Deformation Criteria for Calculation of Composite Constructions

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Abstract. In this paper we consider the calculation of reinforcement and cross-sections of reinforced concrete structures: beams, columns, diaphragms of rigidity, etc. The article is divided into two parts. In the first part, the criterion for limiting deformations in given zones of the most stressed sections of composite systems is proposed as the basis for calculating reinforced concrete structures. The introduction of the appropriate limitations makes it possible to determine, from the equilibrium conditions of forces in a given section, one of the main geometric parameters of this section: height, width, cross-section of the working armature or the maximum possible load. An algorithm for solving the problem of selecting sections of a statically indeterminate system is shown. This approach is called the method of limited deformations (MLD). The method is illustrated by a number of examples. In the second part, the definition of points of nucleation of cracks and trajectories of their development is established by solving the problem of determining the deformed state of a composite under conditions of a plane stress state. Determination of the main deformations is performed by a modernized finite element method. The finite element calculation is based on specially developed composite elements describing the joint work of concrete and reinforcement. Two new (planar and spatial) composite finite elements are proposed. We show the scheme of formation of the general matrix of rigidity of a composite, as superposition of global matrixes of rigidity of a binder and armature.

1. Introduction

Mechanical systems, consisting of composite materials, are becoming more and more popular. They are widely used in construction, aircraft building, shipbuilding. Such materials usually consist of two or more components. The basis of the composites is binder. Strength in given directions is provided by reinforcing material - steel or carbon fiber reinforcement, fibers, threadlike crystals. In this regard, the work of construction from composite highly depends on the location of the reinforcing material and the compound of the composite.

Reinforced concrete can be qualified as a composite, in which a relatively compliant under stretch concrete is reinforced with high-strength fibers in the form of steel (reinforced concrete), carbon plastic or fiberglass reinforcement. The diagrams of steel reinforcement can have an area of fluidity (so-called building steel) or have only a limited descending branch (high-strength reinforcing steel). Composite reinforcement, as a rule, has an elastic-linear relationship between stresses and deformations up to destruction.

Calculation of reinforced concrete constructions, based on the limit equilibrium theory, is extremely

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easy to use, but the complete disregard for the deformations of the mechanical system makes it difficult to design a practical application. Furthermore, actual structures should not form a single plastic hinge even in the static loading conditions, and especially when we have the alternating dynamic system [1-5]. The author for a long time had to deal with the examination of buildings and structures. The presence of cracks in reinforced concrete elements has always been regarded as a circumstance requiring, usually, to conduct additional researches (aggressive environment analysis, evaluation of fire resistance, evaluation of opportunities and the value of dynamic effects, etc.) and to carry out the development of strengthening project. In this regard, there was the intention to make a strain sign a basis for design of reinforced concrete structures. This article provides a simple and reliable method of composite systems calculation, based on the specified limitation of cracks development – limited deformation method (MLD).

Most often, the calculation of composite systems by the finite element method (FEM) contains a procedure of "smearing" the characteristics of reinforcing structures along the body of a finite element [6]. In practice, this consists of selecting orthotropic constants for solving plane problems or anisotropy constants for volume projects. In the ANSYS computational complex [7], it is possible to create combined finite elements from cubic binder elements with hinge assemblies and hinged-rod reinforcing elements.

In addition, in ANSYS it is possible to represent a composite in the form of rectangular parallelepipeds consisting of sets of layers of materials with different orientations and properties of orthotropic material in each layer. A similar approach is proposed in the monograph [8]. In this paper we offer an alternative simple, reliable and physically clear method of finite element analysis of a composite. At the same time, this work continues the development of the method of bounded deformations [9] for the problems of plane stressed state.

2. Methodological basis of calculation MLD and algorithm of the method

The limit equilibrium method of reinforced concrete structures is based on the idea of complete destruction. To prevent destruction of the projected structures, a system of coefficients of reserve is introduced. These coefficients ensure the strength of the structure, but its economy and rationality become extremely doubtful. In contrast to the limit equilibrium method, the basis of the method of limited deformations is based on the opposite idea: the construction should not collapse under any possible loads [4, 5]. With an unfavorable combination of loads in the structure only limited, predefined, surface fractures can occur and that can appear only on one, the most stress-strained section of the element.

The limit equilibrium method could be called a "method of unlimited deformations". In fact: in order for plastic deformations to develop in a middle (in height) zone of a symmetric curved beam, many times greater than maximum possible elastic deformations tens must occur. As a result of this approach, a serious contradiction arises between elastic calculation and the design task.

Brittle fracture of the concrete in the stretched areas of reinforced concrete elements is associated with the formation and development of cracks. Adopting approximating dependencies, we will use the concepts of the limit of compressibility $[\varepsilon^{-}]$ and ultimate tensile of concrete $[\varepsilon^{+}]$, i.e., concrete deformation at the fracture point. Limited compressibility of concrete while bending and eccentric compression can be taken as $[\varepsilon^{-}]\approx(2{,}0\div3{,}5)\cdot10^{-3}$; and the ultimate elongation $[\varepsilon^{+}]\approx(1{,}5\div3)\cdot10^{-4}$, i.e., an order of magnitude less.

Before reaching certain limits in the extreme fiber tensile stress (or strain), we will disclose the internal static indeterminacy basing on the usual hypothesis of plane sections and the equilibrium conditions of the solid composite nonlinear stress. When cracking in cross section j we will assume hypothesis of plane sections as fair, outside of the crack boundary (Figure 1).

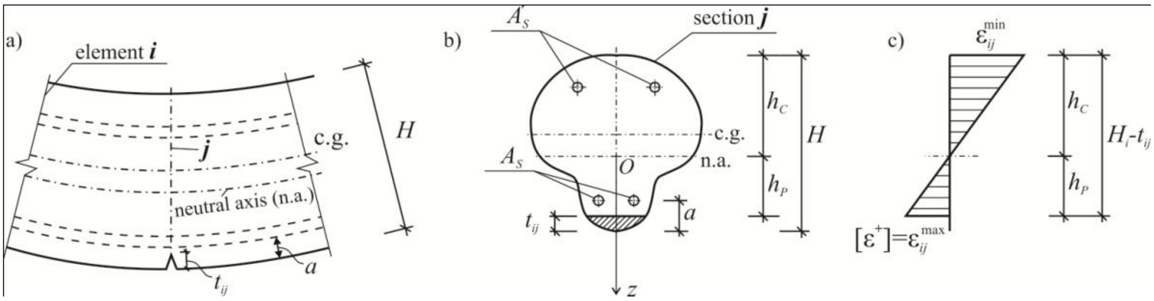


Figure 1. A fragment of a reinforced concrete element

Suppose in this concrete element i we know the most intense cross-section j. We put the condition for this section:

$$t_{ij} \le [t_{ij}], \quad (1)$$

where t_{ij} – depth of the crack in the cross section j of element i; $[t_{ij}]$ – allowable depth of a crack in a cross section j of element i. In the design tasks, it is advisable to take:

$$0 \le [t_{ij}] \le a, \quad (2)$$

where a – protective element layer thickness of i. Then the strain of the stretched composite zone at the crack border will be equal to $[\varepsilon^+]$. In the same cross section, outside of the boundary fracture, deformations are distributed linearly.

Similarly, the restriction to a depth of crushing zone is introduced. However, here it is expedient to introduce the following restrictions:

$$\left.\begin{array}{l}[t_{ij}^-]=0\\ t_{ij}^- \ge 0\end{array}\right\}. \quad (3)$$

For bending elements fiber deformation at a distance from the neutral axis z (Figure 1) is defined by:

$$\varepsilon = z \cdot w''. \quad (4)$$

Here w'' – the curvature of the axis of the element. On the border of the crack the strain is:

$$[\varepsilon^+]_{ij} = (h_p)_{ij} \cdot (w'')_{ij}, \quad (5)$$

where h_P – the height of the tension zone. At any other point of the cross section, the deformation can be calculated by the formula:

$$\varepsilon_{ij} = \left(\frac{z}{h_p}\right)_{ij} \cdot [\varepsilon^+]_{ij}. \quad (6)$$

Note that the maximum compressibility can be achieved only at the height of the compressed zone

$$z = h_c > 10 h_p. \quad (7)$$

What is more usually realized that in the columns than in the girders.

Let longitudinal force N_{ij} and bending moment M_{ij} of external forces in the j cross section correspond to a given strain and generalized load P. We impose the following conditions to effective dimensions (not affected by the crack) of the section:

$$\left.\begin{array}{l}\int\limits_{A_э} \sigma \cdot dA_э = N_{ij}\\ \int\limits_{A_э} \sigma z \cdot dA_э = M_{ij}\end{array}\right\}, \quad (8)$$

where the unknown value is or one of the dimensions that define the area of the cross-section of the concrete A, or a cross-section of tensile reinforcement or generalized load P.

The most rational dimensions of cross sections of reinforced elements are obtained by performing the pre-tensioning of reinforcement. At the same time, the effort in the prestressing rod in the design section is equal to the algebraic sum of the efforts without pre-stress N_1, corresponding deformation $\varepsilon_a(z)$, and tension reinforcement efforts, taking into account losses N_0 (N_0 corresponds to the non-deformed concrete in view of losses).

Algorithm of Limited deformations method of calculation of statically indeterminate systems consists of the following:

1. Preliminary appointing the dimensions of the elements.
2. FEM calculations are performed with the construction of envelope diagrams of moments.
3. Using $M_{ij\ max}$ from equations (8), parameters of the cross-section of *i*-th elements are selected.
4. Introducing improved values of stiffness. The FEM is recalculated with the calculating of new efforts parameters.
5. Refinement the $M_{ij\ max}$ and parameters of the cross-section of *i*-th elements.
6. Introducing improved values of stiffness. Final calculation are performing.

Here is an example of statically indeterminate system (Figure 2a), calculated by this algorithm. The unknown variable is considered to be the width of the cross-section b_1 or b_2.

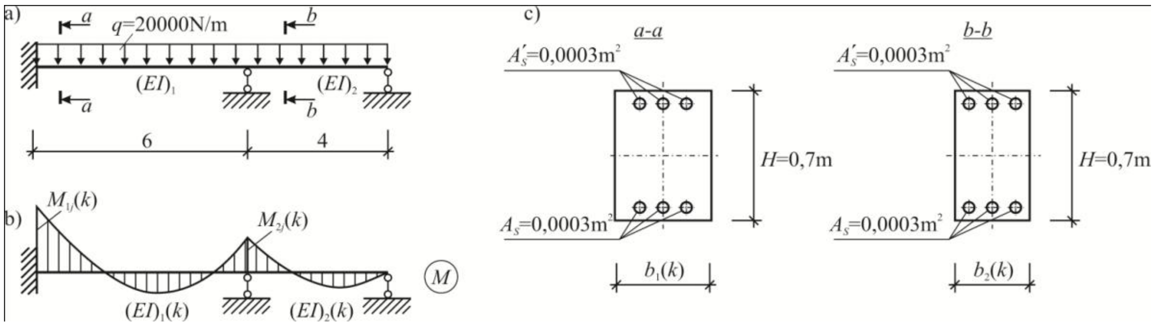


Figure 2. Selection of a statically indeterminate system's cross section

Characteristics of the concrete and reinforcement are the same as in the previous example. The calculations are summarized in Table 1.

Table 1. Results of calculations

The number of calculate ***k***	Stiffness ratio		M_{ij}, N·m		Dimensions of cross-section	
	$(EI)_1$	$(EI)_2$	M_{1j}	M_{2j}	b_1	b_2
1	1	1	64707	50588	155	109
2	1.289	1	65339	49323	157	104
3	1.343	1	65444	49117	157	104

3. Calculation of reinforced concrete systems by FEM

Consider the calculation and selection of reinforcement and cross-sections near the support of reinforced concrete beams or diaphragms of rigidity.

The inclined cracks in the area near the support of the beams originate from the combined effect of cross force and bending moment. Inclined cracks trajectory on the sections of reinforced concrete beams near the supports, which lines are defined by trajectory of the main deformations, present some theoretical interest. Determination of the points of nucleation of cracks and trajectories of their development can be established by solving problems of determination of the deformed state of the composite under conditions of plane stressed state. The initiation of a crack will occur at that point of the support zone of the beam, where the deformations have reached one of the limiting values

$$\varepsilon^{+}_{\max} = [\varepsilon^{+}], \tag{9}$$

$$\varepsilon^{-}_{\min} = [\varepsilon^{-}], \tag{10}$$

where $\varepsilon^{+}_{\max}$, $\varepsilon^{-}_{\min}$ – the main strains of tension and compression at a point, $[\varepsilon^{+}]$, $[\varepsilon^{-}]$ – deformation, corresponding to the breaking and crushing of concrete fibers.
As we know, the main deformations in the plane problem can be determined from the equations:

$$\left.\begin{aligned} \varepsilon^{+}_{\max} &= \frac{1}{2}(\varepsilon_x+\varepsilon_y)+\frac{1}{2}\sqrt{(\varepsilon_x-\varepsilon_y)^2+\gamma^2_{xy}} \\ \varepsilon^{-}_{\min} &= \frac{1}{2}(\varepsilon_x+\varepsilon_y)-\frac{1}{2}\sqrt{(\varepsilon_x-\varepsilon_y)^2+\gamma^2_{xy}} \end{aligned}\right\}. \quad (11)$$

The definition ε_x, ε_y and γ_{xy} can be performed by a modernized finite element method (FEM). Modernization of FEM will be done in a way that it is possible to solve the problems of calculating composite systems. In this case, finite elements will be formed not by choosing orthotropic constants for the description of reinforcement (which is very approximate and conditional), but by developing special composite elements, describing the joint work of concrete and reinforcement. For a plane problem of the theory of elasticity, such a finite element can be obtained by imposing a rod finite element on one side of the triangular finite element of the plane problem (Figure 3). Here the rod element will have the characteristics of the reinforcement, and the flat triangular element will describe the concrete object.

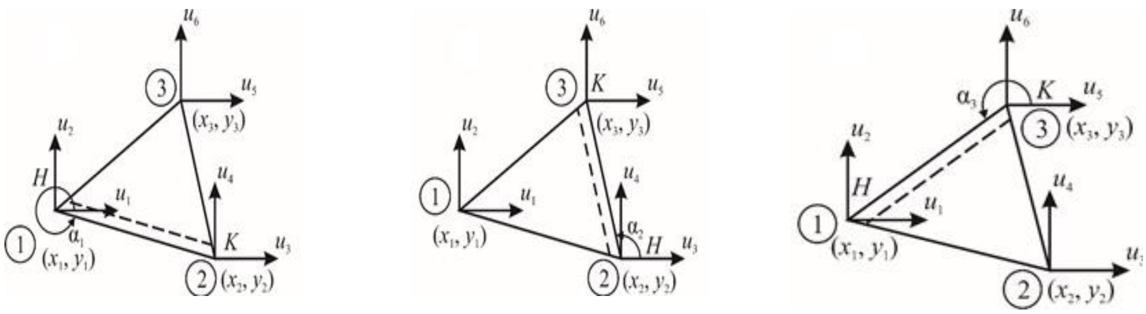


Figure 3. Composite finite element for solving a flat problem for a composite – a triangle describes a concrete object, a rod (dashed line) describes an armature.

The stiffness matrix of a triangular element [10] has the form:

$$K_{pl}=\frac{E_1 h}{4A(1-v_1 v_2)}\begin{bmatrix} y^2_{23}+a_1x^2_{23} & (a_1+v_2)x_{32}y_{23} & y_{23}y_{31}+a_1x_{23}x_{31} & a_1x_{32}y_{31}+v_2x_{13}y_{23} & a_1x_{12}x_{23}+y_{12}y_{23} & a_1x_{32}y_{12}+v_2x_{21}y_{23} \\ (a_1+v_2)x_{32}y_{23} & \frac{v_2}{v_1}x^2_{23}+a_1y^2_{23} & a_1x_{13}y_{23}+v_2x_{32}y_{31} & \frac{v_2}{v_1}x_{23}x_{31}+a_1y_{23}y_{31} & a_1x_{21}y_{23}+v_2x_{32}y_{12} & \frac{v_2}{v_1}x_{12}x_{23}+a_1y_{12}y_{23} \\ y_{23}y_{31}+a_1x_{23}x_{31} & a_1x_{13}y_{23}+v_2x_{32}y_{31} & y^2_{31}+a_1y^2_{31} & (a_1+v_2)x_{13}y_{31} & a_1x_{12}x_{31}+y_{12}y_{31} & a_1x_{13}y_{12}+v_2x_{21}y_{31} \\ a_1x_{32}y_{31}+v_2x_{13}y_{23} & \frac{v_2}{v_1}x_{23}x_{31}+a_1y_{23}y_{31} & (a_1+v_2)x_{13}y_{31} & \frac{v_2}{v_1}x^2_{31}+a_1y^2_{31} & a_1x_{21}y_{31}+v_2x_{13}y_{12} & \frac{v_2}{v_1}x_{12}x_{31}+a_1y_{12}y_{31} \\ a_1x_{12}x_{23}+y_{12}y_{23} & a_1x_{21}y_{23}+v_2x_{32}y_{12} & a_1x_{12}x_{31}+y_{12}y_{31} & a_1x_{21}y_{31}+v_2x_{13}y_{12} & y^2_{12}+a_1x^2_{12} & (a_1+v_2)x_{21}y_{12} \\ a_1x_{32}y_{12}+v_2x_{21}y_{23} & \frac{v_2}{v_1}x_{12}x_{23}+a_1y_{12}y_{23} & a_1x_{13}y_{12}+v_2x_{21}y_{31} & \frac{v_2}{v_1}x_{12}x_{31}+a_1y_{12}y_{31} & (a_1+v_2)x_{21}y_{12} & \frac{v_2}{v_1}x^2_{12}+a_1y^2_{12} \end{bmatrix},$$

where $x_{ij}=x_i-x_j$; $y_{ij}=y_i-y_j$, h – beam or diaphragm thickness, A – area of the final element, $a_1=\frac{G}{E_1}(1-v_1v_2)$, G – shear modulus

The stiffness matrix of the reinforcing rod has the form:

$$K_a=\frac{E_aA_a}{l}\begin{bmatrix} cc & cs & -cc & -cs \\ sc & ss & -sc & -ss \\ -cc & -cs & cc & cs \\ -sc & -ss & sc & ss \end{bmatrix}, \quad (12)$$

where E_a, A_a – the modulus of elasticity and the cross-sectional area of the reinforcement, l –distance between nodes, c, s – cosine and sine of the angle of inclination of the face of the triangle with the armature to the axis x.

The composite stiffness matrix is found by superimposing:

$$K_{COM} = K_{pl} + K_a. \tag{13}$$

To solve the spatial problem, the composite finite element can be obtained in an similar manner – by imposing a rod element on one of the faces of a concrete tetrahedron (Figure 4).

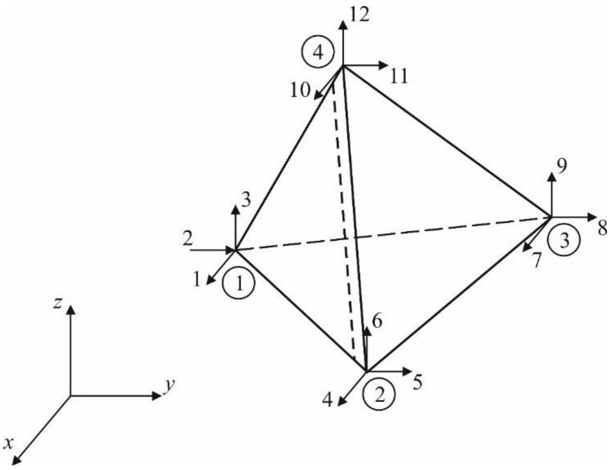


Figure 4. Composite finite element for solving a spatial problem for a composite - a tetrahedron describes a concrete, a rod (dashed line) describes an armature

The stiffness matrix of the composite finite element for solving a spatial problem

$$K = K_T + K_a\,, \tag{14}$$

where K_T for isotropic material is determined by the expression (17.186) [10, page 267], and the stiffness matrix of the reinforcing bar has the form:

$$K_{ai} = \frac{E_a A_{ai}}{l_i}\begin{bmatrix} (c_1c_1)_i & (c_1c_2)_i & (c_1c_3)_i & -(c_1c_1)_i & -(c_1c_2)_i & -(c_1c_3)_i \\ (c_2c_1)_i & (c_2c_2)_i & (c_2c_3)_i & -(c_2c_1)_i & -(c_2c_2)_i & -(c_2c_3)_i \\ (c_3c_1)_i & (c_3c_2)_i & (c_3c_3)_i & -(c_3c_1)_i & -(c_3c_2)_i & -(c_3c_3)_i \\ -(c_1c_1)_i & -(c_1c_2)_i & -(c_1c_3)_i & (c_1c_1)_i & (c_1c_2)_i & (c_1c_3)_i \\ -(c_2c_1)_i & -(c_2c_2)_i & -(c_2c_3)_i & (c_2c_1)_i & (c_2c_2)_i & (c_2c_3)_i \\ -(c_3c_1)_i & -(c_3c_2)_i & -(c_3c_3)_i & (c_3c_1)_i & (c_3c_2)_i & (c_3c_3)_i \end{bmatrix}. \tag{15}$$

where c_j –cosine of the slope angle of the face ***i*** with the ***j*** axis of coordinates.
If the bonding material in the plane problem is represented as an isotropic body, then the approximate solution of the plane problem for the finite element takes the following form:

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} = ((y_3 - y_2)u_1 + (y_1 - y_3)u_3 + (y_2 - y_1)u_5)\,/\,(2A); \\ \varepsilon_y = \dfrac{\partial v}{\partial y} = ((x_2 - x_3)u_2 + (x_3 - x_1)u_4 + (x_1 - x_2)u_6)\,/\,(2A); \\ \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} = ((x_2 - x_3)u_1 + (x_3 - x_1)u_3 + (x_1 - x_2)u_5 + (y_3 - y_2)u_2 + (y_1 - y_3)u_4 + (y_2 - y_1)u_6)\,/\,(2A). \end{cases} \tag{16}$$

4. The general model of a composite

In many cases, it is more convenient to formulate a stiffness matrix not from composite elements, but using the superposition principle (at least in increments) of the global matrixes of stiffness of the binder and reinforcement. Let us imagine separately the kinematic scheme of the reinforcement, which approximates the reinforcing material in the best way, and the kinematic finite element scheme of the body of the binder, which is most suitable for its calculation. We put only one condition: **all** nodes of the kinematic diagram of the reinforcement must coincide with a part of the nodes of the kinematic

scheme of the binder body.
We show this method using the example of a finite element model of a reinforced concrete beam symmetrical relatively to the central vertical axis. Figure 5 shows the kinematic finite element scheme of the concrete beam body. In Figures 6 and 7 shows the kinematic diagram of the reinforcement.

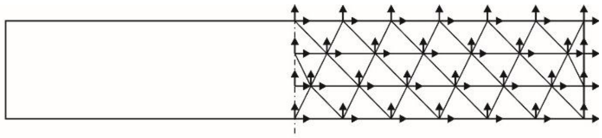


Figure 5. Kinematic model of the concrete body (to the left of the axis of symmetry)

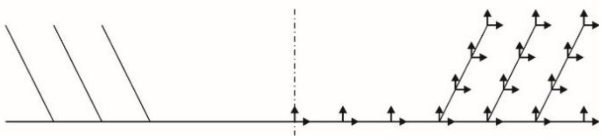


Figure 6. Kinematic model of the reinforcement (to the left of the axis of symmetry) with hinge-rod assemblies

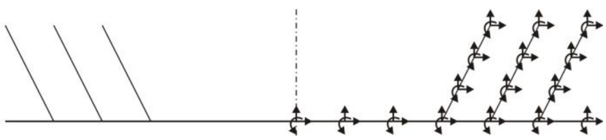
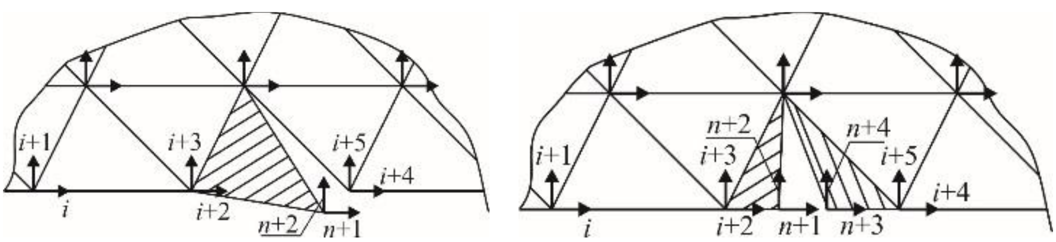


Figure 7. Kinematic model of the reinforcement (to the left of the axis of symmetry) with frame-rod assemblies

The separate global stiffness matrices of a concrete body and reinforcement must have the identical dimension. Then both matrices are summed, thus forming a global stiffness matrix of the composite system - of the reinforce concrete construction. In some cases, it can have null rows and columns. Physically, this means that the reinforcing rods can be nonconnected in one carcass.
When the kinematic schemes are superimposed on each other, the stiffness matrices fold together.
Calculation under cracking conditions is conveniently carried out in the form of a series of numerical experiments with increasing load. If, in performing any calculation, the deformations of the finite element exceed the limitations, then the number of displacements of one of the nodes of this final element of the concrete body should be doubled. In Figure 8a we show the kinematic diagram of the concrete body after breaking of the shaded element. In addition, the process of occurrence and movement of cracks can be modeled by introducing additional two nodes into the body of that concrete element which deformations exceed the limitations (Figure 8b). Trajectories of cracks will be the lines along which we are doubling the number of displacements.



a b

Figure 8. Cracking model for FEM.

Determination of the parameters of beam or diaphragm reinforcement can be performed using the following algorithm. We accept some redundant reinforcement scheme. After the finite-element calculation of the beam (diaphragm), we determine the deformations of the elements ε_x, ε_y, γ_{xy} and the main deformations of the elements $\varepsilon^+_{\max}, \varepsilon^-_{\min}$. If everywhere $\varepsilon^+_{\max} < [\varepsilon^+]$, $\varepsilon^-_{\min} > [\varepsilon^-]$ then reduce the amount of reinforcement. Otherwise, we finish the calculation.

5. Conclusion

Thus, the work deals with the calculation and selection of reinforcement and cross-sections of reinforced concrete beams, columns or diaphragms of rigidity. The main provisions of bounded deformations method are given. The basis for the calculation of this method is the criterion for limiting deformations in given zones of the most stressed normal sections of composite structures. On the basis of the method, the solutions of the problems of selecting the parameters of sections of reinforced concrete beams with a pre-stressed and non-stressed reinforcement are performed. The algorithm of the method for calculating statically indeterminate systems is presented and numerically realized.

When calculating the beams and the stiffening diaphragms, it is proposed to determine the points of nucleation of cracks and the trajectory of their development by solving the problem of determining the deformed state of a composite under conditions of a plane stressed state. Determination of the main deformations is performed by a modernized finite element method. The finite element calculation is based on specially developed composite elements describing the joint work of concrete and reinforcement. Two new (planar and spatial) composite finite elements are proposed. The scheme of formation of the general model of a composite is given. The algorithm for selecting the reinforcement of a composite system is shown.

Using the proposed model, the following design actions can be performed:

- perform a verification calculation with the definition of stresses and deformations in concrete and reinforcement;
- determine the trajectories of the development of cracks;
- optimize the mass and location of the reinforcement;
- determine the deflections of the reinforced concrete mechanical system.

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