

Kinematic analysis of a new 6RSS parallel manipulator performing a certain working operation

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Abstract. This paper presents the kinematic analysis of a new type of 6RSS parallel manipulator with six degrees of freedom. The parallel manipulator consists of a fixed plate and a mobile platform connected through six independent kinematic chains. Each of the six kinematic chains has an actuator revolute joint and two spherical joint. The kinematic analysis was done by applying the restriction method. The motion equations for the six kinematic chains of the parallel mechanism have been defined when it performs a certain technological function. Also kinematic analysis was done using a CAD software and the variation of the angular speeds of the actuating arms were obtained.

1. Introduction

Over the last decade - as a result of the increasing interest in industrial robots - a number of research has been undertaken on the kinematic problems of parallel mechanisms. The attention paid to parallel mechanisms is due to their constructive features that appear to be less susceptible to input errors than their serial counterparts [1].

An important advantage of parallel manipulators, is that superior structural stiffness makes them preferable to serial machines when handling heavy loads or performing high-precision machining [2-5]. Parallel mechanisms also have a better distribution of inertia and are capable of performing accurate and fast movements.

These characteristic make parallel mechanisms find their applicability in various fields: flight simulators and fine positioning devices, high-speed packing machines and milling machines [6,7]. In general, the end goal of any robotic application is to achieve a certain technological function, a first requirement being the exact positioning of the end effector at a point or on a certain trajectory. The study of the parallel mechanisms aims at determining the variable parameters associated to each of their joints so that the coordinates of the effector element verify a given point in the operating space, while ensuring a certain orientation of it. In this way, relations that define kinematic transformations become kinematic control equations.

The kinematic analysis of the parallel mechanisms performs the description of the variation of the scalar parameters of the displacement - between the initial and the final position - in time, without taking into account the forces that intervene during the movement.

Many of researchers approach the kinematics of parallel mechanisms through the reverse kinematic Jacobian. Various solutions for modeling the inverse Jacobian used in expressing speeds are presented in the literature. Thus, three kinematic modeling methods have been identified [8]:



- the vectorial method by which articulated velocities are established by applying vector velocities for velocities [9];

- the kinematic screw method by which the kinematic model is obtained based on the vector transformations applied to the Plücker coordinates of a straight line in space [10-14];

- the partial derivation method which initially presupposes the identification of characteristic geometrical relations for the analyzed parallel structure and their partial derivation in relation to the independent parameters. The obtained model is expressed by the matrix relation $[A] \cdot d\mathbf{q} = [B] \cdot d\mathbf{X}$ where $d\mathbf{q}/dt$ represents the joint speeds and $d\mathbf{X}/dt$ is the operational speeds. In this method, expression of the direct Jacobian involves the inversion of the matrix $[B]$, having the order equal to the number of independent modeling parameters [15].

2. The kinematic model of the 6RSS parallel manipulator

In this paper is presented the kinematic analysis of a parallel manipulator with six degree of freedom (DOF), type RSS (one revolute joint and two spherical joint, Figure 1).

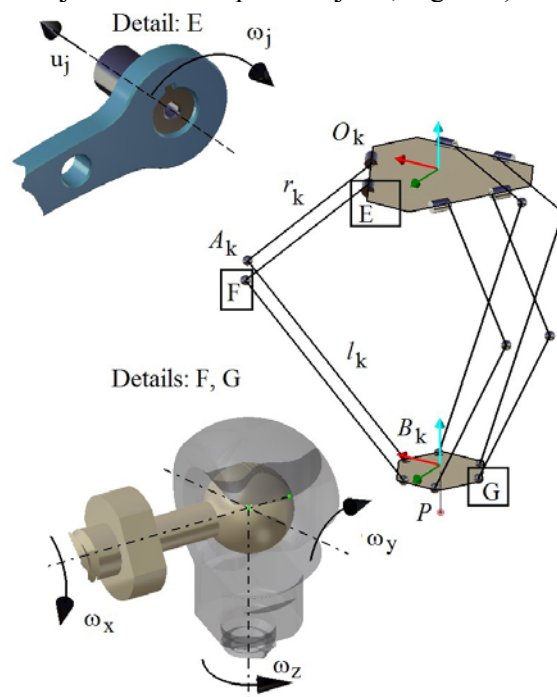


Figure 1. Schematic representation of mechanism 6 RSS.

The 6RSS parallel manipulator has a fixed plate (on which the six drive actuators are mounted) and a mobile platform on which is mounted a seventh servomotor acting a milling tool. One of the six kinematic chains k of the 6RSS parallel manipulator has a kinematic revolute joint in O_k point and a kinematic spherical joint at the A_k respective B_k points. The drive arms $O_k A_k$ are rotated with an angle θ around the axis by unit vector \mathbf{u}_k passing through the O_k point. In Figure 2 is represented the kinematic scheme of one of its six kinematic k chains. The mechanism is characterized by the following features:

- the axis of rotational active joints by unit vector \mathbf{u}_k that generates circular trajectories are coplanar, two coincident and disposed central-symmetrically;
- the actuating arms r_k are of equal length;
- the rods l_k are of equal length.

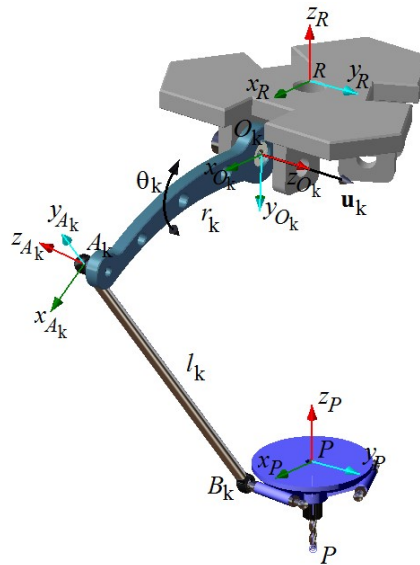


Figure 2. The kinematic scheme of one branch.

Starting from the kinematic scheme presented above, we will define the motion equations for the six kinematic chains of the parallel mechanism when the characteristic point P follows a spatial curve (C) resulting from the intersection of two cylinders with radius $r_1 = 200$ mm and the center at point $Q_1(200, -100, -500)$, respectively $r_2 = 75$ mm and the center at point $Q_2(0, 0, -500)$ shown in Figure 3.

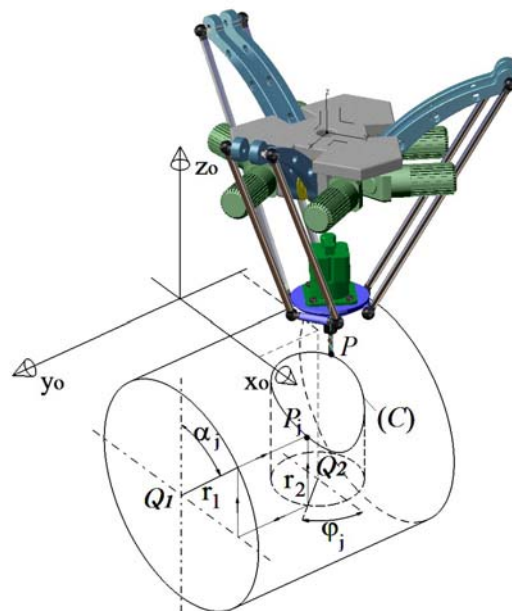


Figure 3. Generated spatial curve.

2.1 Application of the restriction method to determinate the kinematic model

The particularity of the mechanism under consideration is that it has in its structure six parallel and identical geometric actuator arms. This allows us to apply the simplest equation of the restriction

method, the constant length condition. This equation makes direct connection between the kinematic parameters of the motor arms and the platform.

Coordinates on the three directions of points A_k result geometrically as θ_k functions. The position of the platform is determined by the position P and the angle α (rotation of the mobile platform relative to the cylinder axis with radius r_1). The coordinates on the three directions of points B_k in the fixed system are given by the expression:

$$\begin{bmatrix} x_{B_k} \\ y_{B_k} \\ z_{B_k} \\ 1 \end{bmatrix}_{(O)} = \begin{bmatrix} 1 & 0 & 0 & x_P \\ 0 & c\alpha & -s\alpha & y_P \\ 0 & s\alpha & c\alpha & z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{B_k} \\ y_{B_k} \\ z_{B_k} \\ 1 \end{bmatrix}_{(P)} \quad (1)$$

in witch:

$$\cos(\alpha) = c\alpha;$$

$$\sin(\alpha) = s\alpha;$$

The coordinates of points B_k in the local system are also determined geometrically (Figure 4) and are presented in the table 1:

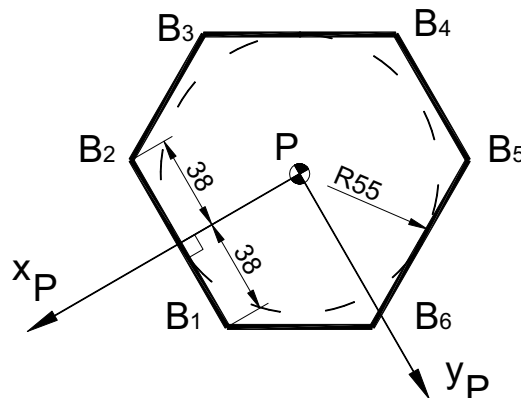


Figure 4. Position of the B_k points of the mobile platform.

Table 1. Cartesian coordinates of B_k points in the local system.

B_k	B_1	B_2	B_3	B_4	B_5	B_6
$x_{B_k(P)}$	55.00	55.00	5.40	-60.40	-60.40	5.40
$y_{B_k(P)}$	-38.00	38.00	66.63	28.63	-28.63	-66.63
$z_{B_k(P)}$	76.00	76.00	76.00	76.00	76.00	76.00

We will begin to write the motion equations for one of the kinematic chains, for the other chains the calculations are analogous.

$$S = \begin{cases} \mathbf{v}_{A_1} = \boldsymbol{\omega}_1 \times \mathbf{O}_1 \mathbf{A}_1 \\ \mathbf{v}_{B_1} = \mathbf{v}_P + \boldsymbol{\omega}_P \times \mathbf{P} \mathbf{B}_1 \end{cases} \quad (2)$$

It is known that:

$$\boldsymbol{\omega}_1 = \omega_1 \cdot \mathbf{u}_{12} \quad (3)$$

From relations (2) and (3), results:

$$S = \begin{cases} \mathbf{v}_{A_1} = \omega_1 \cdot \mathbf{u}_{12} \times \mathbf{O}_1 \mathbf{A}_1 \\ \mathbf{v}_{B_1} = \mathbf{v}_P + \omega_P \times \mathbf{PB}_1 \end{cases} \quad (4)$$

It is known that given a line (Δ) and two points A and B located on it, the projections of the velocity of the two points are equal. The relationship between these two velocity is:

$$\mathbf{v}_B \cdot \mathbf{AB} = \mathbf{v}_A \cdot \mathbf{AB} \rightarrow (\mathbf{v}_B - \mathbf{v}_A) \cdot \mathbf{AB} = 0 \quad (5)$$

The relationship (5) written for points A_1 and B_1 in view of the relationship (4) becomes:

$$(\mathbf{v}_P + \omega_P \times \mathbf{PB}_1 - \omega_1 \cdot \mathbf{u}_{12} \times \mathbf{O}_1 \mathbf{A}_1) \cdot \mathbf{AB} = 0 \quad (6)$$

We make the following notations for simplifying computing:

$$\mathbf{v} = \mathbf{v}_P = v_{P_x} \cdot \mathbf{i} + v_{P_y} \cdot \mathbf{j} + v_{P_z} \cdot \mathbf{k} \quad (7)$$

in witch:

$$v_{P_x} = -r_2 \cdot \varphi_j \cdot \sin(\varphi_j) \quad (8)$$

$$v_{P_y} = r_2 \cdot \varphi_j \cdot \cos(\varphi_j) \quad (9)$$

$$v_{P_z} = \varphi_j \cdot \frac{[r_2 \cdot \cos(\varphi_j)] \cdot [y_{Q_2} - y_{Q_1} + r_2 \cdot \sin(\varphi_j)]}{\left(r_1^2 - [y_{Q_2} - y_{Q_1} + r_2 \cdot \sin(\varphi_j)]^2\right)^{1/2}} \quad (10)$$

where φ_j is the center angle determined by the displacement of the point P on the radius circle r_2 and Q_1 and Q_2 are the centers of the two circles mentioned above.

Are known the expressions for the vectors ω_P and \mathbf{PB}_1 :

$$\omega_P = \omega_{P_x} \cdot \mathbf{i} + \omega_{P_y} \cdot \mathbf{j} + \omega_{P_z} \cdot \mathbf{k} \quad (11)$$

$$\mathbf{PB}_1 = (x_{B_1} - x_P) \cdot \mathbf{i} + (y_{B_1} - y_P) \cdot \mathbf{j} + (z_{B_1} - z_P) \cdot \mathbf{k} \quad (12)$$

We determine the components in the three directions of the points P_j , with the relations below:

$$x_{P_j} = x_{Q_2} + r_2 \cdot \cos(\varphi_j) \quad (13)$$

$$y_{P_j} = y_{Q_2} + r_2 \cdot \sin(\varphi_j) \quad (14)$$

$$z_{P_j} = z_{Q_1} + \left(r_1^2 - (y_{P_j} - y_{Q_1})^2\right)^{1/2} \quad (15)$$

The cross product for the vectors ω_P and \mathbf{PB}_1 is:

$$\mathbf{W} = \omega_P \times \mathbf{PB}_1 = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_{P_x} & \omega_{P_y} & \omega_{P_z} \\ x_{B_1} - x_P & y_{B_1} - y_P & z_{B_1} - z_P \end{bmatrix} \quad (16)$$

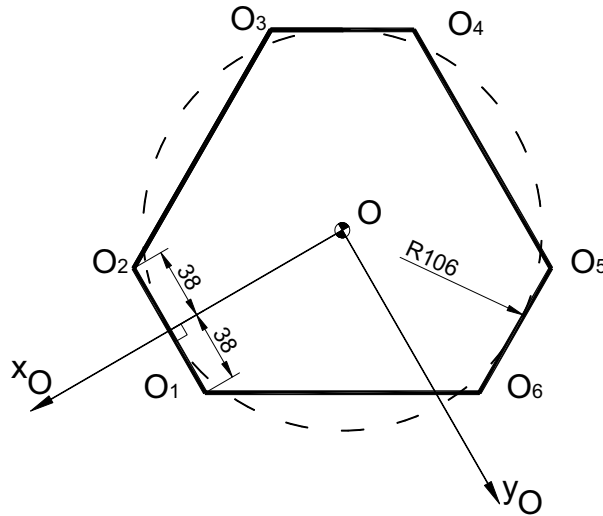
$$\mathbf{m} = \mathbf{AB} = (x_{B_1} - x_{A_1}) \cdot \mathbf{i} + (y_{B_1} - y_{A_1}) \cdot \mathbf{j} + (z_{B_1} - z_{A_1}) \cdot \mathbf{k} \quad (17)$$

Are known the expressions for the vectors \mathbf{u}_{12} and $\mathbf{O}_1 \mathbf{A}_1$:

$$\mathbf{u}_{12} = u_{12_x} \cdot \mathbf{i} + u_{12_y} \cdot \mathbf{j} + u_{12_z} \cdot \mathbf{k} \quad (18)$$

$$\mathbf{O}_1 \mathbf{A}_1 = (x_{A_1} - x_{Q_1}) \cdot \mathbf{i} + (y_{A_1} - y_{Q_1}) \cdot \mathbf{j} + (z_{A_1} - z_{Q_1}) \cdot \mathbf{k} \quad (19)$$

The coordinates of the O_k points of the centers of the revolute joint expressed in the base reference system are determined geometrically (Figure 5) and are represented in table 2.

**Figure 5.** Position of the centers of the revolute joint O_k .**Table 2.** Coordinates of the O_k points in the global system.

O_k	O_1	O_2	O_3	O_4	O_5	O_6
x_{O_k}	106.00	106.00	-20.09	-85.91	-85.91	-20.09
y_{O_k}	-38.00	38.00	110.80	72.80	-72.80	-110.80
z_{O_k}	0.00	0.00	0.00	0.00	0.00	0.00

The components in the three directions of the unit vector \mathbf{u}_{12} are:

$$\begin{aligned} u_{12_x} &= 0 \\ u_{12_y} &= 1 \\ u_{12_z} &= 0 \end{aligned} \quad (20)$$

With expressions (18) and (19) it obtained the cross product for the vectors \mathbf{u}_{12} and $\mathbf{O}_1\mathbf{A}_1$:

$$\mathbf{U} = \mathbf{u}_{12} \times \mathbf{O}_1\mathbf{A}_1 = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_{12_x} & u_{12_y} & u_{12_z} \\ x_{A_1} - x_{O_1} & y_{A_1} - y_{O_1} & z_{A_1} - z_{O_1} \end{bmatrix} \quad (21)$$

By using the above simplification notations and replacing them in relation (6) it becomes:

$$(\mathbf{v} + \mathbf{W} - \omega_1 \cdot \mathbf{U}) \cdot \mathbf{m} = 0 \quad (22)$$

By notation:

$$\mathbf{n} = \mathbf{v} + \mathbf{W} - \omega_1 \cdot \mathbf{U} \quad (23)$$

expression (22) becomes:

$$\mathbf{n} \cdot \mathbf{m} = 0 \quad (24)$$

From the expression of the scalar product of the two vectors, the relationship (24) becomes

$$\mathbf{n} \cdot \mathbf{m} = n_x \cdot m_x + n_y \cdot m_y + n_z \cdot m_z = 0 \quad (25)$$

Next we will determine the components of the above scalar product in which we know:

$$\mathbf{W}_x = \begin{bmatrix} \omega_{P_y} & \omega_{P_z} \\ y_{B_1} - y_P & z_{B_1} - z_P \end{bmatrix} = \omega_{P_y} (z_{B_1} - z_P) - \omega_{P_z} (y_{B_1} - y_P) \quad (26)$$

$$\mathbf{U}_x = \begin{bmatrix} u_{12_y} & u_{12_z} \\ y_{A_1} - y_{O_1} & z_{A_1} - z_{O_1} \end{bmatrix} = u_{12_y} (z_{A_1} - z_{O_1}) - u_{12_z} (y_{A_1} - y_{O_1}) \quad (27)$$

$$n_x \cdot m_x = \left\{ v_{P_x} + \omega_{P_y} (z_{B_1} - z_P) - \omega_{P_z} (y_{B_1} - y_P) - \omega_1 [u_{12_y} (z_{A_1} - z_{O_1}) - u_{12_z} (y_{A_1} - y_{O_1})] \right\} \cdot (x_{B_1} - x_{A_1}) \quad (28)$$

Analogously the components are determined on y and z:

$$\mathbf{W}_y = \begin{bmatrix} \omega_{P_z} & \omega_{P_x} \\ z_{B_1} - z_P & x_{B_1} - x_P \end{bmatrix} = \omega_{P_z} (x_{B_1} - x_P) - \omega_{P_x} (z_{B_1} - z_P) \quad (29)$$

$$\mathbf{U}_y = \begin{bmatrix} u_{12_z} & u_{12_x} \\ z_{A_1} - z_{O_1} & x_{A_1} - x_{O_1} \end{bmatrix} = u_{12_z} (x_{A_1} - x_{O_1}) - u_{12_x} (z_{A_1} - z_{O_1}) \quad (30)$$

$$n_y \cdot m_y = \left\{ v_{P_y} + \omega_{P_z} (x_{B_1} - x_P) - \omega_{P_x} (z_{B_1} - z_P) - \omega_1 [u_{12_z} (x_{A_1} - x_{O_1}) - u_{12_x} (z_{A_1} - z_{O_1})] \right\} \cdot (y_{A_1} - y_{O_1}) \quad (31)$$

$$\mathbf{W}_z = \begin{bmatrix} \omega_{P_x} & \omega_{P_y} \\ x_{B_1} - x_P & y_{B_1} - y_P \end{bmatrix} = \omega_{P_x} (y_{B_1} - y_P) - \omega_{P_y} (x_{B_1} - x_P) \quad (32)$$

$$\mathbf{U}_z = \begin{bmatrix} u_{12_x} & u_{12_y} \\ x_{A_1} - x_{O_1} & y_{A_1} - y_{O_1} \end{bmatrix} = u_{12_x} (y_{A_1} - y_{O_1}) - u_{12_y} (x_{A_1} - x_{O_1}) \quad (33)$$

$$n_z \cdot m_z = \left\{ v_{P_z} + \omega_{P_x} (y_{B_1} - y_P) - \omega_{P_y} (x_{B_1} - x_P) - \omega_1 [u_{12_x} (y_{A_1} - y_{O_1}) - u_{12_y} (x_{A_1} - x_{O_1})] \right\} \cdot (z_{B_1} - z_{A_1}) \quad (34)$$

Substituting (28), (31) and (34) into equation (25) and passing the terms with ω_1 on the right side results:

$$\begin{aligned} & (x_{B_1} - x_{A_1}) v_{P_x} + (y_{A_1} - y_{O_1}) v_{P_y} + (z_{B_1} - z_{A_1}) v_{P_z} \\ & + \left[(y_{B_1} - y_P) (z_{B_1} - z_{A_1}) - (z_{B_1} - z_P) (y_{A_1} - y_{O_1}) \right] \omega_{P_x} \\ & - \left[(x_{B_1} - x_P) (z_{B_1} - z_{A_1}) + (z_{B_1} - z_P) (x_{B_1} - x_{A_1}) \right] \omega_{P_y} \\ & - \left[(y_{B_1} - y_P) (x_{B_1} - x_{A_1}) - (x_{B_1} - x_P) (y_{A_1} - y_{O_1}) \right] \omega_{P_z} \\ & = \left\{ [u_{12_y} (z_{A_1} - z_{O_1}) - u_{12_z} (y_{A_1} - y_{O_1})] (x_{B_1} - x_{A_1}) \right. \\ & + [u_{12_z} (x_{A_1} - x_{O_1}) - u_{12_x} (z_{A_1} - z_{O_1})] (y_{A_1} - y_{O_1}) \\ & \left. + [u_{12_x} (y_{A_1} - y_{O_1}) - u_{12_y} (x_{A_1} - x_{O_1})] (z_{B_1} - z_{A_1}) \right\} \omega_1 \end{aligned} \quad (35)$$

Thus, for each cinematic chain we obtain equations of the form:

$$\begin{aligned} f_1(v_{P_x}, v_{P_y}, v_{P_z}, \omega_{P_x}, \omega_{P_y}, \omega_{P_z}) &= g_1(\omega_1) \\ &\vdots \\ f_6(v_{P_x}, v_{P_y}, v_{P_z}, \omega_{P_x}, \omega_{P_y}, \omega_{P_z}) &= g_6(\omega_1) \end{aligned} \quad (36)$$

If for the equation (35) are done the notations:

$$\begin{aligned} (x_{B_1} - x_{A_1}) &= a_1; \\ (y_{A_1} - y_{O_1}) &= b_1; \\ (z_{B_1} - z_{A_1}) &= c_1; \end{aligned}$$

$$\begin{aligned}
& \left[(y_{B_1} - y_P)(z_{B_1} - z_{A_1}) - (z_{B_1} - z_P)(y_{A_1} - y_{O_1}) \right] = s_1; \\
& \left[(x_{B_1} - x_P)(z_{B_1} - z_{A_1}) + (z_{B_1} - z_P)(x_{B_1} - x_{A_1}) \right] = r_1; \\
& \left[(y_{B_1} - y_P)(x_{B_1} - x_{A_1}) - (x_{B_1} - x_P)(y_{A_1} - y_{O_1}) \right] = t_1; \\
& \left\{ \left[u_{12_y} (z_{A_1} - z_{O_1}) - u_{12_z} (y_{A_1} - y_{O_1}) \right] (x_{B_1} - x_{A_1}) \right. \\
& \quad \left. + \left[u_{12_x} (x_{A_1} - x_{O_1}) - u_{12_z} (z_{A_1} - z_{O_1}) \right] (y_{A_1} - y_{O_1}) \right. \\
& \quad \left. + \left[u_{12_x} (y_{A_1} - y_{O_1}) - u_{12_y} (x_{A_1} - x_{O_1}) \right] (z_{B_1} - z_{A_1}) \right\} = q_1.
\end{aligned}$$

equation (35) becomes:

$$a_1 \cdot v_{P_x} + b_1 \cdot v_{P_y} + c_1 \cdot v_{P_z} + s_1 \cdot \omega_{P_x} + r_1 \cdot \omega_{P_y} + t_1 \cdot \omega_{P_z} = q_1 \cdot \omega_1 \quad (37)$$

For the six kinematic chains it result a system of six equations of form (37) whose matricial representation is:

$$\begin{bmatrix} q_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_6 \end{bmatrix} \cdot \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \\ \omega_6 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 & s_1 & r_1 & t_1 \\ a_2 & b_2 & c_2 & s_2 & r_2 & t_2 \\ a_3 & b_3 & c_3 & s_3 & r_3 & t_3 \\ a_4 & b_4 & c_4 & s_4 & r_4 & t_4 \\ a_5 & b_5 & c_5 & s_5 & r_5 & t_5 \\ a_6 & b_6 & c_6 & s_6 & r_6 & t_6 \end{bmatrix} \cdot \begin{bmatrix} v_{P_x} \\ v_{P_y} \\ v_{P_z} \\ \omega_{P_x} \\ \omega_{P_y} \\ \omega_{P_z} \end{bmatrix} \quad (38)$$

The relationship (38) written in compact form becomes:

$$[\mathbf{B}] \cdot [\mathbf{q}] = [\mathbf{A}] \cdot [\boldsymbol{\tau}] \quad (39)$$

in which:

- $[\mathbf{B}]$ and $[\mathbf{A}]$ represents the direct respectively inverse kinematic Jacobian matrix of the robot;
- $[\dot{\mathbf{q}}]^T = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & \dot{q}_5 & \dot{q}_6 \end{bmatrix}$ represents the generalized velocity vector;
- $[\boldsymbol{\tau}]^T = [v_{P_x} \ v_{P_y} \ v_{P_z} \ \omega_{P_x} \ \omega_{P_y} \ \omega_{P_z}]$ represents the velocity vector of the mobile platform.

To determine the components of the vector $\boldsymbol{\omega}_p$, it is considered the particularity of the application that the platform has a rotation motion only after α components on the other two directions being null. Was done the following notations:

$$\Delta_{x_j} = x_{P_{j+1}} - x_{P_j} \quad (40)$$

$$\Delta_{y_j} = y_{P_{j+1}} - y_{P_j} \quad (41)$$

$$\Delta_{z_j} = z_{P_{j+1}} - z_{P_j} \quad (42)$$

$$\Delta_{\alpha_j} = \alpha_{j+1} - \alpha_j \quad (43)$$

$$\Delta_{s_j} = P_{j+1} - P_j = \left(\Delta_{x_j}^2 + \Delta_{y_j}^2 + \Delta_{z_j}^2 \right)^{1/2} \quad (44)$$

Considering the displacement of the point on the curve with constant velocity $v_m = 10 \text{ mm/s}$ we can write the relationship:

$$\omega_{P_{xj}} = \frac{\Delta_{\alpha_j}}{\Delta_{s_j}} \cdot v_m \quad (45)$$

Based on the above-mentioned algorithm, has been create a program to determine the six angular velocities ω_j of the actuating arms $A_k B_k$ based on the inverse kinematic model, when the characteristic point P it's moving on the spatial curve mentioned above. A function $[A, B, \tau]$ returns the values of the expressions $a_k, b_k, c_k, s_k, r_k, t_k$ that form the matrix $[A]$. Also the same function returns the values of the expressions q_k that form the matrix $[B]$ and the values of the components $v_{P_x}, v_{P_y}, v_{P_z}, \omega_{P_x}, \omega_{P_y}, \omega_{P_z}$ of the matrix $[\tau]$. It has been defined in the program, a matrix $[W]$ in which the values of the angular velocities are found. Using the elements of the matrices $[A]$, $[B]$, $[\tau]$ the angular velocities of the actuating arms for the $P_j, (j=1...72)$ positions of the characteristic point on the curve were determined.

For one of the 72 positions of the characteristic point P we have six angular variations θ of different actuating arms so six different positions of the points $A_k, (k=1...6)$ belonging to the spherical joints. Also the six positions A_k determine six different positions for the points B_k belonging to the spherical joints of the mobile platform. As each position A_k and B_k is determined by three Cartesian coordinates x, y, z the matrix size $[A]$ and $[B]$ (72×18) result. We have defined a matrix $[P]$ of P points of dimension 72×3 and a matrix $[K]$ of angles α of size 72×1 . Within the computational program the length L of the curve generated by the intersection of the two cylinders was established. Was determined the length of the arc d_s between two consecutive points P_j on the curve. Also, the $[U]$ matrix of unit versor u_{12}, u_{34}, u_{56} with dimension 6×3 was defined.

Based on the obtained values $\omega_{1...6}$ for the P_j positions of the characteristic point, their variation graphs were done (Figure 6).

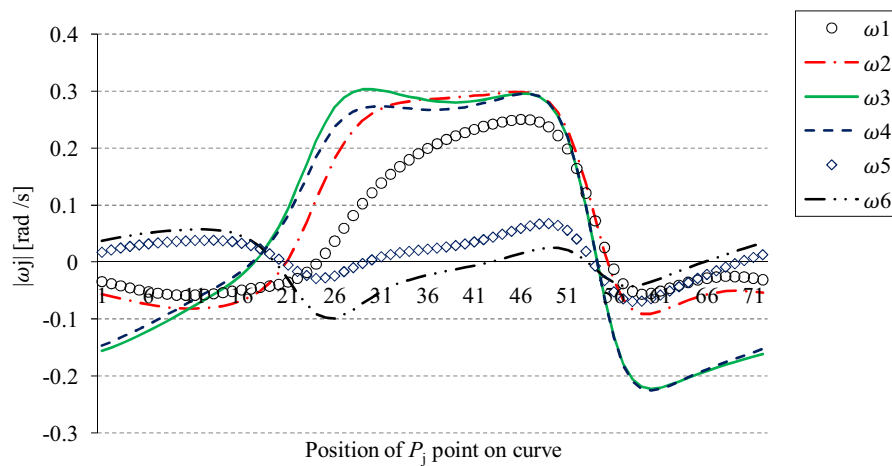


Figure 6. Variation of angular velocities $\omega_{1...6}$ for the positions P_j of the characteristic point (computational program).

2.2 Kinematic analysis for parallel mechanism 6RSS using a CAD software

For the same displacement of the characteristic point on the curve described by the intersection of the two cylinders, the angular speeds of the actuating arms were determined by using a CAD software. For this application, the space curve was divided into 72 equal arcs, the displacement being carried out at constant speed. The angular velocity of a point belonging to one of the k actuating arms of the

manipulator was analyzed. This operation was repeated for each of the six actuating arms, resulting the magnitude of angular speed variation graphs from Figure 7.

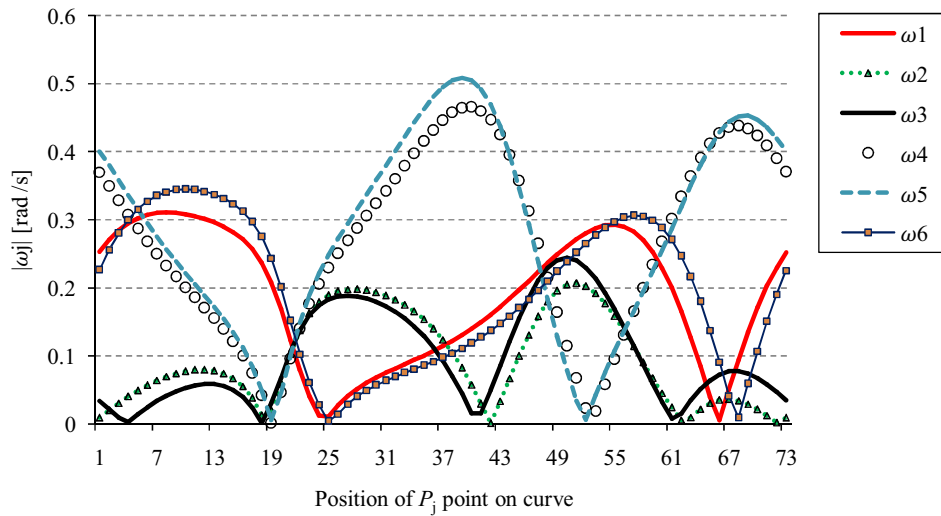


Figure 7. Variation of angular velocities $\omega_{1...6}$ for the positions P_j of the characteristic point (CAD software).

The graphical interpretation for the two applications described above is shown in Figures 8. The figure represents the difference division path between the two methods of generation.

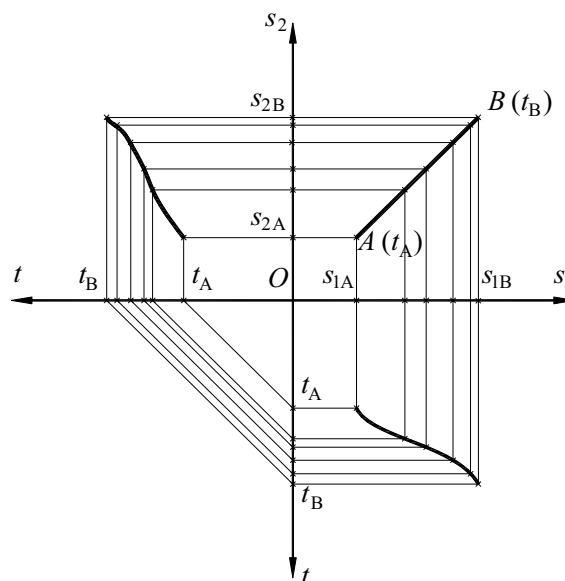


Figure 8. Graphical interpretation of the manipulator kinematics for the two applications.

Let s_1 and s_2 the curve arcs lengths traveled by the characteristic point at time t_i for the first and the second application respectively. In other words, each arc of curve corresponds to a time t_i . All points on the curve belonging the plan $[s_1, s_2]$ are obtained as an intersection of two straight lines. The curve in the upper right plane with equation $f(s_1, s_2) = 0$ is obviously a straight line when the two arcs are the same. The curve from the plane $[s_1, s_2]$ deviates more from the bisecting line of quadrant I, the larger the division differences. The points A and B on the curve represent the start and end position of the movement of the characteristic point on the curve, and t_A and t_B are the start and end times of the movement.

3. Conclusions

In the present paper two methods for determining the angular speeds of actuated joints of a six-degree parallel manipulator have been presented. The kinematic analysis for both methods was done under the condition that the manipulator performs a certain technological operation. In the first case the P points were unevenly distributed on the space curve, in the second case the points divide the curve into 72 equal arcs. Based on the two methods, the angular speeds of the drive arms were achieved.

As can be seen from Figures 6 and 7, variation of the graphs depend on the particularity of each application. Thus, in the first case when the P points were unevenly distributed on the space curve, the angular velocities of the actuating arms vary in the range $[0.3, -0.23]$ and in the second case when the points divide the curve into equal arcs 72, the angular velocities of the actuating arms vary in the range $[0.51, 0]$.

4. References

- [1] Bonev I A 2007 Are parallel robots more accurate than serial robots *Transactions of the Canadian Society for Mechanical Engineering*. **31** pp 445–455
- [2] Huang T, Li Z, Li M, Chetwynd D, Gosselin C M 2004 Conceptual design and dimensional synthesis of a novel 2DOF translational parallel robot for pick-and-place operations *ASME Journal of Mechanical Design* **126** pp 449–455
- [3] Zhang D, Gosselin C M 2002 Kinetostatic modeling of parallel mechanisms with a passive constraining leg and revolute actuators *Mechanism and Machine Theory* **37** pp 599–617
- [4] Xu Q, Li Y 2008 An investigation on mobility and stiffness of a 3-DOF translational parallel manipulator via screw theory *Robotics and Computer-Integrated Manufacturing* **24** pp 402–414
- [5] Liu S, Huang T, Mei J, et al. 2012 Optimal design of a 4-DOF SCARA type parallel robot using dynamic performance indices and angular constraints *Journal of Mechanisms and Robotics* **4** (3) p 031005
- [6] Huang Z, Cao Y 2005 Property identification of the singularity loci of a class of Gough-Stewart manipulators *The International Journal of Robotics Research* **24** pp 675–685
- [7] Pierrot F, Reynaud C, Fournier A 1990 Delta: A simple and efficient parallel robot. *Robotica* **8** pp105–109
- [8] Neagoe M, Diaconescu D, Săulescu R G 2002 O nouă abordare a modelării preciziei structurilor de tip paralel *Product Design, Robotics, Advanced Mechanical & Mechatronic Systems and Innovation Conference 2002* **1** pp 169-170
- [9] Merlet J P 1990 Les robot parallèles Ed. Hermes, Paris 1990
- [10] Glazunov A K, Rashoyan V A and Offer S 2016 Analysis of Kinematic Screws That Determine the Topology of Singular Zones of Parallel-Structure Robots *Journal of Machinery Manufacture and Reliability* **45** pp 291–296
- [11] Kong X, Gosselin C M 2004 Type synthesis of 3T1R 4-DOF parallel manipulators based on screw theory *IEEE Transactions on Robotics and Automation* **20** pp 181–190
- [12] Guo S, Fang Y, Qu H 2012 Type synthesis of 4-DOF non overconstrained parallel mechanisms based on screw theory *Robotica* **30** pp 31–37
- [13] Xu Q, Li Y 2008 An investigation on mobility and stiff-ness of a 3-DOF translational parallel manipulator via screw theory *Robotics and Computer-Integrated Manufacturing* **24** pp 402–414
- [14] Kong X, Gosselin C M 2004 Type synthesis of 3-DOF translational parallel manipulators based on screw theory *Journal of Mechanical Design* **126** pp 83–92
- [15] Glazunov V 2006 Twists of movements of parallel mechanisms inside their singularities *Mechanism and Machine Theory* **41** pp 1185–1195

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