

Numerical study on the influence of primary suspension damping upon the dynamic behaviour of railway vehicles

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Abstract. The paper deals with a numerical study on the dynamic behaviour of the railway vehicle while running on a track with vertical irregularities in a particular case of a damper failure in the primary suspension. The study is based on model of rigid-flexible coupled vehicle, with seven degrees of freedom, where the carbody is modelled as an Euler-Bernoulli type equivalent beam. The dynamic behaviour of the vehicle is evaluated based on the dynamic response of the carbody bogie, expressed as the power spectral density of the vertical acceleration, and on the root mean square of the acceleration for various cases of reduction in the damping constant of the primary suspension in an axle, compared with the reference value. The results of the numerical simulations highlight the significant increase in the level of bogie vibration at decreasing the damping of the primary suspension of an axle. In the carbody, the level of vibrations is less influenced, mainly at its centre.

1. Introduction

The railway vehicle is a complex oscillating system, with specific vibration characteristics. The major cause of the vibrations in these vehicles is to be found in the phenomena of interaction between the vehicle and the track [1 - 3]. In its architecture, the rolling track has a set of deviations from the ideal geometric shape, on the one hand, and also defects of the rolling surfaces of the rails, on the other hand. These two can be added to the construction discontinuities of the rail, thus all three leading to preeminent causes of vibration in the vehicle [4]. In the wheel, flaws such as eccentricity, ovality, polygonization, flat rolling surface or flattening are also reasons for the vibrations in the vehicle.

The suspension system plays a decisive role in regards to the capability of the vehicle to provide ride quality and safety, comfort of the passengers and to keep the goods being transported in good condition [5 - 8]. Similarly, the suspension assists with the reduction in the dynamic load to which the carrying structure of the vehicle is subjected to, on the one hand, and the track [9], on the other hand.

The suspension damages generally lead to decrease or increase of the nominal parameters' values - damping and stiffness coefficients [10]. Any deviation from nominal characteristics of suspension influence vehicle's dynamic behaviour or, in extreme cases, it can jeopardize the security of passengers.

The paper deals with a numerical study on the dynamic behaviour of the railway vehicle while running on a track with vertical irregularities in a particular case of a damper failure in the primary suspension. As a rule, the vertical bounce and pitch vibrations of the bogie are considered decoupled by cause of the use of elastic and damping components with identical characteristics in the primary suspension of each axle in a bogie and to the fact that the bogie is symmetrical from geometry and



mass perspectives, compared to the vertical-transversal plan. Nevertheless, there is a weak coupling between the bounce and pitch of the bogie via the carbody bending vibration, a phenomenon neglected due to a practical reason.

Following the reduction of damping derived from the damper failure in the suspension of one of the two axles, an imbalance occurs in the system that results in dynamic interferences between the bounce and pitch vibrations of the bogie [11]. The level of vibrations in the bogie will increase, thus also triggering an amplification of the behaviour of vibrations in the vehicle carbody, mainly in the bending resonance frequency. The results from the numerical simulations will prove the above. The applications of numerical simulation are developed in Matlab, based on a rigid-flexible coupled vehicle model, in which the carbody is modelled as a free-free equivalent beam of Euler-Bernoulli type, with a constant section and uniformly distributed mass.

The dynamic behaviour of the vehicle is evaluated based on the dynamic response of the carbody bogie, expressed as the power spectral density of the vertical acceleration and on the root mean square of the acceleration for various cases of reduction in the damping constant of the primary suspension in an axle, compared with the reference value. Similarly, the root mean square of the bogie vertical acceleration and of the carbody is calculated for a wide velocity range. As a principle, the results herein shown prove the significant increase in the level of vibrations in the bogie along with the damping decrease in a primary suspension of an axle, mainly at the resonance frequencies of the bounce and pitch in the bogie. In the carbody, the level of vibrations amplifies at its centre and it has a lower value for the bending resonance frequency. Such results are further visible in the values of the root mean square of the vertical acceleration, where these values have a considerable rise in the bogie and less in the carbody.

2. Railway vehicle mechanical model

To study the dynamic behaviour of a railway vehicle at damper failure in the primary suspension, this is considered to travel at a constant velocity V on a track, deemed perfectly rigid with vertical irregularities. The track irregularities are described against each axle by the functions η_j , with $j = 1 \dots 4$.

The vehicle is represented by a rigid-flexible coupled model (see fig 1) that includes a body with parameters distributed for the carbody and six rigid bodies representing the suspended masses of the bogies and the four axles, connected among them by Kelvin-Voigt type systems, by which the two suspension stages are modelled.

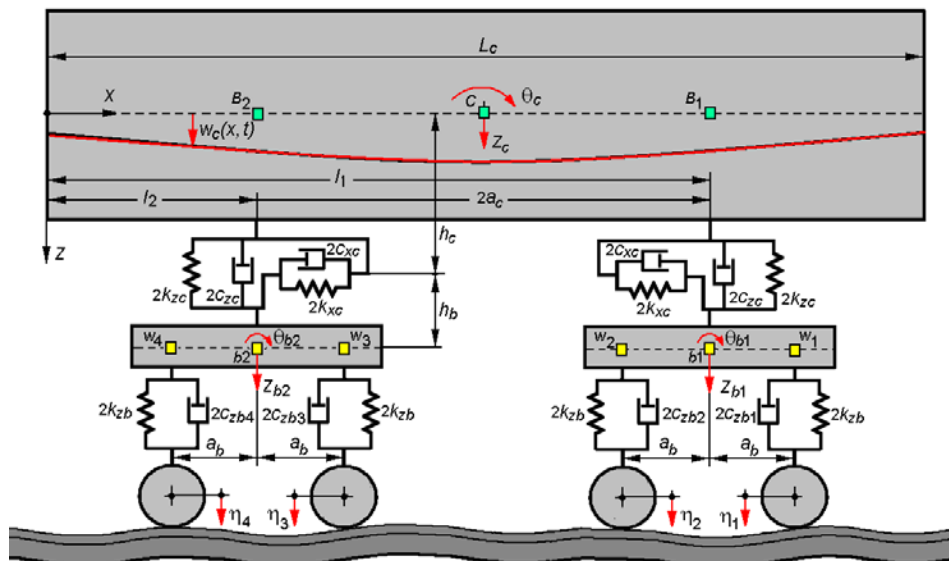


Figure 1. The mechanical model of the railway vehicle.

The modelling of the secondary suspension corresponding to a bogie is carried out by a Kelvin-Voigt system, for the vertical translation. The primary suspension for an axle is also modelled via a Kelvin-Voigt system operating on the translation in the vertical direction. The longitudinal traction system between the carbody and bogie is modelled via a Kelvin-Voigt system operating on the translation in the longitudinal direction. The parameters of the vehicle model are found in table 1.

Table 1. The parameters of the vehicle model.

Symbol	Definition	Symbol	Definition
m_c	carbody mass	$2c_{zc}$	vertical damping of the secondary suspension (per bogie)
m_b	bogie suspended mass	$2k_{zc}$	vertical stiffness of the secondary suspension (per bogie)
J_c	carbody inertia moment	$2c_{xc}$	damping of the longitudinal traction system between the carbody and bogie
J_b	bogie inertia moment	$2k_{xc}$	stiffness of the longitudinal traction system between the carbody and bogie
L_c	carbody length	$2c_{zb1\dots4}^*$	vertical damping of the primary suspension (per axle); $^* 2c_{zb1} = 2c_{zb2} = 2c_{zb3} = 2c_{zb4}$ should no damper is faulty
$2a_c$	carbody wheelbase	$2k_{zb}$	vertical stiffness of the primary suspension (per axle)
$2a_b$	bogie wheelbase	h_c / h_b	levels in the longitudinal traction system between the carbody and bogie compared to the carbody medium fiber of the carbody / the centre of mass of the bogie

The vehicle carbody is represented by a free-free equivalent beam, a constant section and uniformly distributed mass, of Euler-Bernoulli type. The beam parameters are defined in dependence on the carbody's, namely: L_c – beam length; $\rho_c = m_c/L_c$ – beam mass per length unit; μ – structural damping coefficient; EI – bending modulus, where E stands for the longitudinal modulus of elasticity, and I is the inertia moment of the beam's transversal section.

The carbody rigid vibration modes – bounce z_c and pitch θ_c , and the first carbody flexible eigenmode in a vertical plan – vertical bending, will be taken into account.

The carbody displacement $w_c(x, t)$ is the result of overlapping between the rigid modes of vibration – bounce z_c and pitch θ_c and the first natural mode of carbody vertical bending. This writes as:

$$w_c(x, t) = z_c(t) + \left(x - \frac{L_c}{2}\right)\theta_c(t) + X_c(x)T_c(t), \quad (1)$$

where $T_c(t)$ is the coordinate of the carbody vertical bending and $X_c(x)$ represents the natural function of this vibration mode, described in the equation:

$$X_c(x) = \sin \beta x + \sinh \beta x - \frac{\sin \beta L_c - \sinh \beta L_c}{\cos \beta L_c - \cosh \beta L_c} (\cos \beta x + \cosh \beta x) \quad (2)$$

$$\text{with} \quad \beta = \sqrt[4]{\omega_c^2 \rho_c / (EI)} \quad (3)$$

$$\text{and} \quad \cos \beta L_c \cosh \beta L_c - 1 = 0, \quad (4)$$

where ω_c is the natural pulsation of the carbody bending.

For each bogie, a two mode of vibration is considered, namely the bounce z_{bi} and pitch θ_{bi} , with $i = 1, 2$.

According to figure 1, three reference carbody points are defined, located along the carbody medium fibre, such as C – at the carbody centre, $B_{1,2}$ – above the bogies, against the leaning points of the carbody on the secondary suspension. Similarly, three reference points are defined for each bogie, situated along the axis going through the bogie's centre of mass - against the suspension corresponding to each axle, noted with $w_{1...4}$, and at the centre of the bogie, noted with $b_{1,2}$.

3. Vehicle motion equations

The motion equation of the carbody vehicle has the general form:

$$EI \frac{\partial^4 w_c(x,t)}{\partial x^4} + \mu I \frac{\partial^5 w_c(x,t)}{\partial x^4 \partial t} + \rho_c \frac{\partial^2 w_c(x,t)}{\partial t^2} = \sum_{i=1}^2 F_{zci} \delta(x-l_i) + \sum_{i=1}^2 h_c F_{xci} \frac{d\delta(x-l_i)}{dx} \quad (5)$$

where $\delta(\cdot)$ is Dirac delta function, distances l_i (for $i = 1, 2$) fix the supporting points position of the carbody on the secondary suspension; F_{zci} și F_{xci} stand for the forces due to the secondary suspension of bogie i ,

$$F_{zci} = -2c_{zc} \left(\frac{\partial w_c(l_i, t)}{\partial t} - \dot{z}_{bi} \right) - 2k_{zc} [w_c(l_i, t) - z_{bi}] \quad (6)$$

$$F_{xci} = 2c_{xc} \left(h_c \frac{\partial^2 w_c(l_i, t)}{\partial x \partial t} + h_b \dot{\theta}_{bi} \right) + 2k_{xc} \left(h_c \frac{\partial w_c(l_i, t)}{\partial x} + h_b \theta_{bi} \right). \quad (7)$$

By applying the modal analysis method and considering the orthogonality property of the eigenfunction in the carbody vertical bending, the equation of motion (5) turns into three two-order differential equations with ordinary derivatives, which describe the movements of bounce, pitch and bending in the carbody:

$$m_c \ddot{z}_c = \sum_{i=1}^2 F_{zci} \quad (8)$$

$$J_c \ddot{\theta}_c = \sum_{i=1}^2 F_{zci} \left(l_i - \frac{L_c}{2} \right) - \sum_{i=1}^2 h_c F_{xci} \quad (9)$$

$$m_{mc} \ddot{T}_c + c_{mc} \dot{T}_c + k_{mc} T_c = \sum_{i=1}^2 F_{zci} X_c(l_i) - \sum_{i=1}^2 h_c F_{xci} \frac{dX_c(l_i)}{dx} \quad (10)$$

where k_{mc} , c_{mc} and m_{mc} are stiffness, damping and the carbody modal mass are given in the below equations

$$k_{mc} = EI \int_0^L \left(\frac{d^2 X_c}{dx^2} \right)^2 dx \quad c_{mc} = \mu I \int_0^L \left(\frac{d^2 X_c}{dx^2} \right)^2 dx \quad m_{mc} = \rho_c \int_0^L X_c^2 dx. \quad (11)$$

If equations (6) and (7) are replaced in the carbody movement equations and based on the symmetry properties of the eigenfunction $X_c(x)$, the following notations are introduced:

$$X_c(l_1) = X_c(l_2) = \varepsilon \quad (12)$$

$$\frac{dX_c(l_1)}{dx} = -\frac{dX_c(l_2)}{dx} = \lambda \quad (13)$$

and hence written:

$$m_c \ddot{z}_c + 2c_{zc}[2\dot{z}_c + 2\varepsilon \dot{T}_c - (\dot{z}_{b1} + \dot{z}_{b2})] + 2k_{zc}[2z_c + 2\varepsilon T_c - (z_{b1} + z_{b2})] = 0 \quad (14)$$

$$J_c \ddot{\theta}_c + 2c_{zc}a_c[2a_c\dot{\theta}_c - (\dot{z}_{b1} - \dot{z}_{b2})] + 2k_{zc}a_c[2a_c\theta_c - (z_{b1} - z_{b2})] + 2c_{xc}h_c[2h_c\dot{\theta}_c + h_b(\dot{\theta}_{b1} + \dot{\theta}_{b2})] + 2k_{xc}h_c[2h_c\theta_c + h_b(\theta_{b1} + \theta_{b2})] = 0 \quad (15)$$

$$m_{mc} \ddot{T}_c + c_{mc} \dot{T}_c + k_{mc} T_c + 2c_{zc}\varepsilon[2\dot{z}_c + 2\varepsilon \dot{T}_c - (\dot{z}_{b1} + \dot{z}_{b2})] + 2k_{zc}\varepsilon[2z_c + 2\varepsilon T_c - (z_{b1} + z_{b2})] + 2c_{xc}h_c\lambda[2h_c\lambda\dot{T}_c + h_b(\dot{\theta}_{b1} - \dot{\theta}_{b2})] + 2k_{xc}h_c\lambda[2h_c\lambda T_c + h_{b2}(\theta_{b1} - \theta_{b2})] = 0. \quad (16)$$

The equations describing the bounce and pitch motions of the bogies are:

$$m_b \ddot{z}_{b1} = \sum_{j=1}^2 F_{zbj} - F_{zc1}; \quad m_b \ddot{z}_{b2} = \sum_{j=3}^4 F_{zbj} - F_{zc2} \quad (17)$$

$$J_b \ddot{\theta}_{b1} = a_b \sum_{j=1}^2 (-1)^{j+1} F_{zbj} - h_b F_{xc1}; \quad J_b \ddot{\theta}_{b2} = a_b \sum_{j=3}^4 (-1)^{j+1} F_{zbj} - h_b F_{xc2} \quad (18)$$

where F_{zbj} represent the forces due to the primary suspension corresponding to the axles j ,

$$F_{zb1,2} = -2c_{zb1,2}(\dot{z}_{b1} \pm a_b \dot{\theta}_{b1} - \dot{\eta}_{1,2}) - 2k_{zb}(z_{b1} \pm a_b \theta_{b1} - \eta_{1,2}), \text{ for } j = 1, 2 \quad (19)$$

$$F_{zb3,4} = -2c_{zb3,4}(\dot{z}_b \pm a_b \dot{\theta}_b - \dot{\eta}_{3,4}) - 2k_{zb}(z_{b2} \pm a_b \theta_{b2} - \eta_{3,4}), \text{ for } j = 3, 4. \quad (20)$$

After processing, the equations (17) and (18) are as such:

$$m_b \ddot{z}_{b1} + 2c_{zb1}(\dot{z}_{b1} + a_b \dot{\theta}_{b1} - \dot{\eta}_1) + 2c_{zb2}(\dot{z}_{b1} - a_b \dot{\theta}_{b1} - \dot{\eta}_2) + 2k_{zb}[2z_{b1} - (\eta_1 + \eta_2)] + 2c_{zc}(\dot{z}_{b1} - \dot{z}_c - a_c \dot{\theta}_c - \varepsilon \dot{T}_c) + 2k_{zc}(z_{b1} - z_c - a_c \theta_c - \varepsilon T_c) = 0 \quad (21)$$

$$m_b \ddot{z}_{b2} + 2c_{zb3}(\dot{z}_{b2} + a_b \dot{\theta}_{b1} - \dot{\eta}_3) + 2c_{zb4}(\dot{z}_{b2} - a_b \dot{\theta}_{b2} - \dot{\eta}_4) + 2k_{zb}[2z_{b2} - (\eta_3 + \eta_4)] + 2c_{zc}(\dot{z}_{b2} - \dot{z}_c + a_c \dot{\theta}_c - \varepsilon \dot{T}_c) + 2k_{zc}(z_{b2} - z_c + a_c \theta_c - \varepsilon T_c) = 0 \quad (22)$$

$$J_b \ddot{\theta}_{b1} + 2a_b c_{zb1}(\dot{z}_{b1} + a_b \dot{\theta}_{b1} - \dot{\eta}_1) - 2a_b c_{zb2}(\dot{z}_{b1} - a_b \dot{\theta}_{b1} - \dot{\eta}_2) + 2a_b k_{zb}[2a_b \theta_{b1} - (\eta_1 - \eta_2)] + 2c_{xc}h_b[h_b \dot{\theta}_{b1} + h_c(\dot{\theta}_c + \lambda \dot{T}_c)] + 2k_{xc}h_b[h_b \theta_{b1} + h_c(\theta_c + \lambda T_c)] = 0 \quad (23)$$

$$J_b \ddot{\theta}_{b2} + 2a_b c_{zb3}(\dot{z}_{b2} + a_b \dot{\theta}_{b2} - \dot{\eta}_3) - 2a_b c_{zb4}(\dot{z}_{b2} - a_b \dot{\theta}_{b2} - \dot{\eta}_4) + 2a_b k_{zb}[2a_b \theta_{b2} - (\eta_3 - \eta_4)] + 2c_{xc}h_b[h_b \dot{\theta}_{b2} + h_c(\dot{\theta}_c - \lambda \dot{T}_c)] + 2k_{xc}h_b[h_b \theta_{b2} + h_c(\theta_c - \lambda T_c)] = 0 \quad (24)$$

The bounce and pitch vibrations of each bogie are coupled for $2c_{zb1} \neq 2c_{zb2}$ and $2c_{zb3} \neq 2c_{zb4}$. In case there is no faulty damper in the primary suspension of the vehicle ($2c_{zb1} = 2c_{zb2} = 2c_{zb3} = 2c_{zb4}$), a weak coupling remains between the bounce and pitch vibrations of each bogie; the bounce and pitch vibrations of the bogie are coupled via the bending vibration of the carbody (see. eq. 25 - 28):

$$m_b \ddot{z}_{b1} + 2c_{zb}[2\dot{z}_{b1} - (\dot{\eta}_1 + \dot{\eta}_2)] + 2k_{zb}[2z_{b1} - (\eta_1 + \eta_2)] + 2c_{zc}(\dot{z}_{b1} - \dot{z}_c - a_c \dot{\theta}_c - \varepsilon \dot{T}_c) + 2k_{zc}(z_{b1} - z_c - a_c \theta_c - \varepsilon T_c) = 0 \quad (25)$$

$$m_b \ddot{z}_{b2} + 2c_{zb}[2\dot{z}_{b2} - (\dot{\eta}_3 + \dot{\eta}_4)] + 2k_{zb}[2z_{b2} - (\eta_3 + \eta_4)] +$$

$$+ 2c_{zc}(\dot{z}_{b2} - \dot{z}_c + a_c \dot{\theta}_c - \varepsilon \dot{T}_c) + 2k_{zc}(z_{b2} - z_c + a_c \theta_c - \varepsilon T_c) = 0 \quad (26)$$

$$J_b \ddot{\theta}_{b1} + 2c_{zb}a_b[2a_b \dot{\theta}_{b1} - (\dot{\eta}_1 - \dot{\eta}_2)] + 2k_{zb}a_b[2a_b \theta_{b1} - (\eta_1 - \eta_2)] +$$

$$+ 2c_{xc}h_b[h_b \dot{\theta}_{b1} + h_c(\dot{\theta}_c + \lambda \dot{T}_c)] + 2k_{xc}h_b[h_b \theta_{b1} + h_c(\theta_c + \lambda T_c)] = 0 \quad (27)$$

$$J_b \ddot{\theta}_{b2} + 2c_{zb}a_b[2a_b \dot{\theta}_{b2} - (\dot{\eta}_3 - \dot{\eta}_4)] + 2k_{zb}a_b[2a_b \theta_{b2} - (\eta_3 - \eta_4)] +$$

$$+ 2c_{xc}h_b[h_b \dot{\theta}_{b2} + h_c(\dot{\theta}_c - \lambda \dot{T}_c)] + 2k_{xc}h_b[h_b \theta_{b2} + h_c(\theta_c - \lambda T_c)] = 0 \quad (28)$$

Hence results a 7-equation system with ordinary derivatives (eq. 14-16 and 21-24), written in a matrix-like form as:

$$\mathbf{M}\ddot{\mathbf{p}} + \mathbf{C}\dot{\mathbf{p}} + \mathbf{K}\mathbf{p} = \mathbf{P}\dot{\mathbf{\eta}} + \mathbf{R}\mathbf{\eta} \quad (29)$$

where M, C and K are the inertia, damping and stiffness matrices, and P, R are the track displacement and velocity input matrices. The motion equations can be solved numerically using compiled MATLAB codes.

4. The calculation of the root mean square of the vertical acceleration

To calculate the root mean square, the frequency response functions that are specific to the harmonic permanent vibration behaviour should be first defined, based on which the frequency response functions that correspond to the random behaviour of vibrations and the power spectral density of the acceleration are then calculated. These are obtained from the power spectral density of the vertical irregularities in the track and from the frequency response functions of the vehicle. Further on, the root mean square of the acceleration is calculated based on the power spectral density of the acceleration, which can help with the assessment of the dynamic behaviour of the railway vehicle.

4.1. The frequency response functions in a permanent harmonic behaviour of vibration

To calculate the frequency response functions, the vertical irregularities of the track are considered to be in a harmonic shape with the wavelength Λ and amplitude η_0 that are outphased against the axles corresponding to the distances between them, $2a_c$ and $2a_b$, respectively. Therefore, the functions η_j describing the irregularities of the track against the four axles are in the form of:

$$\eta_{1,2}(x) = \eta_0 \cos \frac{2\pi}{\Lambda}(x + a_c \pm a_b) \quad \eta_{3,4}(x) = \eta_0 \cos \frac{2\pi}{\Lambda}(x - a_c \pm a_b) \quad (30)$$

where $x = Vt$ is the coordinate for the carbody centre.

The functions η_j can also be written as time harmonic functions, namely:

$$\eta_{1,2}(t) = \eta_0 \cos \omega \left(t + \frac{a_c \pm a_b}{V} \right) \quad \eta_{3,4}(t) = \eta_0 \cos \omega \left(t - \frac{a_c \mp a_b}{V} \right) \quad (31)$$

where $\omega = 2\pi V/\Lambda$ represents the means the angular frequency induced by the track excitation.

As for the vehicle response, this is assumed to be harmonic, with the same frequency as the track excitation induced frequency. The coordinates describing the motions of the vehicle are written under the general form as:

$$p_k(t) = P_k \cos(\omega t + \varphi_k) \quad \text{for } k = 1 \div 7 \quad (32)$$

where P_k is the amplitude, and φ_k represents the phase of the coordinate k compared to the track vertical irregularities with respect to the vehicle centre.

The complex units associated with real ones are introduced in the system of equations (29):

$$\bar{\eta}_{j,j+1}(t) = \bar{\eta}_{j,j+1} e^{i\omega t} \quad \text{for } j = 2i - 1 \text{ and } i = 1, 2 \quad (33)$$

$$\bar{p}_k(t) = \bar{p}_k e^{i\omega t}. \quad (34)$$

A lineary system of non-homogeneous algebraic equations is obtained, whose solution allows the calculation of the frequency response functions of the vehicle.

The acceleration response function at the carbody centre is:

$$\bar{H}_{cm}(\omega) = \bar{H}_c\left(\frac{L_c}{2}, \omega\right) = \omega^2 \left[\bar{H}_{zc}(\omega) + X_c\left(\frac{L_c}{2}\right) \bar{H}_{Tc}(\omega) \right] \quad (35)$$

and above the two bogies

$$\bar{H}_{cb1,2}(\omega) = \bar{H}_c(l_{1,2}, \omega) = \omega^2 \left[\bar{H}_{zc}(\omega) \pm a_c \bar{H}_{\theta_c}(\omega) + X_c(l_{1,2}) \bar{H}_{Tc}(\omega) \right] \quad (36)$$

where $\bar{H}_{zc}(\omega)$, $\bar{H}_{\theta_c}(\omega)$, $\bar{H}_{Tc}(\omega)$ are the response functions corresponding to the rigid, bounce and pitch modes of vibration (z_c și θ_c) and to the vertical bending of the carbody (T_c).

Similarly, the response functions of the bogie acceleration in the centre are:

$$\bar{H}_{bm1,2}(\omega) = \omega^2 \bar{H}_{zb1,2}(\omega) \quad (37)$$

and against the axles

$$\bar{H}_{bw1,2}(\omega) = \omega^2 [\bar{H}_{zb1}(\omega) \pm a_b \bar{H}_{\theta_{b1}}(\omega)] \quad \bar{H}_{bw3,4}(\omega) = \omega^2 [\bar{H}_{zb2}(\omega) \pm a_b \bar{H}_{\theta_{b2}}(\omega)] \quad (38)$$

in which $\bar{H}_{zb1,2}(\omega)$ and $\bar{H}_{\theta_{b1,2}}(\omega)$ are the response functions that correlate to the bounce and pitch of the two bogies.

4.2. The power spectral density of the vertical acceleration

To calculate the power spectral density of the vertical acceleration, it deems that the track vertical irregularities are considered to represent a stationary stochastic process, which can be described via the power spectral density given by the relation

$$S(\Omega) = \frac{A\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} \quad (39)$$

where Ω is the wavelength, $\Omega_c = 0.8246$ rad/m, $\Omega_r = 0.0206$ rad/m, and A is a coefficient depending on the track quality. For a high level quality track, $A = 4.032 \cdot 10^{-7}$ radm, whereas for a low level quality, the coefficient A is $1.080 \cdot 10^{-6}$ radm [12].

As a function of the angular frequency $\omega = V\Omega$, the power spectral density of the track irregularities can be written as in the general relation

$$G(\omega) = \frac{S(\omega/V)}{V}. \quad (40)$$

What results is the power spectral density of the track irregularities in the form of

$$G(\omega) = \frac{A\Omega_c^2 V^3}{[\omega^2 + (V\Omega_c)^2][\omega^2 + (V\Omega_r)^2]}. \quad (41)$$

To calculate the power spectral density of the vertical acceleration at the centre of the carbody, the starting point is the response function of the acceleration in the equation (35) and the power spectral density of the track irregularities in equation (41), such as

$$G_{cm}(\omega) = G(\omega) \left| \bar{H}_{cm}(\omega) \right|^2. \quad (42)$$

The same pattern is followed by the power spectral density of the carbody acceleration above the two bogies,

$$G_{cb1,2}(\omega) = G(\omega) \left| \bar{H}_{cb1,2}(\omega) \right|^2. \quad (43)$$

The calculation of the power spectral density of the vertical acceleration at the centre of the bogie and against the axles comes from the relations below,

$$G_{bm1,2}(\omega) = G(\omega) \left| \bar{H}_{bm1,2}(\omega) \right|^2 \quad (44)$$

$$G_{bw1,2}(\omega) = G(\omega) \left| \bar{H}_{bw1,2}(\omega) \right|^2 \quad G_{bw3,4}(\omega) = G(\omega) \left| \bar{H}_{bw3,4}(\omega) \right|^2. \quad (45)$$

4.3. The root mean square of the vertical acceleration

The root mean square of the vertical acceleration in the carbody reference points is calculated based on the dynamic response of the carbody, expressed in the form of the power spectral density of the acceleration, as follows

- at the carbody centre,
$$a_{cm} = \sqrt{\frac{1}{\pi} \int_0^{\infty} G_{cm}(\omega) d\omega} \quad (46)$$

- above the bogies,
$$a_{cb1,2} = \sqrt{\frac{1}{\pi} \int_0^{\infty} G_{cb1,2}(\omega) d\omega}. \quad (47)$$

The root mean square of the vertical acceleration at the centre of the bogies and against the axles comes from:

$$a_{bm} = \sqrt{\frac{1}{\pi} \int_0^{\infty} G_{bm}(\omega) d\omega} \quad (48)$$

$$a_{bw1,2} = \sqrt{\frac{1}{\pi} \int_0^{\infty} G_{bw1,2}(\omega) d\omega} \quad a_{bw3,4} = \sqrt{\frac{1}{\pi} \int_0^{\infty} G_{bw3,4}(\omega) d\omega}. \quad (49)$$

Both the power spectral density of the acceleration and the root mean square of the acceleration will be used in the next section so as to evaluate the dynamic behaviour of the vehicle.

5. The results of the numerical simulations

This section features the results of the numerical simulation regarding the influence of the damping in the primary suspension on the dynamic behaviour of the vehicle. To this purpose, different cases of reducing the damping constant of the suspension of axle 1 compared with the reference value are taken into account, due to the damper failure. There will be calculation of the power spectral density of acceleration and the root mean square of acceleration in the front bogie and the carbody, in the reference points defined in section 2 (see figure 1), while considering that the vehicle is running on a

low-quality track ($A=1.080 \cdot 10^{-6}$ radm). The parameters of the vehicle used in the numerical simulations are introduced in the table 2. The natural frequencies of the vibration modes in the carbody and bogies, corresponding to the parameters in table 2, are featured in table 3.

Table 2. The parameters of the vehicle numerical model.

$m_c = 34.0 \cdot 10^3$ kg	$k_{zc} = 0.60$ MN/m
$m_b = 3.20 \cdot 10^3$ kg	$k_{xc} = 2.00$ MN/m
$J_c = 1.96 \cdot 10^6$ kg·m ²	$c_{zc} = 17.15$ kNs/m
$J_b = 2.05 \cdot 10^3$ kg·m ²	$c_{xc} = 25.0$ kNs/m
$EI = 3.16 \cdot 10^9$ Nm ²	$k_{mc} = 89.0$ MN/m
$m_{mc} = 35.2 \cdot 10^3$ kg	$c_{mc} = 53.1$ kNm/s
$L_c = 26.4$ m	$k_{zb} = 1.10$ MN/m
$2a_c = 19.0$ m; $2a_b = 2.56$ m	$c_{zb} = 13.05$ kNs/m
$h_c = 1.30$ m	$h_b = 0.20$ m

Table 3. The natural frequencies of the vehicle vibration modes.

Vibration mode	Frequency
Carbody bounce	1.17 Hz
Carbody pitch	1.46 Hz
Carbody bending	8.20 Hz
Bogie bounce	6.61 Hz
Bogie pitch	9.63 Hz

Figure 2 shows the power spectral density of the vertical acceleration in the reference points of the front bogie – at the centre and above the two axles, for the velocity of 200 km/h and various values of the damping constant in the suspension of the front axle. The amplification of the behaviour of vibrations in the bogie at the resonance frequencies of bounce and pitch in the bogie is highlighted on while c_{zb1} is reduced. At the centre of the bogie, the level of vibrations rises both at the resonance frequency of the bogie bounce (6.61 Hz) and at the resonance frequency of the bogie pitch (9.63 Hz). Above the axles, the significant increase in the level of vibrations in the bogie is visible at the pitch resonance frequency.

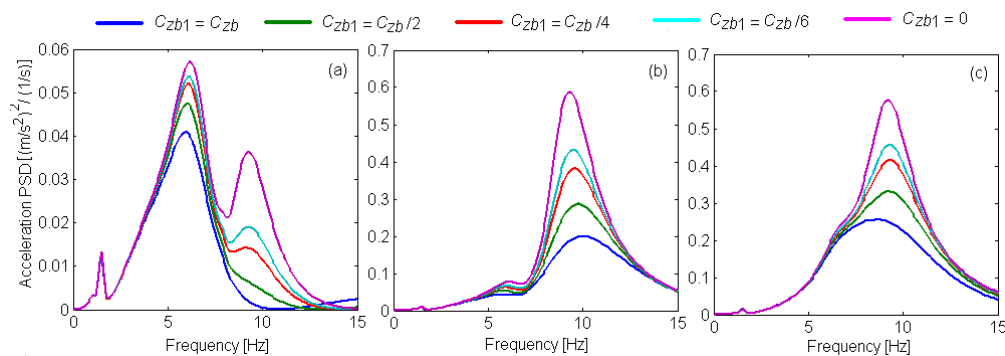


Figure 2. Acceleration power spectral density:
(a) at the bogie centre; (b) above the wheelset 1; (c) above the wheelset 2.

The diagrams in figure 3, showing the power spectral density of the vertical acceleration in the carbody reference points – at the centre and above the two bogies – validate that the damping of the

primary suspension has a little influence upon the level of vibrations in the carbody. An explanation lies in the fact that the bogie mass interposes itself between the primary suspension and the carbody, and its inertia reduces the impact of the damping in the primary suspension. While the damping constant of the suspension in axle 1 decreases, the level of vibrations only goes up at the carbody centre at the carbody bending frequency, as a result of the amplification in the bogie pitch.

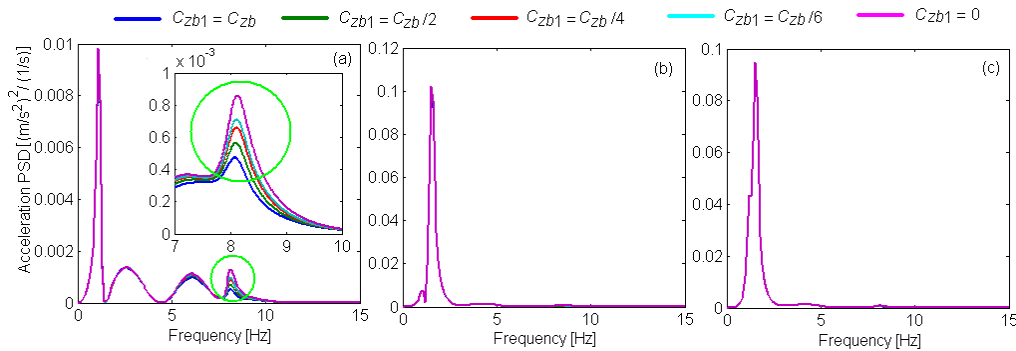


Figure 3. Acceleration power spectral density:
(a) at the carbody centre; (b) above the front bogie; (c) above the rear bogie.

Figure 4 features the root mean square of the vertical acceleration at the bogie centre and against the two axles for velocities ranging from 60 to 200 km/h and various values of the damping constant in the suspension of the front axle. It can be firstly noticed that the acceleration has a uniform increase along with the velocity. Secondly, the reduction in the damping constant c_{zb1} is seen to trigger a general surge of the level of vibrations in the bogie; the acceleration rises in all the reference points of the bogies. At velocity 200 km/h, a_{b1} goes up by 27.43%, a_{bw1} by 35.50%, and a_{bw2} by 25.89%, should the damping constant goes down from the reference value ($c_{zb1}=c_{zb}=13.05$ kNs/m) to $c_{zb1}=0$.

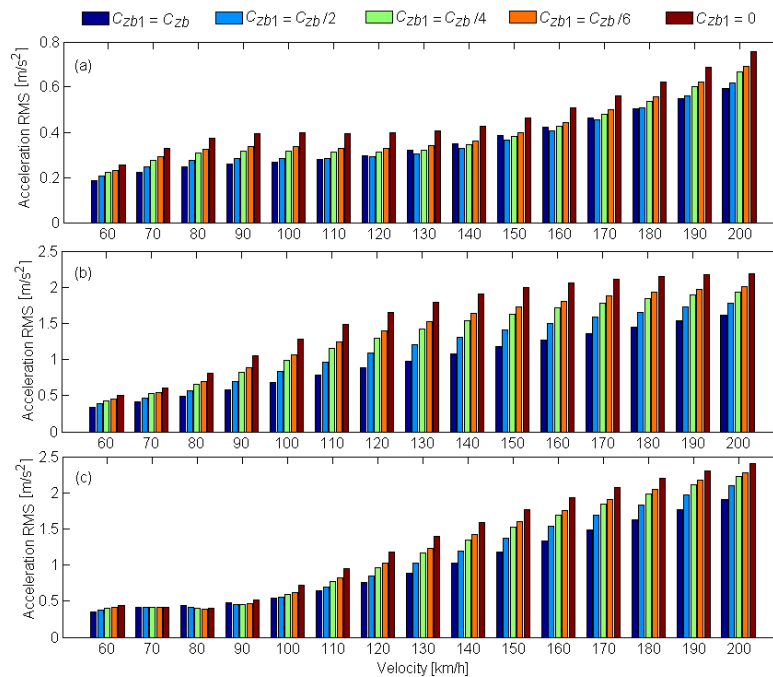


Figure 4. Root mean square of the vertical acceleration:
(a) at the bogie centre; (b) above the wheelset 1; (c) above the wheelset 2.

As expected, the root mean square of the carbody acceleration is less affected at the fault of a damper in the primary suspension of the vehicle, since the damping in the primary suspension has a lower influence on the carbody behaviour of vibrations. The above is visible in the results in figure 5. For example, a_{cm} rises by cca. 5%, a_{cb1} by cca. 3%, and a_{cb2} by cca. 1.5%, at the velocity of 200 km/h when the damping constant lowers from the reference value ($c_{zb1}=13.05$ kNs/m) to $c_{zb1}=0$. It can also be noticed that the acceleration increase in the carbody is not uniform, unlike the bogie acceleration that rises uniformly along with the velocity. This is a consequence of the geometric filtering effect given by the wheelbases. The geometric filtering effect is an essential feature of the behavior of vertical vibrations in the railway vehicles, mainly due to the manner in which the excitations coming from the track are conveyed to the suspended masses via the axles, irrespective of the suspension characteristics. Practically speaking, the geometric filtering effect is the result of the displacement between the vertical movements in the axles coming from moving on a track with irregularities, a displacement that derives from the axle position in the assembly of the running gear and the vehicle velocity. This fact gives the geometric filtering a selective nature, depending on the vehicle wheelbases and on velocity and, to that end, a differentiated efficiency along the vehicle carbody on the one hand and the movement behavior, on the other hand [13, 14].

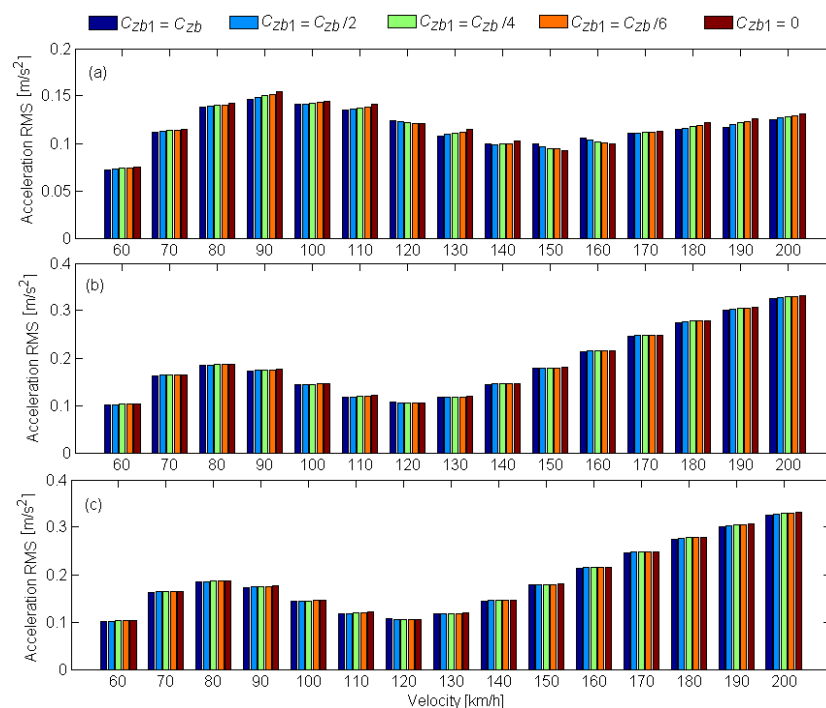


Figure 5. Root mean square of the vertical acceleration: (a) at the carbody centre; (b) above the front bogie; (c) above the rear bogie.

6. Conclusions

The paper deals with a numerical study based on which the dynamic behaviour of the railway vehicle while running on a track with vertical irregularities is evaluated, in a particular case of a damper failure in the primary suspension. The failure of the damper in the primary suspension of an axle is simulated via various degrees of reducing the damping constant, versus the reference value.

The study relies on the results from numerical simulations developed on a rigid-flexible vehicle model. The former set of results represents the dynamic response of the bogie and of the carbody's in

the form of the power spectral density of the vertical acceleration calculated in the reference points of the bogie – at the centre and against the two axles and of the carbody's – at the centre and against the two bogies. The reduction in the damping constant of the primary suspension in an axle has been said to have an effect as a significant amplification of the behaviour of vibrations in all the reference points of the bogie, at the resonance frequencies of bounce and pitch in the bogie. For the carbody, the amplification of the behaviour of vibrations is reduced and only manifests itself in the reference point at its centre, at the resonance frequency of bending. The latter set of results shows an increase in the root mean square of the vertical acceleration in the reference points of the bogie, which can reach up to circa 35% at the velocity of 200 km/h, during the total failure of the damper – simulated by the damping constant that is equal with zero. In the carbody, the root mean square of the acceleration exhibits a much lower increase at the velocity of 200 km/h, which varies from 1.5% in the reference point above the rear bogie to 5% in the reference point at its centre.

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