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Compliant mechanism for ankle rehabilitation device; Part I: modelling and design

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Abstract. Robotic rehabilitation devices are able to meet certain requirements for long time rehabilitation, such as compact design, repeatability and storage patients' evolution. As ankle injuries have a high incidence among the population, regardless of their age or activity, the need for an easy to manufacture rehabilitation device emerges. The proposed design offers two degrees of freedom, required for a full recovery of the ankle joint and programmable exercises. Although a safety button can be included in the electronic system that can be operated by the patient, extra safety measures are required in order not to damage furthermore the rehabilitated limb. A compliant mechanism that can bear the extra load from the motors' crank, in case of overloading of the joint, is required. In this paper some design requirements and systematic synthesis methods for a compliant mechanism are presented.

1. Introduction

The ankle joint provides stability and mobility for everyday activities, thus becoming one of the most prone to injuries structure of the leg. Lower limb injuries wary from sprains to fractures, each of them require resting periods and special treatments. After the immobility period recovery is necessary in order to enhance the mobility and strength of the ligaments. This is achieved trough classical rehabilitation ankle exercises, which require a lot of patients' time and effort, as well as a trained physical therapist that monitors the evolution of the patient. An alternative to classical rehabilitation is provided by robotic rehabilitation devices [1], that provide the require exercises trough various methods of actuation. A novel device for ankle rehabilitation was proposed in previous work ([2]- [4]) that offers two degrees of freedom, necessary for a complete recovery of the joint. Although a stop button is implemented that allows the patient (or the therapist) to interrupt the rehabilitation exercise, extra safety measures are required. For this purpose a compliant mechanism that will receive the extra load is studied; it will command the motors to stop and will not allow further damage to the ankle.

A compliant mechanism is a jointless, monolithic, flexible structure that gains its mobility trough elastic deformation of its flexible elements. Such structure replaces multiple rigid parts, pin joints, springs. Compliant mechanisms have become more popular in the last years versus rigid body systems connected by conventional joints, with applications that range from mechanical systems, to medical use and even space devices ([8]-[13]).

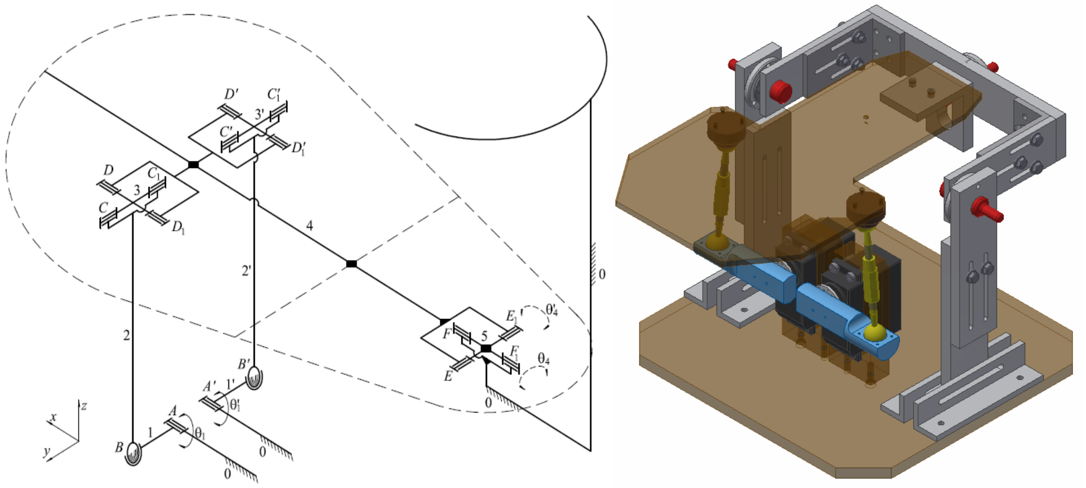
The benefits of compliant mechanisms over conventional rigid-link mechanisms are given by the lack of relative motion among pieces and lack of overlapping pieces. The advantages of these mechanisms can be summarised as follows:

- cost reduction due to ease of fabrication (single-piece injection moulding, extrusion, rapid prototyping, silicon surface micromachining, electroplating techniques);
- monolithic design resulting in reduced maintenance (absence of wear, noise, vibration and need of lubrication due to absence of sliding friction);
- compactness, capacity to be utilized in small-scale applications (miniaturization);
- backlash is eliminated, resulting in reduced positioning error and increased precision.

As disadvantages of these mechanisms one can mention: reduced mobility (due to limit of deflection of the material), energy retention and design complexity. The design process is intricate and must consider stress and strain relationships , because they determine the deformed shape of the elements.

2. Design requirements

The developed ankle rehabilitation model is presented in Figure 1. The patient will be sited on a chair and the foot will be rested on element 4 (that supports the sole). The ankle will execute exercises for dorsiflexion/plantar flexion movements as well as inversion/eversion movements; hence the mechanism possesses two degrees of freedom. A compliant mechanism is required to be positioned between the crank and the motor, in order to prevent the overloading of the ankle joint.



(a) (b)

Figure 1. Ankle rehabilitation device: (a) kinematic model; (b) 3D model

For a compliant element the mechanism performance requirements are: range of motion, torque margin, operating speed, operating life, accuracy. The compliant device must be able to be integrated into other subsystems, hence some system requirements appear: light weight, stiffness, no environment effect over functionality (temperature, vibrations).

The values for developed ankle torque, to a healthy subject, vary with age, sex and everyday activity. Hence for dorsiflexion/plantar flexion maximum voluntary developed torque the following values were found: 43 *Nm* for dorsiflexion and 137 *Nm* for plantar flexion ([14],[15]). The inversion/eversion values were not studied, since these are supplementary motions and have smaller impact. An ankle model for the passive moment-displacement relationships under static conditions was developed by Jamwal et. al. [16] and maximum ankle moments were found between -10 *Nm* and 7 *Nm*. Based on this studies the compliant mechanism must offer a maximum of 15*Nm* torque, after this value one can assume that the exercise is becoming painful for the patient. The compliant element is required to withstand the maximum output torque supplied by the actuator and to provide a linear torque vs. rotation relationship. The performance is limited by the material's Young's modulus (low value allows an easier bending of the element) and yield stress (determines how far the element can bend before failure). Also the elastic component should be connected directly from the motor to the crank, in order to avoid the reduction of force fidelity caused by transmissions non-linearity. Finally, actuator dimensions and weight must be reduced as much as possible. All these requirements result from systematic synthesis and structural optimization of the element.

3. Systematic synthesis and modeling of the compliant mechanism

In order to obtain a desired design, three main different synthesis methods are used [17], presented in Figure 2. It can be observed that in kinematics based approaches two methods are most used: the rigid-body-replacement models and the freedom and constraints topologies. For the building blocks approach two main methods are identified: the instant centres and the flexible building blocks.

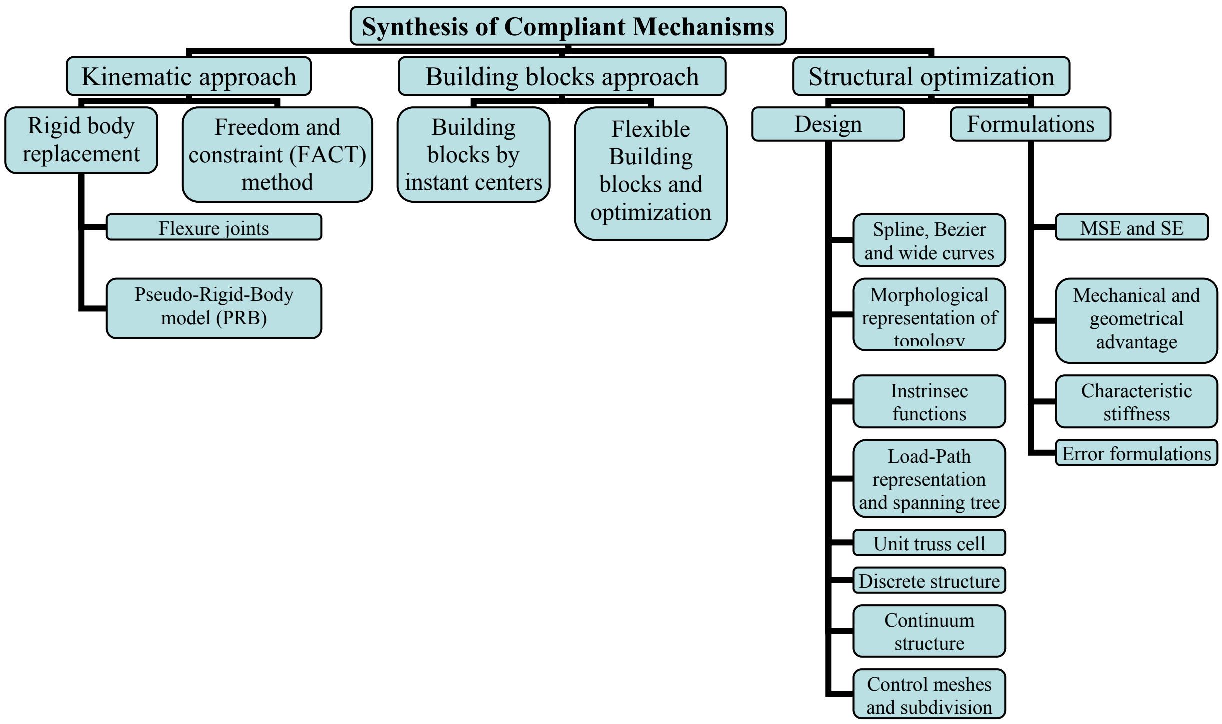


Figure 2. Synthesis of compliant mechanisms [17]

The first method from the kinematic approaches is represented by the rigid-body-replacement method that consists in finding a rigid body mechanism that accomplishes the desired function. This conversion can be achieved through the Pseudo-Rigid-Body (PRB) model, or by replacing the conventional joints with flexure joints.

3.1. Flexure hinges

A flexure element can be defined as a complex spring that transmits rotation and translation movements and can undergo large deflections relative to rigid adjacent regions. This element can be separated into three categories, function of its deflection axes: single-axis (with constant width), multiple-axis (revolute geometry) and two-axis. The single-axis category of flexure hinges enables rotation only around one axis, allowing relative rotation between two adjacent members. This axis is called the sensitive axis and defines the main function and operational motion of a flexure hinge.

For the designed device a single-axis flexure hinge compliant mechanism is the best option, since the rotation of the crank it's allowed only around a single axis. The section if a flexure hinge can be rectangular, corner filleted, circular, parabolic, hyperbolic, elliptical, inverse parabolic, secant or hybrid [16]. The rectangular cross-section hinge with constant width and variable thickness (Figure 3) is the most used since the sensitive axis lies in the cross-section of minimum thickness where maximum bending compliance is present [18].

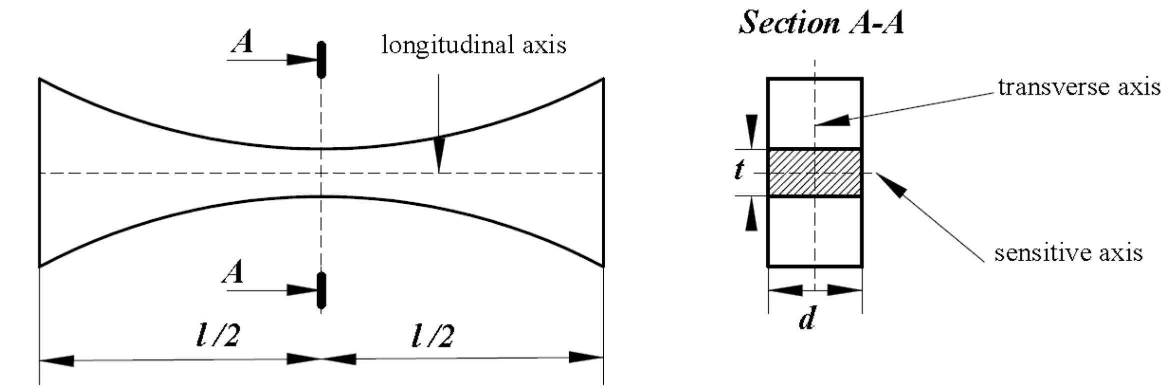


Figure 3. Single-axis flexure hinge of constant width and rectangular cross-section [18]

In order to characterize the mechanical properties of flexure hinges some attributes are necessary to be determined, such as its capacity of rotation (displacements, maximum deformations), precision of rotation and maximum stress levels (mechanical stresses). These properties were analytical calculated by Dirksen and Lammering [19] based on Timoshenko's beam theory.

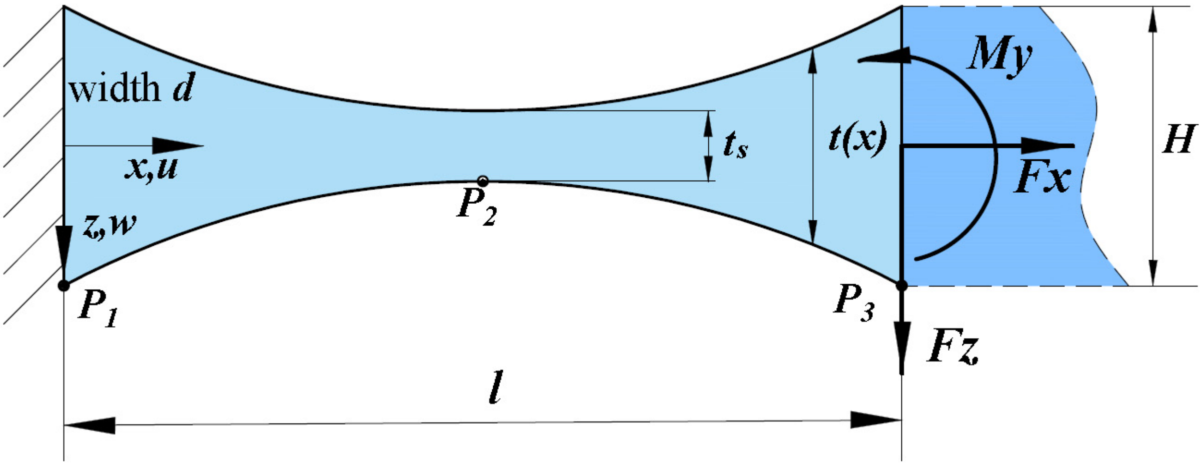


Figure 4 . Planar flexure hinge [19]

In order to resist external loads F_x, F_z, M_y a planar flexure hinge (Figure 4), characterized by length l, width(or depth) $d=10\ mm$, height H, variable thickness $t(x) \geq t_s$ and common points $P_1(0,H/2)$, $P_2(l/2,t_s/2)$, $P_3(l,H/2)$, requires the following mechanical properties:

- relevant normal stress:

$$\sigma_x(z) = \frac{1}{dt_s} F_x + \frac{12(x-l)z}{dt_s^3} F_z + \frac{12z}{dt_s^3} M_y \tag{1}$$

- shear stress:

$$\tau_{xz}(z) = \left(\frac{-6z^2}{dt_s^3} + \frac{3}{2dt_s} \right) F_z \tag{2}$$

- displacements

$$u(x,z) = \frac{x}{Edt_s} F_x - \frac{(12lx - 6x^2)z}{Edt_s^3} F_z + \frac{12xz}{Edt_s^3} M_y \tag{3}$$

$$w(x) = \frac{12(1+v)t_s^2 x + 30lx^2 - 10x^3}{5Edt_s^3} Fz - \frac{6x^2}{Edt_s^3} M_y$$

$$\psi(x) = -\frac{12lx - 6x^2}{Edt_s^3} F_z + \frac{12x}{Edt_s^3} M_y \tag{4}$$

- maximum equivalent stresses

$$\sigma_{V,max} = \sqrt{\left(\frac{F_x}{dt_s} K_{tx} + \frac{6\left(M_y - lF_z\right)}{dt_s^2} K_{td} \right)^2 + 3\left(\frac{F_z}{2dt_s} \right)^2} \tag{5}$$

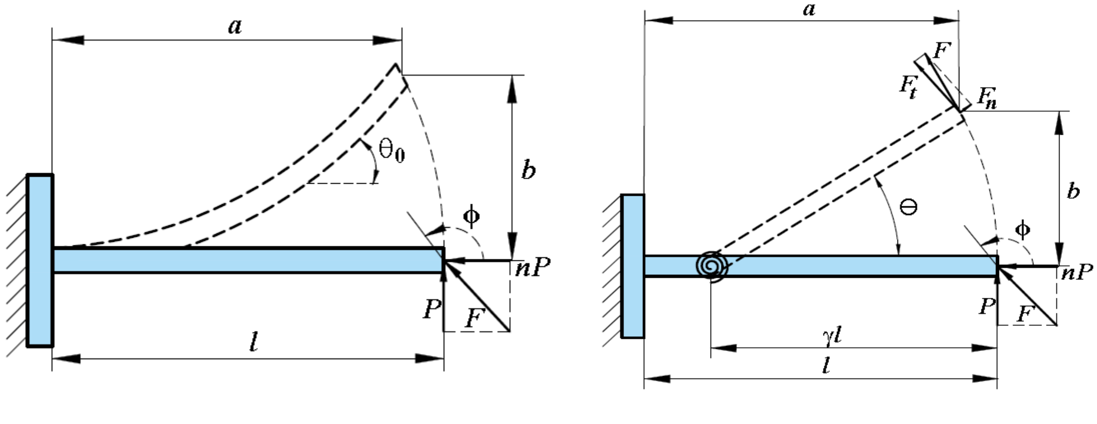
where K_{tx}, K_{td} are stress concentration factors for rectangular leaf type hinges [20].

Numerical simulation of these equations [19] show a low bending stiffness and very high rotational deflection for rectangular geometry flexure hinges, perfect qualities for the design of a compliant mechanism. Furthermore these equations allow the selection of the appropriate dimensions for the flexure hinge, prior to any modelling process.

3.2. Pseudo-Rigid-Body Model

This model allows the replacement of the elastic elements with a rigid-body element that emulates their behaviour. More precisely the force-deflection relation is approximated by springs, while the kinematics of the rigid-body provides the deflection path. Although flexure hinges may have variable cross-sections this method assumes that the element has a constant cross-section.

The foundation of this model was created by Howell and it's based on the premise that the compliant beam can be simulated by rigid segments, connected through a pin joint [21]. To simulate the beam compliance a spring is placed at the pin joint (Figure 5), located along the undeformed beam geometry with the help of γ factor (characteristic radius).



(a) (b)

Figure 5. Pseudo-rigid-body model for a cantilever beam (fixed-pinned condition): (a) deflected beam; (b) equivalent PRB model [21]

The θ_0 angle represents the beam end angle, Θ the pseudo-rigid-body angle, b is the vertical displaced position of the beam end and a is the horizontal displaced position. The characteristic stiffness for the spring is given [16] by γ and a stiffness coefficient K_θ:

$$K = \frac{\gamma K_\Theta EI}{l} \tag{6}$$

where γ and K_θ are function of the n parameter (that sets the orientation of the applied force at the free end as a proportion of its components) and have the following values:

$$\begin{cases} \gamma = 0.855651 - 0.016438n, for -4 < n \le -1.5 \\ \gamma = 0.852138 - 0.01861n, for -1.5 < n \le -0.5 \\ \gamma = 0.851892 - 0.020805n + 0.005967n^2 - 0.000895n^3 + 0.000069n^4 - 0.000002n^5, for -0.5 < n \le 10 \end{cases}$$

$$\begin{cases} K_\Theta = 2.66041 - 0.069005n - 0.002268n^2, \ for -4 < n \le -0.5 \\ K_\Theta = 2.648834 - 0.074727n + 0.026328n^2 - 0.004609n^3 + 0.00039n^4 - 0.000013n^5, \ for -0.5 < n \le 10 \end{cases}$$

A pseudo-rigid-body model for a cartwheel hinge was elaborated by Pei et. al.[22] with the following dimensional variables: l the length of one flexible element, H half-height of the hinge, φ half-angle between two flexible elements (45º) and t is thickness of the element. The model can be further simplified due to its symmetric structure to a single cantilever beam, and the spring stiffness (the spring constant) was obtained [22] as:

$$K = \frac{4EI}{H}\cos\varphi \quad (7)$$

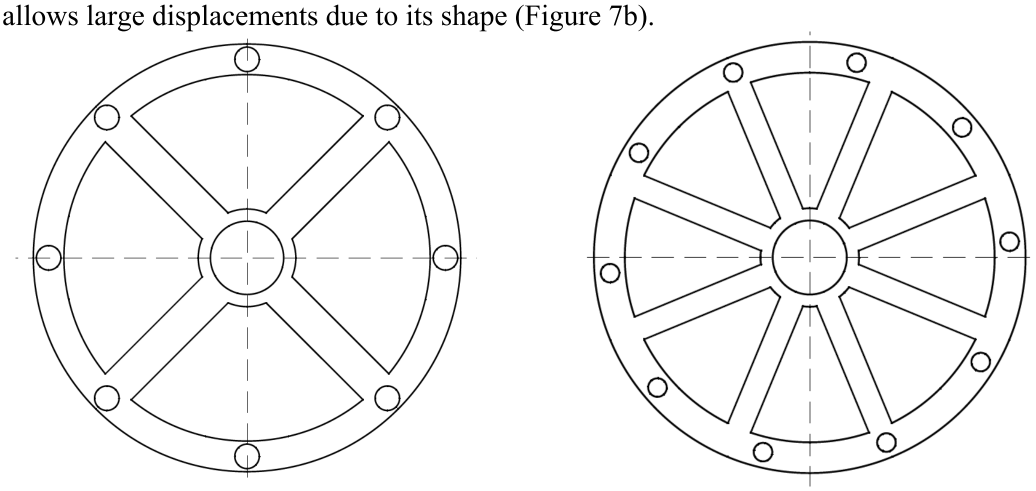
The maximum stress was found as:

$$\sigma_{\max} = \frac{2Et}{l}\alpha = \frac{Et}{l}\theta \quad (8)$$

The φ angle (half the angle between two flexures) is a very important parameter because it affects the stiffness and precision of the hinge. As demonstrated in [22] the stiffness of the cartwheel hinge could be reduced by increasing this angle and the H parameter (height). Stiffness is also related with the material of the hinge and the section dimension of the element.

3.3. Proposed designs

Based on the pseudo-rigid model a cartwheel hinge was proposed, consisting in a pair of leaf-type flexures intersecting at right angles (Figure 6a), with the point of intersection being considered the pivot point. The structure of this mechanism is symmetric and the deflection motion is modelled by rigid links attached to four pin-joints. Since the φ angle affects the stiffness of the hinge another model was designed, doubling the number of elastic flexures (**Figure 6**b). Derived from the mathematical model of a flexure hinge with constant width and variable thickness another compliant design was proposed, presented in Figure 7a. The last studied model is based on double spring Archimedes's coil, which allows large displacements due to its shape (Figure 7b).



(a) (b)

Figure 6. 2D model of a compliant mechanism: (a) cartwheel hinge with four flexures [22]; (b) cartwheel hinge with eight flexures

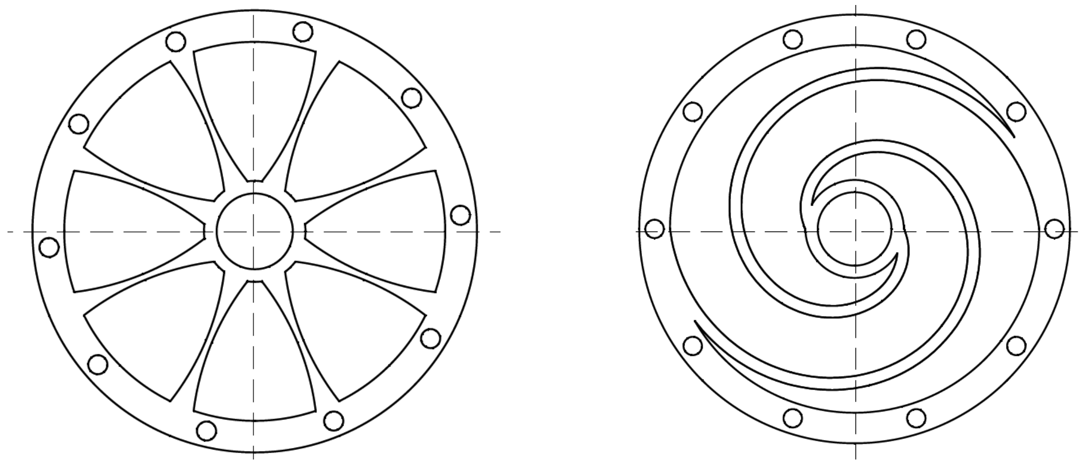


Figure 7. 2D model of a compliant mechanism: (a) cartwheel hinge with constant width and variable thickness; (b) compliant mechanism based on Archimedes's coil

4. Conclusions

In this paper (Part I) some preliminary designs of a monolithic compliant mechanism were presented. The elastic element will be used for safety measures in an ankle rehabilitation device. The main purpose of the element is to act as a safety mechanism, preventing the overload of the ankle joint. First design is represented by a cartwheel hinge, based on flexure elements. Flexure elements have great advantages that can be used for compliant mechanism (for example their behaviour is predictable). Nevertheless the analysis of a complex compliant mechanism is difficult and lacks the required tools. A partial answer of the problem is given by the flexure hinge model and the pseudo-rigid-body model. The pseudo-rigid-model reduces the study of complex compliant mechanisms to that of ordinary mechanism, but it cannot be applied for all models. The alternative to these methods is the use of finite-element analysis where loads and displacements can be studied. In Part II of the paper design options trough structural optimization approaches will be studied, including finite-element analysis.

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