IOP Conf. Series: Materials Science and Engineering **447** (2018) 012079 doi:10.1088/1757-899X/447/1/012079

Structural and mathematical modeling of Cosserat lattices composed of particles of finite size and with complex connections

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Abstract. This article provides a brief review of the work and discussion of the problems related to the discrete structural and generalized continuum models of the Cosserat lattices consisting of particles of finite size and with complex connections. A simple Cosserat lattice, a Cosserat lattice with auxetic properties, and a chiral microstructure are considered. Various approaches to constructing a hierarchy of multi-field models for materials having a square lattice are presented. These approaches are based on the introduction of macrocells of different types and, accordingly, on the use of a larger number of fields describing deformations. A combination of micropolar and multi-field theories with the development of models is discussed.

1. Introduction

In the ordinary theory of lattices and in classical models of the mechanics of continuous media, deformations of solids are characterized only by translational displacements of particles. However, in some cases, the correct description of the deformations requires a consideration of rotational degrees of freedom. In particular, this applies to solids composed of rigid rotating elements, as well as solids with a beam microstructure. The description of deformation of such bodies, as granular media, complex molecular lattices, nanomaterials, biomaterials, stone masonry, beam lattice constructions, liquid crystals etc, needs taking into account the rotation of their constituent structural elements.

Beam lattices and their models play an important role in the interpretation and application of the micropolar theory [1, 2]. Lattices with a beam-like microstructure are used in the mechanics of metamaterials for development of auxetic and chiral structures [3-5]. The models of lattices composed of particles of finite size and with complex connections [6-9] are discussed in this article.

2. Discrete and micropolar models of materials, taking into account the rotational degrees of freedom of particles

We consider the Cosserat square lattice, i.e., the lattice composed of particles with positions determined not only by the displacements u_n and v_n, but also by the rotations φ_n, see figure 1 a.

In the expression of kinetic energy, the inertia of rotation of particles is taken into account.

$$2E_{kin}^{n} = M\dot{u}_n^2 + M\dot{v}_n^2 + J\dot{\varphi}_n^2. \tag{1}$$

The potential energy of the elastic connections of the elements m and k can be written in the form

$$2E_{pot}^{k,m} = K_n^{k,m}\Delta u^2 + K_s^{k,m}\gamma^2 + G_r^{k,m}\Delta\varphi^2, \tag{2}$$

where the following notations are introduced:

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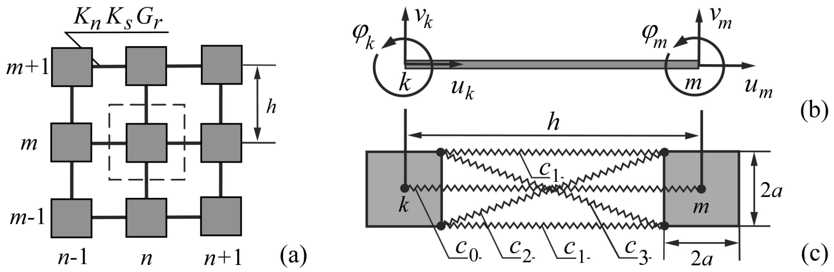

Figure 1. The Cosserat square lattice (a). The models of the two variants of connections in the Cosserat lattices: (b) a beam-type connection, (c) a complex connection of particles of finite size.

$$\Delta u = u_m - u_k\,,\ \gamma = v_m - v_k - h(\varphi_m + \varphi_k)/2\,,\ \Delta\varphi = \varphi_m - \varphi_k\,.$$

This form of the potential of interaction was introduced in [10] for simulation of granular media. It should be noted that in the micropolar theory of elasticity, the potential also includes three similar terms. Particular cases of the potential (2) are the potential of a beam-type joint (figure 1b) that is used in modeling of materials with a beam microstructure and the potential of a complex symmetrical $(c_2 = c_3)$ connection of particles (figure 1 c), which is employed for development of continuum models of lattices consisting of particles of finite size [8, 9, 11].

On the basis of the Lagrangian, using the kinetic energy (1) and the potential of the connections (2), the equations of motion of a particle (n, m) of a unit cell are derived. Taylor series expansion up to the second order of the discrete equations with respect to the displacement components and rotations leads to continuum equations. They are analogs of the equations of the micropolar theory of elasticity. The derivation of continuum models from discrete models allows, in particular, establishing a relation between micro- and macroparameters.

3. Multi-field approach in modeling of microstructured solids. Combination of approaches of multi-field and micropolar mechanics

For construction of a micropolar model, in contrast to the classical theory of elasticity, the rotational degrees of freedom of the particles are taken into account. However, the drawback of the micropolar model is that it is not capable of describing rapidly changing short-wave-length deformations of the lattice.

The classical micropolar model is constructed on the basis of a primitive cell (figure 1 a), using the minimal number of fields corresponding to the degrees of freedom of the cell. Such an approach is a certain hypothesis, the rejection of which leads to the multi-field theory. The construction of multi-field models is based on the consideration of macrocells composed of several primitive cells, and,

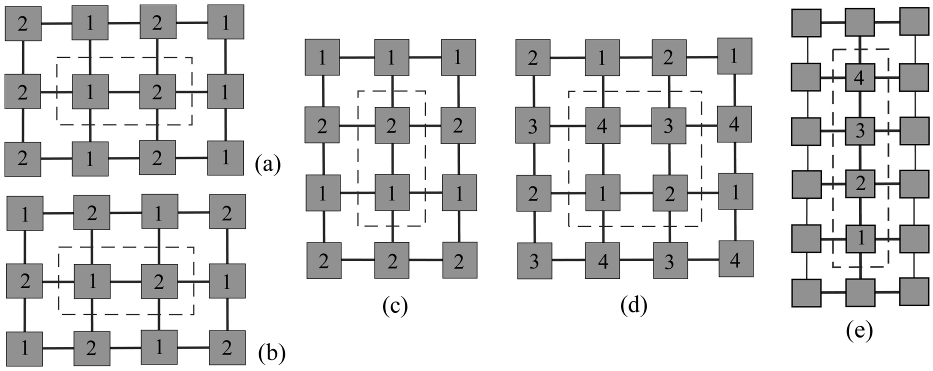

Figure 2. Variants of macrocells for construction of two-field (a-c) and four-field (d, e) models.

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accordingly, more fields are used to describe deformations. Such an approach provides, particularly, a description of short-wavelength deformations. A review of the results on the development of the multi-field theory of bodies with a microstructure is given in [12].

Using various macrocells (figures 2 a-e) the multi-field approach enables one to construct a hierarchy of models describing the dynamic properties of the lattice with increasing accuracy [13, 14]. The classical single-field micropolar model is valid for approximation of lattices for low-frequency long-wavelength oscillations. Multi-field models improve the single-field model for high-frequency short-wavelength oscillations. These models can be useful, in particular, for modeling lattices as directed frequency filters. The area of the Brillouin zone in which the model is improved depends on the choice of a macrocell.

In [15,16], it was shown that there are systems with the long-wavelength and short-wavelength static boundary effects. The classical micropolar model describes long-wavelength boundary effects, but it is not suitable for systems with short-wavelength boundary effects, since for their description one function must rapidly vary, and the long-wavelength model containing only second-order derivatives is not applicable. Both long-wavelength and short-wavelength deformations are effectively described by two slowly varying functions of the two-field model. Therefore, the two-field model is applicable to describe the effects of both types. Consideration of such deformations can be important in the problems of destruction.

4. Cosserat lattices with special properties

4.1. Lattice of particles of finite size with auxetic properties

Materials with a negative Poisson's ratio are called auxetics. The property of auxetics to expand in the lateral direction under uniaxial tension makes them attractive for technological applications.

In [6], for the structural model presented in figure 3 a, discrete and micropolar models were developed. Further, the model of the ordinary elasticity theory is obtained by eliminating the rotational degree of freedom. This allows us to find the relationship between micro- and macroparameters, in particular, to express the Poisson's ratio in terms of microstructural lattice parameters and to find parameters for which the Poisson's ratio is negative. It is shown that the negative Poisson's ratio is realized only if the particle sizes are not equal to zero, i.e. the finiteness of the particle size plays an important role for the manifestation of the auxetic property. This result was confirmed in [11] for a similar lattice consisting of round particles with symmetric spring connections of three different types.

The short-wavelength rotation fields are typical for an auxetic lattice presented in figure 3 a. They are realized even in the case of simple stretching. However, as mentioned above, short-wavelength deformations are not described by the ordinary micropolar theory. In [7], two-field and four-field models were developed. It is shown that multi-field models describe short-wavelength oscillations and static short-wavelength boundary effects in an auxetic lattice.

4.2. The Cosserat lattice with a chiral microstructure

The potential (1) corresponds to the connection of particles shown in figure 1 c, in the case when $c_2 = c_3$. To study chiral structures, we consider the opposite case, when $c_2 \neq c_3$. In this case, the interaction potential takes the form

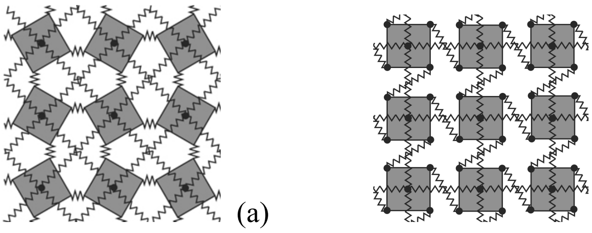
$$2E_{pot}^{k,m} = C_{11}^{k,m}\Delta u^2 + C_{22}^{k,m}\gamma^2 + C_{33}^{k,m}\Delta\varphi^2 + C_{12}^{k,m}\gamma\Delta u, \qquad (3)$$

where the following notations are used:

$$C_{11} = c_0/h^2 + 2c_1/r^2 + r^2(c_2 + c_3)/d^4 \,,\; C_{22} = 4a^2(c_2 + c_3)/d^4 \,,$$

$$C_{33} = 2a^2 c_1/r^2 \,,\; C_{12} = 4ar(c_2 - c_3)/d^4 \,,\; r = h - 2a \,,\; d = \sqrt{h^2 + (2a)^2} \,.$$

It can be shown that the first three terms of the potential (3) can be obtained by choosing proper coefficients of the potential (1). The potential (3) contains the fourth additional component, which enables one to take chirality into account. Accounting for this component leads to changes in the



(a) (b)

Figure 3. The Cosserat square lattices with particles of finite size: auxetic (a) and chiral (b).

equations of the micropolar theory. For a square lattice of finite size particles (figure 1 a) with complex asymmetric connections (figure 1 cc), they take the form

$$\rho u_{tt} = C_{11}u_{xx} + C_{22}\left(u_{yy} + \varphi_y\right) + C_{12}\left(v_{xx} - v_{yy} - \varphi_x\right)/2 + f_x(x,y) = 0,$$

$$\rho v_{tt} = C_{11}v_{yy} + C_{22}\left(v_{xx} - \varphi_x\right) + C_{12}\left(u_{xx} - u_{yy} - \varphi_y\right)/2 + f_y(x,y) = 0, \quad (4)$$

$$j\varphi_{tt} = \left(C_{33} - h^2C_{22}/4\right)\left(\varphi_{xx} + \varphi_{yy}\right) + C_{22}\left(v_x - u_y - 2\varphi\right) + C_{12}\left(u_x + u_y\right)/2 + f_\varphi(x,y) = 0.$$

Here $f_x(x,y) = F_x^{(m,k)}/h^2$, $f_y(x,y) = F_y^{(m,k)}/h^2$, and $f_\varphi(x,y) = F_\varphi^{(m,k)}/h^2$ are the components of the vector of distributed forces and moments, $\rho = M/h^2$ is the density of the medium, $j = J/h^2$ is the moment of inertia of a particle per unit cell, M is the mass of the particle, J is its moment of inertia. For the chiral lattice with $c_2 \neq c_3$ the elasticity modulus is $C_{12} \neq 0$. If $C_{12} = 0$, i.e. for symmetric connections with $c_2 = c_3$, equations (4) are structurally similar to the equations of the micropolar elasticity theory. If $C_{12} \neq 0$, the equations (4) contain additional components determined by chirality.

The chiral lattice possesses some interesting properties. For example, when a concentrated force acts on an ordinary lattice, the deformation field is symmetric with respect to the axis of the force action. In the case of a chiral medium, the axis of the force application is displaced and the deformation field is not symmetric with respect to this axis. The load is transmitted in the lateral direction from the force action axis. Moreover, the particles of the chiral lattice deviate in different directions, depending on the type of force acting - compressive or tensile.

Acknowledgements
The research was carried out under the financial support of the Russian Foundation for Basic Research (project N 16-08-00971-a) and the Ministry of Education and Science of Russian Federation within the framework of the basic part of State Work for scientific activity (Work No. 9.7446.2017/8.9).

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