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Grapho-analytical kinematic analysis for plane cam mechanisms and follower with finite curvature

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Abstract. The paper presents a general grapho-analytical methodology for kinematical analysis of whichever plane cam mechanism. The main constructive solutions of plane cam mechanisms are reviewed and the solutions of kinematical analysis are briefly presented. The most important difficulty in cam mechanisms analysis is the fact that when the follower has a curved face with random shape, the contact point between the cam and the follower has unknown trajectory. At the same time, the contact point moves both on the cam and on the follower contour, respectively. These displacements are required in the study of accelerations. The paper presents a method of analysis directly applicable for actual mechanism which allows finding all kinematical parameters of the mechanism. For structural reasons, there are separately approached the curved-face follower and the flat-face follower cases.

1. Introduction

The cam mechanisms are mechanisms characterized by the fact that the motion is transmitted by means of a higher pair from a driving element, the cam, to a driven element, the follower, [1-2]. According to this definition, it is obvious that the gear mechanisms are a special class of cam mechanisms. The presence of higher pair makes more complex the kinematical study of such a mechanism. To sustain this affirmation, the comparison between relative motions in lower pair and higher pair is presented. In the case of lower pair, revolute or prismatic, by fixing one of the elements any of the points from the mobile element will describe the same type of curve, circle and straight line respectively, regardless of which of the elements of the pair is fastened. Considering the pure rolling contact between a circle and a straight line, the contact point will describe different trajectories depending on which of the pair's elements is considered fixed. Thus, when the circle is fastened, a point attached to the straight line will describe an evolvent curve while when the straight line is fixed, the points attached to the mobile circle will trace a family of cycloid. From the above considerations, it is expected that the kinematical study of a mechanism with higher pairs should be more intricate than the one regarding the lower pair mechanism [3]. To this end, if for the mechanisms with lower pairs of cylindrical pair type, Hartenberg and Denavit [4] set the fundamentals of a general method for the kinematical analysis based on matrix calculus, for higher pairs mechanisms a general method for kinematical approach was not elaborated and therefore in the monographs of mechanisms theory there are specific chapters concerning the cam mechanisms and gear mechanisms, respectively, even if as principle, the two types of mechanisms belong to the class of higher pair mechanisms. For the case of plane mechanisms, an acknowledged kinematical analysis method for mechanisms with higher pairs consists in substituting these with replacing mechanisms that have in structure only lower pairs. Based on structural-kinematical considerations the replacement of the higher pair can be made. In principle, a higher pair is replaced by a kinematical element and two lower pairs.

2. Theoretical considerations

The usual plane mechanisms with rotating cam are presented in figure 1. Depending on the follower's motion and construction, they can be categorized as knife edge follower (as in figure 1a and figure1c, [5]) or flat face follower (figure 1b and figure 1d, [5]), case with the particular form of rectilinear follower which is widespread in engineering.

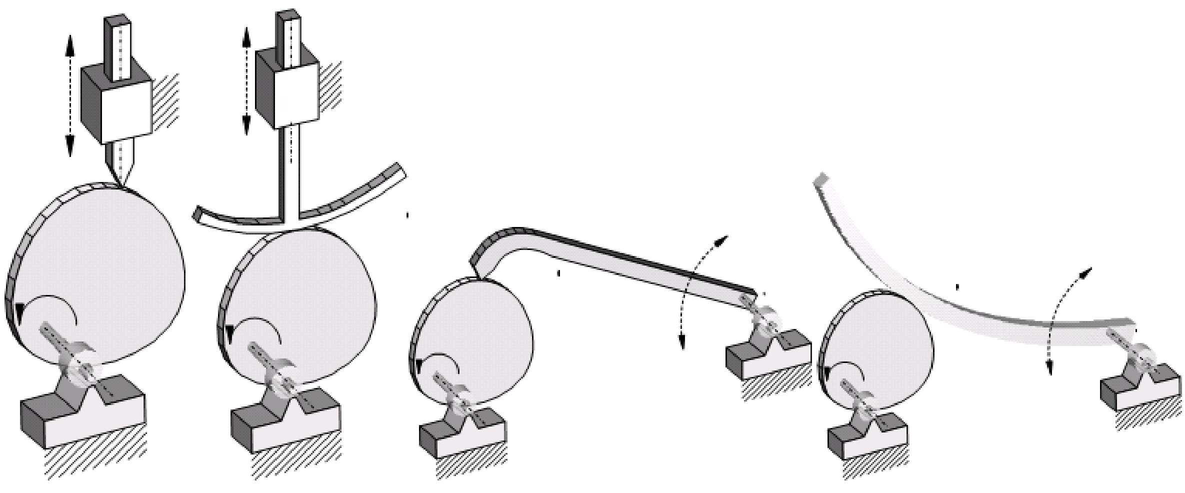

Figure 1. Usual types of plane cam mechanisms [5]

The mechanisms from figure 1 have as common characteristic the point contact between the cam and the follower. The contact point moves on the contour of the cam and it can be immobile on the follower for the knife edge follower or mobile, for the flat face follower. Thus, relations describing the variation of velocity and acceleration of a point moving on a mobile curve are required.

A mobile frame $O'x'y'$ with respect to a fixed plane Oxy is considered as shown in figure 2.

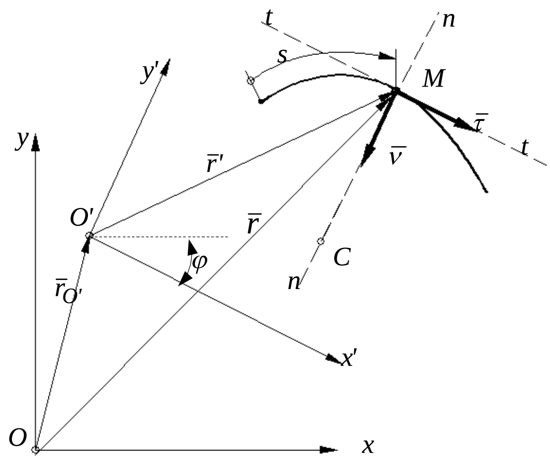

Figure 2. Motion of a point on a mobile curve

In the mobile plane a curve attached to the fix plane is considered. A point M moves on the curve and its position is set by the intrinsic coordinate s. Denoting by $\bar{r}$ the position vector of the point with respect to origin O, by $\bar{r}'$ the position vector with respect to origin of the mobile plane and by $\bar{r}_{O'}$ the position vector of the mobile origin with respect to the fixed origin, the basic relation exists:

$$\bar{r} = \bar{r}_{O'} + \bar{r}' \tag{1}$$

As in [6], by differentiating the relation (1) with respect to time once and twice respectively and conveniently arranging the terms, the absolute velocity and acceleration of point M are obtained, respectively:

$$\bar{v}_M = \frac{d\bar{r}}{dt} = \bar{v}_{O'} + \bar{\omega} \times \bar{r}' + \frac{\partial \bar{r}'}{\partial t} = [\bar{v}_{O'} + \bar{\omega} \times \bar{r}'] + \dot{s}\bar{\tau} = \bar{v}_{M'} + v_{MM'}\bar{\tau} \tag{2}$$

$$\begin{aligned}\bar{a}_M &= \frac{d^2\bar{r}}{dt^2} = \bar{a}_0 + \bar{\omega} \times (\bar{\omega} \times \bar{r}') + \bar{\varepsilon} \times \bar{r}' + 2\bar{\omega} \times \frac{\partial \bar{r}'}{\partial t} + \frac{\partial^2 \bar{r}'}{\partial t^2} = \\ &= [\bar{a}_0 + \bar{\omega} \times (\bar{\omega} \times \bar{r}') + \bar{\varepsilon} \times \bar{r}'] + 2\bar{\omega} \times \dot{s}\bar{\tau} - \frac{\dot{s}^2}{\rho}\bar{\nu} + \ddot{s}\tau = \bar{a}_{M'} + \bar{a}^c_{MM'} + \bar{a}^{t\nu}_{MM'} + \bar{a}^{\tau}_{MM'}\end{aligned} \tag{3}$$

In relation (3), $\bar{a}^c_{MM'}$ is the complementary Coriolis acceleration, $\bar{a}^{t\nu}_{MM'}$ is the normal component of relative acceleration and $\bar{a}^{\tau}_{MM'}$ is the tangential component of relative acceleration; the velocity and acceleration of mobile origin are $\bar{v}_{O'}$ and $\bar{a}_{O'}$ respectively. Considering $s = const$ in relations 3, the relationship between velocities and accelerations of two points from a rigid are found which are useful in characterising the motion of the tip of oscillating follower. In order to obtain the relations characteristic to the flat edge follower it is sufficient to consider $\rho = \infty$.

3. Illustration of method. Kinematics analysis for cam mechanisms with curved face follower

To illustrate the method, the case of mechanism with oscillating curved face follower is considered. This mechanism was chosen because in the vector equations of velocity and acceleration all terms occur.

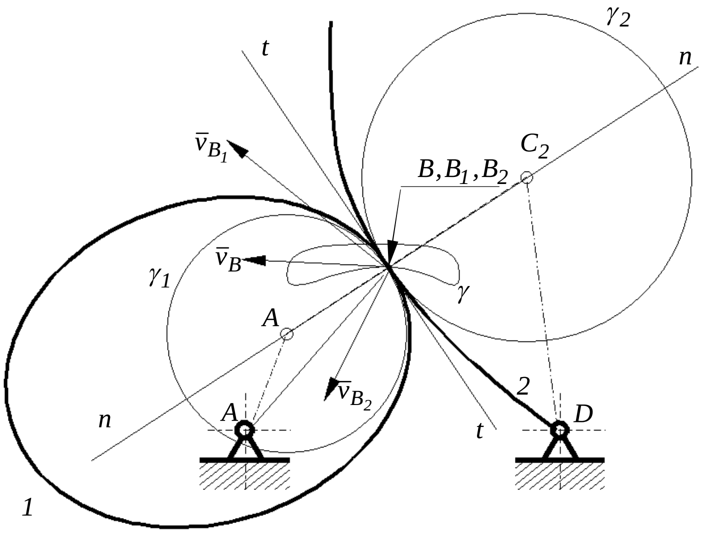

Figure 3. Mechanism with oscillating follower with curved face

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The kinematics analysis of the mechanisms presented in figure 3 assumes that for given position and stipulated motion of the cam, the motion of the follower and the motion from higher pair B must be found. In figure 3, the curvature centers of the cam and follower in the contact points are C_1 and C_2 respectively. The replacing mechanisms is AC_1C_2D, the substituting element being the rod connecting the centers of curvature having at the ends revolute pairs. In the higher pair the contact point B and the points B_1 and B_2 from the cam and follower, respectively, overlie. Supposing known position of the mechanism, [7] the relation between he velocities of the points B and B_1 on one side, and B and B_2 on the other side, according to relation (2) are written:

$$\begin{cases} \bar{v}_B = \underline{\underline{\bar{v}_{B_1}}} + \underset{||tt}{\underline{\bar{v}_{BB_1}}} \\ \bar{v}_B = \underset{\perp DB}{\underline{\bar{v}_{B_2}}} + \underset{||tt}{\underline{\bar{v}_{BB_2}}} \end{cases} \tag{4}$$

As it can be noticed, the system (4) is undetermined since by equaling the right members of the equations a plane scalar equation is obtained, with two scalar equations of projection, while the number of the unknowns is three (v_{BB_1}, v_{BB_2} and v_{B_2}). This fact is due to the unknown form of the trajectory of the contact point B. If the trajectory of the contact point is identified, as for the knife edge follower, then the direction of the velocity of point B would be known and the two equations (4), considered separately, are sufficient for finding the motion from the higher pair and the motion of the follower, respectively.

As it can be observed from figure 3, the point B is attached to the replacing element, the coupler of a four-bar mechanism, and therefore the trajectory is a coupler curve, γ. Based on the remark that the replacing mechanism AC_1C_2D is the same for any mechanism obeying the relation:

$$\rho_1 + \rho_2 = ct \tag{5}$$

Pelecudi [8] proposes that, from substituting mechanisms, the one for which the coupler curve takes particular form should be used. Specifically, Pelecudi recommends the employment of the mechanism where the cam has the curvature radius $\rho_1 + \rho_2$ and the follower has the curvature radius $\rho = 0$. Actually, from all mechanisms that have AC_1C_2D as replacing mechanism, Pelecudi chooses the mechanism with cam and knife edge follower, the follower being represented by the DC_2 segment. In this case the curve γ is reduced to an arch of a circle, trailed on both ways. The method is useful when only the motion of the follower is aimed.

In order to characterize the motion from higher pair, the equations (3) are subtracted member by member and the next relation is obtained:

$$\underset{\perp DB}{\underline{\bar{v}_{B2}}} = \underline{\underline{\bar{v}_{B_1}}} + \underset{||tt}{\underline{\bar{v}_{B_2B_1}}} \tag{6}$$

By solving the equation (6) both the motion of the follower v_{B_2} and the relative motion from higher pair $v_{B_2B_1}$ are obtained. But the motion of the contact point from the cam and the follower remains undetermined. It can be remarked that, after finding v_{B_2} any of the equations (4) contains three unknowns: the absolute velocity of the contact point, completely unknown and the velocities of the point moving on the cam $\bar{v}_{BB_1}$ and follower v_{BB_2}, respectively, parallel to the tangent tt. As seen from relation (3), these velocities are required for finding the Coriolis acceleration:

$$\bar{a}^c_{BB_{1,2}} = 2\bar{\omega}_{1,2} \times \bar{v}_{BB_{1,2}} \tag{7}$$

and the normal components of relative accelerations:

$$\bar{a}^{tv}_{BB_{1,2}} = -\frac{v^2_{BB_{1,2}}}{\rho_{1,2}}\bar{v}_{1,2} \tag{8}$$

Next, the graphical methodology for kinematical analysis of a mechanism is presented. In figure 4 is presented the mechanism with cam and curved face follower to be analyzed, for which the replacing element is the rod C_1C_2 with the immobile contact point B on it. The equation 6, written for the velocity polygon, has the form:

$$\overline{p_v b_2} = \overline{p_b b_1} + \overline{b_1 b_2} \tag{9}$$

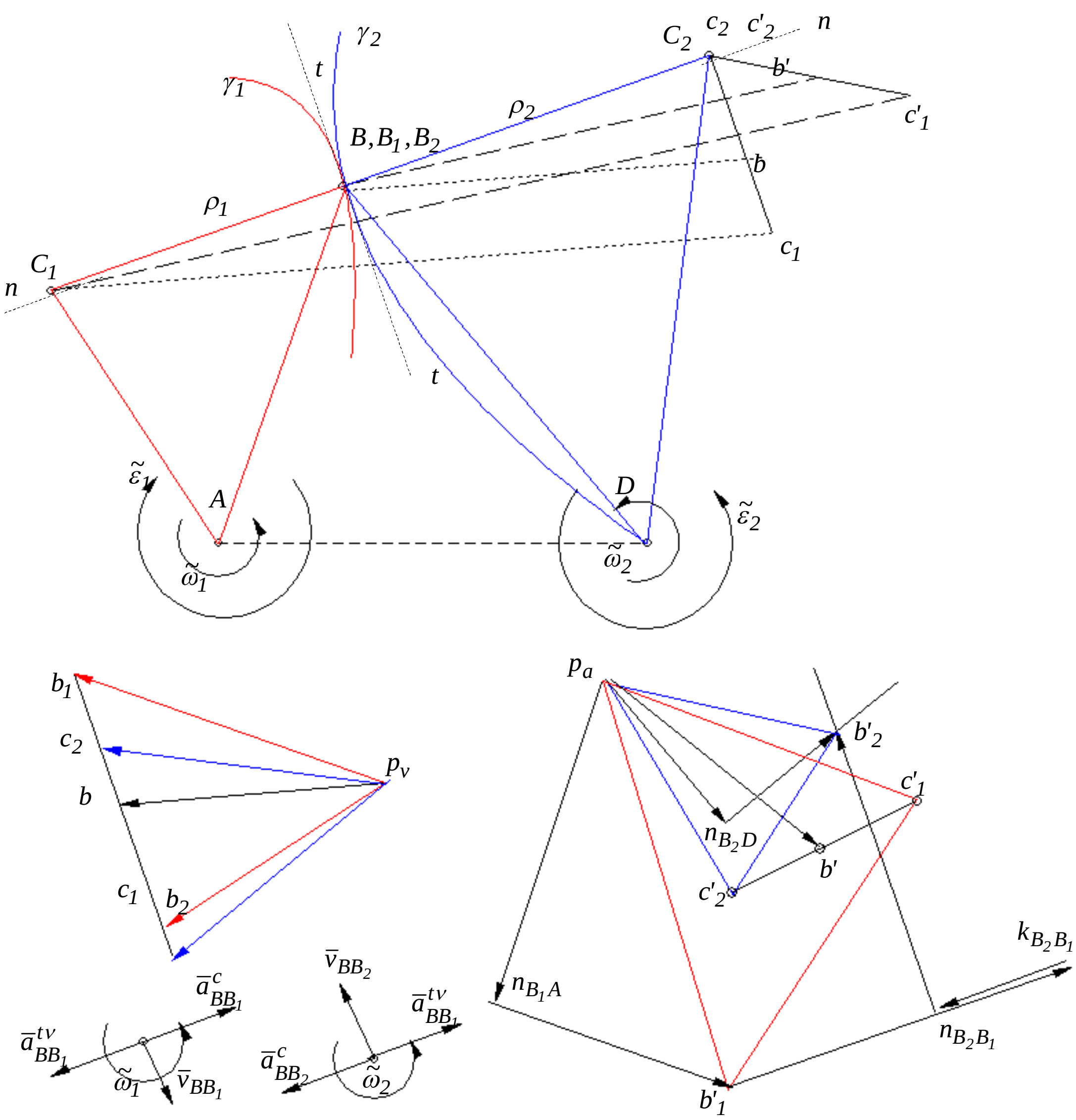

Figure 4. The mechanism and the velocities and accelerations polygons

In the velocity polygon the points b_1 and b_2 can be found, the velocity of the point B_2 is known and implicitly the angular velocity $\tilde{\omega}_2$ of the follower, together with the relative velocity from the higher pair $v_{B_2B_1}$. Applying the kinematic similarity theorem for the points A, C_1 and B_1 the point c_1 is found from similarity of triangles AC_1B and ac_1b_1. In an analogous manner the position of the point c_2 from the follower is found from the similarity between DC_2B and dc_2b_2 triangles.

Applying the method for an actual case lead to the remark that the points c_1, c_2 b_1 b_2 are collinear, belonging to a straight line parallel to the tangent tt. The result is expected considering the fact that the points are situated on the normal nn and on this direction the distanced between points are constant. With the points c_1 and c_2 known in velocity polygon, the position of the point c is found based on the observation that the points C_1, C_2 and B are attached to the replacing element. It results the conclusion that the point b is in the same relation and ratio as the ones for the points C_1, C_2 and B. To this end, the segment c_1c_2 is placed in a random position but with the end c_2 in the point C_2. A straight line parallel to C_1c_1 is traced through point B and it intersects the segment c_2c_1 in the point b; the construction manner ensures the proportionality of the segments c_1b and c_2b, on one side and of the segments C_1B and C_2B on the other side. Now, the velocity distribution is completely known.

The equation (4) is applied twice, for points B and B_1 first and B and B_2 respectively, for finding the accelerations distribution:

$$\begin{cases} \overline{a}_B = \underset{=}{\overline{a}^n_{B_1A}} + \underset{=}{\overline{a}^t_{B_1A}} + \underset{=}{\overline{a}^c_{BB_1}} + \underset{=}{\overline{a}^{t\nu}_{BB_1}} + \underset{\|tt}{\overline{a}^{\tau}_{BB_1}} \\ \overline{a}_B = \underset{\perp DB}{\overline{a}^t_{B_2D}} + \underset{=}{\overline{a}^n_{B_2D}} + \underset{=}{\overline{a}^c_{BB_2}} + \underset{=}{\overline{a}^{t\nu}_{BB_2}} + \underset{\|tt}{\overline{a}^{\tau}_{BB_2}} \end{cases} \tag{10}$$

The Coriolis accelerations $\overline{a}^C_{BB_{1,2}}$ and the normal transport accelerations $\overline{a}^{t\nu}_{BB_{1,2}}$ are now known, being directed along the common normal, as shown in figure 4. The right members of the two equations are equaled and after some arrangements it results:

$$\underset{=}{\overline{a}^n_{B_2D}} + \underset{\perp DB}{\overline{a}^t_{B_2D}} = \underset{=}{\overline{a}^n_{B_1A}} + \underset{=}{\overline{a}^t_{B_1A}} + (\underset{=}{\overline{a}^c_{BB_1}} - \underset{=}{\overline{a}^c_{BB_2}}) + (\underset{=}{\overline{a}^{t\nu}_{BB_1}} - \underset{=}{\overline{a}^{t\nu}_{BB_2}}) + (\underset{\|tt}{\overline{a}^{\tau}_{BB_1}} - \underset{\|tt}{\overline{a}^{\tau}_{BB_1}}) \tag{11}$$

$$\overline{a}^c_{B_2B_1} = \underset{=}{\overline{a}^c_{BB_1}} - \underset{=}{\overline{a}^c_{BB_2}} \tag{12}$$

$$\overline{a}^{t\nu}_{B_2B_1} = \underset{=}{\overline{a}^{t\nu}_{BB_1}} - \underset{=}{\overline{a}^{t\nu}_{BB_2}} \tag{13}$$

$$\overline{a}^{\tau}_{BB_1} - \overline{a}^{\tau}_{BB_2} = (\overline{a}^{\tau}_B - \overline{a}^{\tau}_{B_1}) + (\overline{a}^{\tau}_B - \overline{a}^{\tau}_{B_2}) = \overline{a}^{\tau}_{B_2} - \overline{a}^{\tau}_{B_1} = \overline{a}^{\tau}_{B_2B_1} \tag{14}$$

The equation (13) is written under the form:

$$\underset{=}{\overline{a}^n_{B_2D}} + \underset{\perp DB}{\overline{a}^t_{B_2D}} = \underset{=}{\overline{a}^n_{B_1A}} + \underset{=}{\overline{a}^t_{B_1A}} + \underset{=}{\overline{a}^c_{B_2B_1}} + \underset{=}{\overline{a}^{t\nu}_{B_2B_1}} + \underset{\|tt}{\overline{a}^{\tau}_{B_2B_1}} \tag{15}$$

and is formally identical to the equation of accelerations corresponding to the knife edge follower but with the remark that $\bar{a}^{c}_{B_2B_1}$, $\bar{a}^{n\tau}_{B_2B_1}$ are just notations, the relations for calculus being different from the one known fro the two terms. The correlated equation from the acceleration polygon is:

$$\overline{p_a n}_{B_2D} + \overline{n_{B_2D}b'_2} = \overline{p_a n}_{B_1A} + \overline{n_{B_1A}b'_1} + \overline{b'_1 k}_{B_2B_1} + \overline{n_{B_2B_1}b'_2} \tag{16}$$

In figure 4 is presented the method of working out the equation in acceleration polygon. There are found $a^{t}_{B_2D}$ and implicitly the angular acceleration $\tilde{\varepsilon}_2$ together to the tangential component of relative acceleration $a^{\tau}_{B_2B_1}$. Having the points b'_1 and b'_2 in the polygon, the points c'_1 and c'_2 are found analogous to the manner the points c_1 and c_2 from velocity polygon were found, by constructing in the acceleration polygon the triangle $a' b'_1 c'_1 \sim ABC_1$, and $d' b'_2 c'_2 \sim DBC_2$ respectively. Lastly, the point b' is found from the condition similar to the one from velocities, specifically the point b' must divide $c'_1 c'_2$ in the same ratio as the point B divides the segment C_1C_2 . At this moment the kinematical analysis is completed. The other mentioned cases are solved in similar manners but with a series of particularly simplifications. For instance, for the flat face oscillating follower, the acceleration $\bar{a}^{\tau\nu}_{BB_2}$ is zero because the curvature radius of the follower is $\rho_2 = \infty$. In the case of translating curved face follower, the acceleration $\bar{a}^{c}_{BB_2}$ is zero due to lack of rotation motion of the follower, $\omega_2 \equiv 0$. Obviously, for the flat face translating follower the analysis is further simplified since both components $\bar{a}^{\tau\nu}_{BB_2}$ and $\bar{a}^{c}_{BB_2}$ vanish, from the above reasons.

4. Conclusions

The paper presents a grapho-analytical method for kinematical analysis of a cam plane mechanism of whatever type. If for the case of tip follower the grapho-analytical method is relatively simple to apply because the contact point between the cam and the follower is immobile on the follower's profile and with a known trajectory, in the case of curved face follower, the contact point moves on both profiles, the cam's and the follower's profile, too. The absolute trajectory of the contact point is a complex curve, practically unknown, and therefore the direction of the velocity of the contact point is also unidentified.

The direct application of the equations which express the velocity/acceleration of the same point in two ways, provide a system of two plane vector equations with three unknowns, being therefore undetermined. The principle of the method consists in simultaneous consideration of both the actual mechanism and the replacing lower pair mechanism.

Thus, in the cam-follower contact point there always be superposed three points, first on the cam, the second on the follower and the third is attached to the replacing element. Applying the equations for velocity/acceleration of the point from the replacing element with respect to the velocity/acceleration of the points from the cam and follower, vector equation systems are obtained. After using appropriate mathematical manipulation, these equations allow for finding all kinematical parameters of the mechanism.

The method is exemplified for an actual case and is fairly expedite and therefore recommended for didactical activity, too.

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