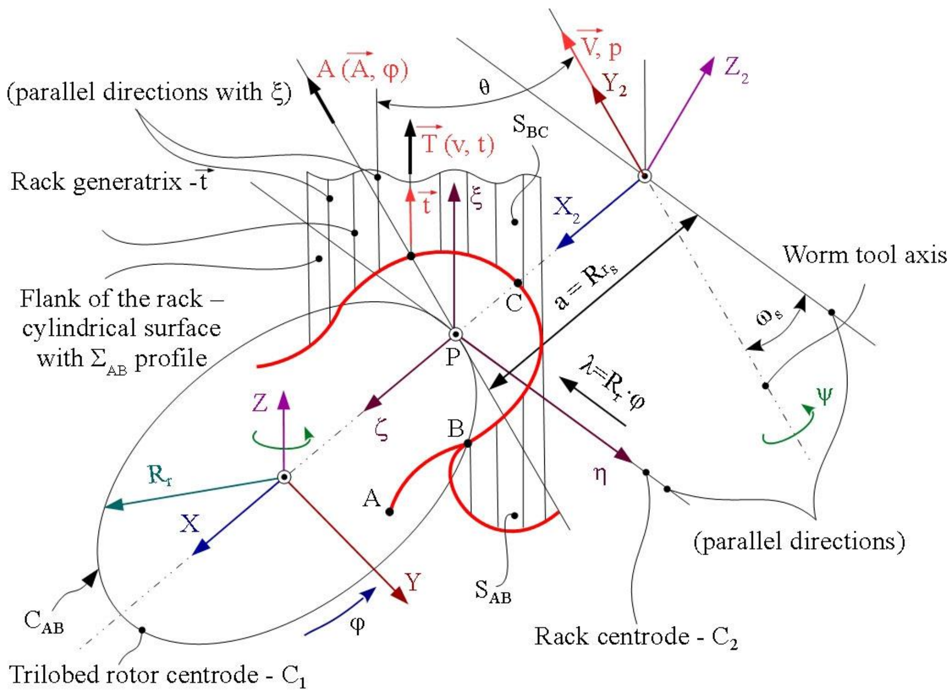

Figure 3. Hob mill axis; reference systems; helical movement decomposition, $\left(\vec{V}, p\right)$

The angle of hob mill helix is:

$$tg\omega_s = \frac{P_{ax_m}}{2\pi R_{r_s}}, \tag{22}$$

with P_{ax_m} axial pitch of hob mill and R_s rolling radius of hob mill.

In figure 3, is obviously the equality:

$$\theta = \frac{\pi}{2} - \omega_s. \tag{23}$$

From the condition that the normal pitch of rack-gear to be equal to the normal pitch of the hob mill, it results:

$$\sin\omega_s = \frac{R_{r_p}}{R_{r_s}} \cdot \frac{1}{z_{lobs}} \cdot \cos\beta_d. \tag{24}$$

The helical parameter of the hob mill is defined as:

$$p = \frac{P_{ax_m}}{2\pi} = \frac{R_{r_p}}{z_{lobs}} \cdot \frac{\cos\beta_d}{\cos\omega_s} \cdot \frac{1}{2\pi}, \tag{25}$$

from where it results:

$$a = p \cdot tg\theta = \frac{R_{r_p}}{z_{lobs}} \cdot \frac{\cos\beta_d}{\sin\omega_s} \tag{26}$$

and

$$tg\omega_s = \frac{p}{R_{r_s}}, \tag{27}$$

or

$$a = R_{r_s}. \tag{28}$$

In this manner, it is obviously that the rolling radius value depends on the characteristics of helical motion decomposition.

Determining the primary peripheral surface of the hob mill, enwrapping for generating rack-gear, previously determined for section BC (and similarly for circle's arc AB) leads to determining the characteristic curve at contact between rack-gear and helical surface, with respect to the helical motion decomposition particularities (see equation (19)).

3.1. Enwrapping condition at contact between rack gear and hob mill's helical surface

The enwrapping condition will depends only by rotation around the $\vec{A}$ axis.

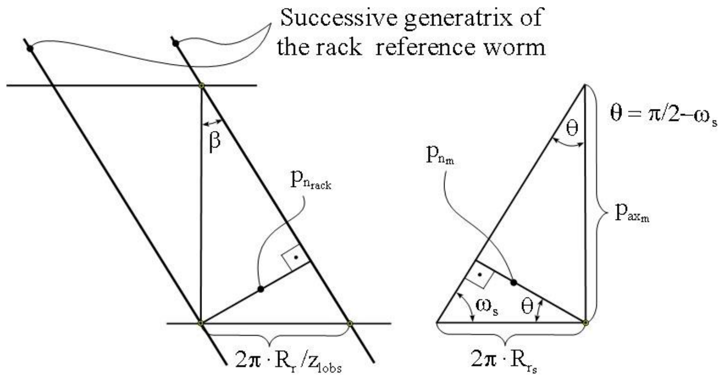

Figure 4. Normal and circular pitch of rack-gear; characteristic angles

So, for generating rack-gear's surfaces it can be used equations:

$$S\begin{vmatrix}\xi=\xi(\upsilon);\\ \eta=\eta(\upsilon);\\ \zeta=\zeta(t),\end{vmatrix} \tag{29}$$

with *v* and *t* independent parameters, expressed in *ξηζ* reference system, see figure 4, where the $\vec{A}$ and $\vec{V}$ axis are presented, as the reference systems:

- *ξηζ* is the reference system joined with the generating rack-gear;
- $\xi_1\eta_1\zeta_1$ – reference system joined with $\vec{A}$ axis.

The normal to the *S* surface (rack gear flank) has directrix parameters:

$$\vec{N}_S=\begin{vmatrix}\vec{i} & \vec{i} & \vec{k}\\ \dot{\xi}_{(v)} & \dot{\eta}_{(v)} & 0\\ 0 & 0 & 1\end{vmatrix} \tag{30}$$

or

$$\vec{n}_s=\dot{\eta}_{(v)}\vec{i}-\dot{\xi}_{(v)}\vec{j}. \tag{31}$$

In this way, the normals in the current points onto *S*, has equations:

$$\vec{N}_S=\left[\xi(v)+\lambda\dot{\eta}_{(v)}\right]\vec{i}+\left[\eta(v)-\lambda\dot{\xi}_{(v)}\right]\vec{j}, \tag{32}$$

which, by coordinates transformation,

$$\begin{Vmatrix}\xi_1\\ \eta_1\\ \zeta_1\end{Vmatrix}=\begin{Vmatrix}1 & 0 & 0\\ 0 & \cos\omega_s & \sin\omega_s\\ 0 & -\sin\omega_s & \cos\omega_s\end{Vmatrix}\cdot\begin{Vmatrix}\xi\\ \eta\\ \zeta\end{Vmatrix}, \tag{33}$$

may lead to the following form of normals to S profile:

$$N_S\ \left|\begin{array}{l}\xi_1=\xi(v)+\lambda\dot{\eta}_{(v)};\\ \eta_1=\left[\eta(v)+\lambda\dot{\xi}_{(v)}\right]\cos\omega_s;\\ \zeta_1=-\left[\eta(v)+\lambda\dot{\xi}_{(v)}\right]\sin\omega_s.\end{array}\right. \tag{34}$$

The trajectories family of N_S normals in the rotation around $\vec{A}$ axis is:

$$\begin{Vmatrix}\xi_1\\ \eta_1\\ \zeta_1\end{Vmatrix}=\begin{Vmatrix}\cos v & 0 & -\sin v\\ 0 & 1 & 0\\ \sin v & 0 & \cos v\end{Vmatrix}\cdot\begin{Vmatrix}\xi(v)+\lambda\dot{\eta}_{(v)}\\ \left[\eta(v)+\lambda\dot{\xi}_{(v)}\right]\cos\omega_s\\ -\left[\eta(v)+\lambda\dot{\xi}_{(v)}\right]\sin\omega_s\end{Vmatrix}. \tag{35}$$

It is imposed that the normals family intersect the $\vec{A}$ axis:

$$\vec{A}=\vec{j}. \tag{36}$$

Therefore, the equations system results

$$\begin{cases}\xi_{(v)}\cos v+\eta_{(v)}\sin\omega_s\sin v=-\lambda\left[\dot{\eta}_{(v)}\cos v-\dot{\xi}_{(v)}\sin\omega_s\sin v\right]\\ \xi_{(v)}\sin v-\eta_{(v)}\sin\omega_s\cos v=-\lambda\left[\dot{\eta}_{(v)}\sin v+\dot{\xi}_{(v)}\sin\omega_s\cos v\right],\end{cases} \tag{37}$$

and, starting from this, by eliminating the λ parameter, the following equation can be obtained:

$$\xi(v)\dot{\xi}_{(v)}+\eta(v)\dot{\eta}_{(v)}=0, \tag{38}$$

which represents the specific enwrapping condition.

The condition (38) has the geometric significance: the points belonging to the characteristic curve, represents the $\vec{A}$ axis projection onto the S surface (surface of the generating rack gear).

The $\xi_{(v)}$ and $\eta_{(v)}$ equations represents a generic form of the rack gear flank.

3.2. Primary peripheral surface of hob mill

In principle, the equations of the characteristic curve onto the generic surface of generating rack gear S (29) is a curve representing the projection of the $\vec{A}$ axis, in principle known in form:

$$C_S \left| \begin{aligned} &\xi_1 = \xi_1(v_A); \\ &\eta_1 = \eta_1(v_A); \\ &\zeta_1 = \zeta_1(v_A), \end{aligned} \right. \tag{39}$$

with v_A parameter for which is accomplished the condition(38).

Giving to the curve (39) a helical motion with axis $\vec{V}$ and helical parameter p (see equation (25)), the primary peripheral surface of the hob mill is generating.

In figure 4, it is defined, the $X_2Y_2Z_2$ reference system, with Y_2 axis overlapped to the axis $\vec{V}$ and with axis parallel to the $\xi_1\eta_1\zeta_1$ reference system:

$$\begin{Vmatrix} X_2 \\ Y_2 \\ Z_2 \end{Vmatrix} = \begin{Vmatrix} \xi_1 \\ \eta_1 \\ \zeta_1 \end{Vmatrix} - \begin{Vmatrix} -R_{r_s} \\ 0 \\ 0 \end{Vmatrix}. \tag{40}$$

By transforming equation (40), the C_S curve (39) is defined in the new reference system joined with hob mill:

$$C_{S_{X_2Y_2Z_2}} \left| \begin{aligned} &X_2 = X_2(v_A); \\ &Y_2 = Y_2(v_A); \\ &Z_2 = Z_2(v_A). \end{aligned} \right. \tag{41}$$

In the helical motion,

$$E_{X_2Y_2Z_2} \begin{Vmatrix} X_2 \\ Y_2 \\ Z_2 \end{Vmatrix} = \omega_2^T(\psi) \begin{Vmatrix} X_2(\upsilon_A) \\ Y_2(\upsilon_A) \\ Z_2(\upsilon_A) \end{Vmatrix} + P\psi\vec{j}, \tag{42}$$

with p helical parameter defined with equation (25).

The axial section of the helical surface $E_{X_2Y_2Z_2}$ with a plane ($X_2 = 0$) represents the axial profile of the hob mill.

4. Graphical method in CATIA

In order to be able to profile the hob mill using the graphic method, the intermediate surface method will be used.

This method consists in determining an intermediate surface which is the primary peripheral surface of a rack type tool.

Subsequently, on the intermediate surface, the characteristic curve by projecting the hob mill axis on this surface is determined.

The characteristic curve is generated with a helical movement with $\vec{V}$ axis and p helical parameter, resulting in the primary peripheral surface of the hob mill.

The determination of the rack gear tool's profile using the method of "generating trajectories family" consists in generating in CATIA the mechanism that reproduces the kinematic of the rake tool generation.

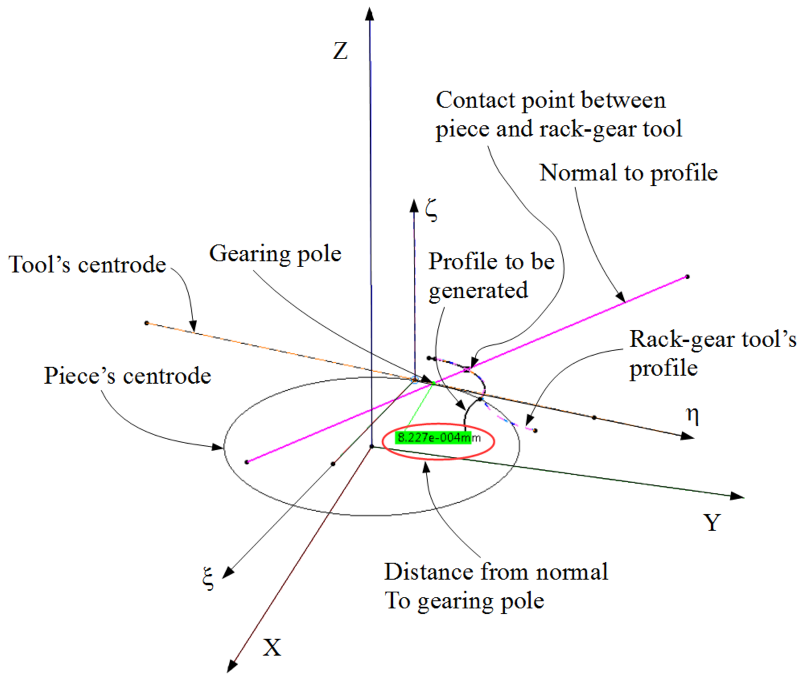

Figure 5. Profiling the rack gear tool

The mechanism is composed of three elements. The first element is the reference file containing the profile to be generated, the axis of the reference system associated with the part and the centrode of the piece, which in this case is a circle.

The second element is also a part file, but it will contain the axis of the reference system associated to the tool.

The third element has only a basic role and contains the axes of the reference systems associated to the piece and the tool, represented in the fixed reference system. In this way, the relative positioning of the two mobile systems can be made.

Onto the curve representing the profile to be generated a current point by which a straight line is drawn perpendicular to that profile is defined. In the assembly drawing, the operation of the mechanism is simulated, considering the rack reference as the fixed element, and the distance from the perpendicular to the pole of the gear is monitored. When this distance is zero, the contact condition between the track profile and the tool profile is met. The current point on the track profile is transferred to the tool reference system, obtaining a point on its profile.

Change the position of the current point and resume the mechanism simulation until sufficient points on the tool profile are obtained so that it can be obtained as a spline curve. This curve will represent the profile of the rack.

Subsequently, in the file where the toothed rack profile was obtained, the primary peripheral surface is thereof obtained by using the command "*extrusion*", which is applied to the curve in the direction of the rack generator version, see figure 3.

On the surface thus obtained, the tool axis is projected (see figure 3), resulting in the characteristic curve, which is the contact curve between the primary peripheral surface of the rack and the primary peripheral surface of the worm mill.

Applying the characteristic curve to a helical motion with $\vec{V}$ axis and p helical parameter, it produces the primary peripheral surface of the worm gear.

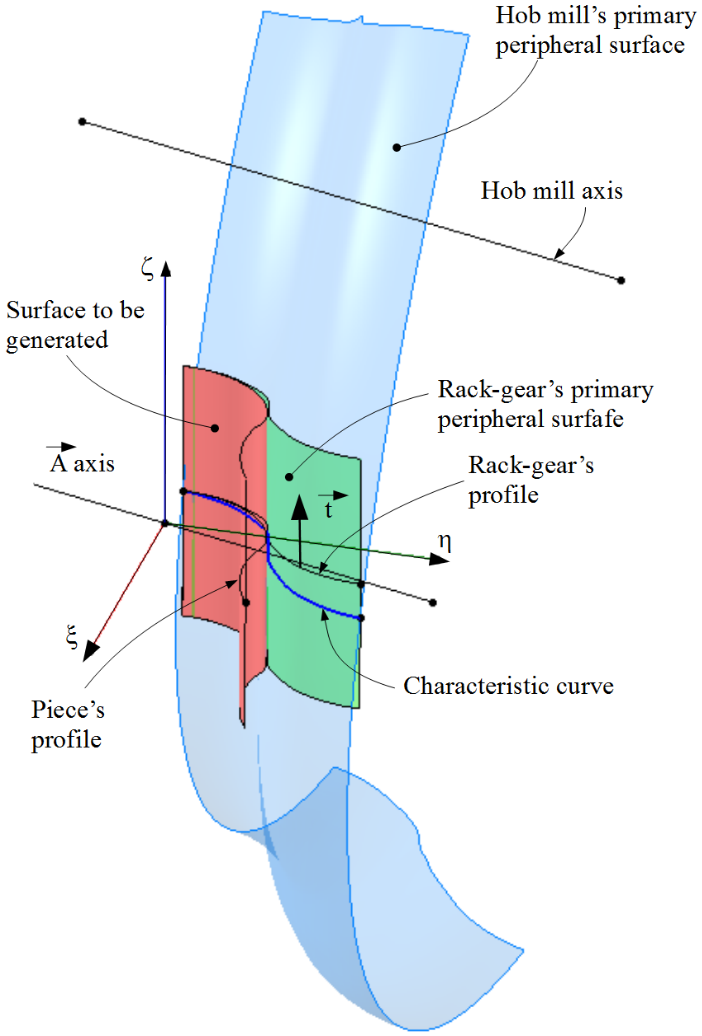

Figure 6. Hob mill's primary peripheral surface

The advantage of using CATIA software in order to determine the rack gear flank relies in the availability, inside CATIA, of all instruments that are needed for tool profiling.

In addition, the method for graphical design, through its specific characteristics and using numerical algorithms especially dedicated to graphical manipulation, helps avoiding the use of large mathematical equations which would normally lead to a large computer time.

Moreover, any error may occur in graphical design would be very visible and easy to identify, the program also offers solutions for identifying the correction possibilities.

5. Conclusions

The graphical method, developed in the CATIA graphics design environment, based on the complementary theorem of generating trajectories, leads to a simple expression of the enwrapping condition – the projection of the $\vec{A}$ axis on the component surfaces of the generating rack gear flank.

The method of determining the characteristic curve at the contact of the primary peripheral surface of the hob mill with the rack flank is rigorous and easy to achieve using a CATIA-specific command.

The characteristic point of contact of the helical surfaces of the hob mill with the helical flank of the rotor to be generated is obtained as the intersection point of the C_S characteristic curve, mainly in the shape (39), with the rectilinear generatrix of the rack.

From numerical point of view, this method is simple, intuitive and rigorous.

Appendix

It is proposed an analysis of the constitutive elements from the relation (19), regarding the decomposition of the helical motion, see Figure 4.

The circular pitch of the tooth, p_C, of the compressor rotor, in the frontal plane,

$$P_c = \frac{2\pi R_{r_p}}{z_{lobs}}, \tag{43}$$

where z_{lobs} is the number of lobs of the rotor ($z_{lobs} = 3$, for the driving rotor and $z_{lobs} = 6$ driven rotor);

R_{r_p} – radius of the associated centrode: $R_{r_p} = R_{r_{p1}}$, for driving rotor and $R_{r_p} = R_{r_{p2}}$, for driven rotor.

The rack gear normal pitch is defined,

$$p_{n_c} = p_c \cdot \cos\beta_d, \tag{44}$$

hence

$$p_{n_c} = \frac{2\pi R_{r_p}}{z_{lobs}} \cdot \cos\beta_d. \tag{45}$$

The angle of the hob mill helix

$$tg\omega_s = \frac{p_{ax_m}}{2\pi R_{r_s}}, \tag{46}$$

where: p_{ax_m} is the axial pitch of the hob mill;

R_{r_s} - rolling radius of the hob mill.

Considering the relation (18), from the decomposition of the helical motion, it results:

$$a = p \cdot tg\theta, \tag{47}$$

where: p is the helical parameter of the hob mill;

θ – the angle of cylindrical surface generatrix with the hob mill axis,

$$\theta = \frac{\pi}{2} - \omega_s. \tag{48}$$

From the equality condition between the generating rack gear normal pitch and the normal pitch of the hob mill, the following condition results:

$$\frac{2\pi R_{r_p}}{z_{lobs}} \cdot \cos\beta_d = 2\pi R_{r_s} \cdot tg\omega_s \cdot \cos\omega_s. \tag{49}$$

From the relation (49) results:

$$\sin\omega_s = \frac{R_{r_p}}{R_{r_s}} \cdot \frac{1}{z_{lobs}} \cdot \cos\beta_d. \tag{50}$$

The helical parameter of the hob mill is calculated as:

$$p = \frac{p_{ax_m}}{2\pi} = \frac{p_{n_m}}{\cos\omega_s} \cdot \frac{1}{2\pi} = \frac{R_{r_p}}{z_{lobs}} \cdot \frac{\cos\beta_d}{\cos\omega_s} \cdot \frac{1}{2\pi}, \tag{51}$$

where p_{n_m} is the normal pitch of the hob mill helix.

The a distance is calculated, from (18), as:

$$a = \frac{R_{r_p}}{z_{lobs}} \cdot \frac{\cos\beta_d}{\cos\omega_s} \cdot \frac{\sin\left(\frac{\pi}{2} - \omega_s\right)}{\cos\left(\frac{\pi}{2} - \omega_s\right)}, \tag{52}$$

resulting:

$$a = \frac{R_{r_p}}{z_{lobs}} \cdot \frac{\cos\beta_d}{\sin\omega_s}. \tag{53}$$

Considering the definition of the ω_s, the a dimension has the expression:

$$a = \frac{R_{r_p}}{z_{lobs}} \cdot \cos\beta_d \cdot \frac{R_{r_s} \cdot z_{lobs}}{R_{r_p} \cdot \cos\beta_d}, \tag{54}$$

hence,

$$a = R_{r_s}. \tag{55}$$

Obviously, the value of rolling radius R_{r_s} is equal to the dimension a resulted from the decomposition of the helical motion.

The equality of the normal pitch:

- rack gear's circular pitch:

$$p_c = \frac{2\pi R_{r_p}}{z_{lobs}}; \tag{56}$$

- rack gear's normal pitch:

$$p_{n_c} = p_c \cdot \cos\beta_d, \tag{57}$$

hence:

$$p_{n_c} = \frac{2\pi R_{r_p}}{z_{lobs}} \cdot \cos\beta_d ; \tag{58}$$

- hob mill's axial pitch:

$$p_{ax_m} = 2\pi R_{r_s} tg\omega_s ; \tag{59}$$

- hob mill's normal pitch:

$$p_{n_m} = p_{ax_m} \cdot \cos\omega_s , \tag{60}$$

hence:

$$p_{n_m} = 2\pi R_{r_s} tg\omega_s \cdot \cos\omega_s . \tag{61}$$

If the normal pitches are equals, then it results:

$$\frac{2\pi R_{r_p}}{z_{lobs}} \cdot \cos\beta_d = 2\pi R_{r_s} tg\omega_s \cdot \cos\omega_s . \tag{62}$$

Therefore,

$$\sin\omega_s = \frac{R_{r_p}}{R_{r_s}} \cdot \frac{1}{z_{lobs}} \cdot \cos\beta_d \tag{63}$$

and

$$a = p \cdot tg\theta, \tag{64}$$

where θ is the angle of the $\vec{t}$ generatrix with the axis of hob mill,

$$\theta = \omega + \frac{\pi}{2} - \beta_d ; \tag{65}$$

p – helical parameter of the hob mill,

$$p = \frac{P_{ax_m}}{2\pi} , \tag{66}$$

Finally,

$$p = \frac{P_{n_m}}{\cos\omega_s} \cdot \frac{1}{2\pi} = \frac{2\pi R_{r_p}}{z_{lobs}} \cdot \cos\beta_d \cdot \frac{1}{\cos\omega_s} \cdot \frac{1}{2\pi} , \tag{67}$$

resulting:

$$p = \frac{R_{r_p}}{z_{lobs}} \cdot \frac{\cos\beta_d}{\cos\omega_s}. \tag{68}$$

Therefore,

$$a = p \cdot tg\theta = \frac{R_{r_p}}{z_{lobs}} \cdot \frac{\cos\beta_d}{\cos\omega_s} \cdot \frac{\sin\left(\frac{\pi}{2} - \omega_s\right)}{\cos\left(\frac{\pi}{2} - \omega_s\right)}, \tag{69}$$

and simplifying, the following is obtained:

$$a = \frac{R_{r_p}}{z_{lobs}} \cdot \frac{\cos\beta_d}{\sin\omega_s}. \tag{70}$$

If it is considered the definition of ω_s,

$$\sin\omega_s = \frac{R_{r_p}}{R_{r_s}} \cdot \frac{1}{z_{lobi}} \cdot \cos\beta_d, \tag{71}$$

so,

$$a = \frac{R_{r_p}}{z_{lobi}} \cdot \cos\beta_d \cdot \frac{R_{r_s}}{R_{r_p} \cdot \cos\beta_d} \cdot z_{lobi}, \tag{72}$$

so,

$$a = R_{r_s}. \tag{73}$$

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