Boltzmann principle of superposition in the theory of wood creep for deformations in time

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Abstract. A number of theoretical orientations can be distinguished in the field of wood creep study. The first orientation includes models based on different combinations of the simplest linear models: Hooke model of elastic body, viscous element of Newton, element of dry friction of Coulomb. The second orientation is based on the parallel connection of the elastic element with the viscous element. In the third orientation the elastic and viscous element are arranged sequentially. In the fourth orientation the Boltzmann theory of elastic inheritance is used. This article considers the feasibility and practicability of the study of wood creep within a framework of the latter of these orientations. The evidence of the erroneous of using this principle is given.

1. introduction

In accordance with the Boltzmann principle [1, 2], the linear creep equation of wood was recorded in 1954 in the following form (in the final notations):

$$\varepsilon(t) = \sigma(\tau_1)\delta(t,\tau_1) + \int_{\tau_1}^{t} \frac{d\sigma(\tau_1)}{d\tau}\delta(t,\tau)d\tau, \tag{1}$$

where $\delta(t,\tau) = \frac{1}{E(\tau)} + \left(C_0 + \frac{A_1}{\tau}\right)\left[1 - e^{-\gamma(t-\tau)}\right]$;

$\frac{1}{E(\tau)}$ is elasto-instantaneous deformation,

$C_0 = 0{,}32 \cdot 10^{-5}\ \mathrm{cm}^2/_{\mathrm{kgf}}$, $A_1 = 5 \cdot 10^{-5}\ \mathrm{cm}^2/_{\mathrm{kgf}}$, $\gamma = 0{,}04$ are wood creep constants derived from extensive experimental studies.

Prokopovich I.E. and Zedgenidze V.A. [3] recorded the measure of wood creep on the basis of experiments in the form:

$$C(t,\tau) = (C_0 + Ae^{-\gamma\tau})\left(1 - B_1 e^{-\gamma_1(t-\tau)}\right),$$

где $C_0 = 2{,}87 \cdot 10^{-5}(\mathrm{MPa})^{-1}$;
$A = 10{,}95 \cdot 10^{-5}(\mathrm{MPa})^{-1}$;
$\gamma_1 = \gamma = 0{,}15\left(1/_{\mathrm{day}}\right)$;
$B_1 = 1$.

After integrating by parts, the basic equation of wood creep takes the form:

$$\varepsilon(t) = \frac{\sigma(\tau_1)}{E(t)} - \int_{\tau_1}^{t} \sigma(\tau)\ \frac{d}{d\tau}\delta(t,\tau)d\tau, \tag{2}$$

where $\varepsilon(t)$ is the full deformation.

In the reasoning of equation (2), Prokopovich I.E. and Zedgenidze V.A. consider it possible to observe the influence of wood moisture "'as a result of the peculiar aging associated with the drying of wood". Such variant of the theory is developed by a number of scientists on the basis of experimental investigations of Ivanov U.M. At direct consideration of humidity parameter $\omega(t)$, integral equation (2) is written in the form:

$$\varepsilon(t) = \frac{\sigma(\tau_1)}{E[\omega(t)]} - \int_{\tau_1}^{t} \sigma(\tau)\ \frac{d}{d\tau}\delta[\omega(\tau),t,\tau]d\tau, \tag{3}$$

where $\delta[\omega(\tau),t,\tau] = \frac{1}{E[\omega(t)]} + \frac{\varphi[\omega(\tau),t,\tau]}{E[\omega(t)]}$,

$\varphi[\omega(\tau),t,\tau]$ is creep characteristic.

For example, Orlovich R.B. [4] gets:

$$\varphi[\omega(\tau),t,\tau] = \left\{\frac{\varphi_{\infty(\omega(\tau))}}{t-\tau}\int_0^t F[\omega(\tau+\rho)]d\rho\right\}\left[1-e^{-\beta(\omega)(t-\tau)}\right].$$

In a simpler assumption (Prokopovich I.E., Zedgenidze V.A.) it can be written:

$$\varphi[\omega(\tau),t,\tau] = \varphi_\infty[\omega(\tau)]\left[1-e^{-\beta(\omega)(t-\tau)}\right]. \tag{4}$$

The equations (1) and (2) are based on two fundamental assumptions:

-the principle of classical linear connection:

$$\varepsilon(t,\tau) = \sigma(\tau)\cdot\delta(t,\tau)\ \text{, or } \varepsilon[\omega(\tau),t,\tau] = \sigma(\tau)\cdot\delta[\omega(\tau),t,\tau]; \tag{5}$$

-the Boltzmann principle of linear superposition.

2. Application of Boltzmann principle in wood creep theory

Classical linear connection (5) is usually established by means of thorough experiments (simple loading) of the process of wood deformation with long time constant strain $\sigma_0 = \sigma(\tau_1)$. This is due to the fact that to detect a similar connection at variable strains $\sigma(t)$ the known behaviour of wood deformation at single constant strains is used. This known behaviour is described by a creep measure $C[\omega(\tau),t,\tau] = \frac{\varphi[\omega(\tau),t,\tau]}{E[\omega(t)]}$. We emphasize that there are non-stationary variables, because of the dependence on humidity - the function of wood aging $\varphi_\infty[\omega(\tau)]$ and modulus of elasticity $E[\omega(\tau)]$.

The Boltzmann principle of linear superposition is true only at very strong limitations on the function of aging and modulus of elasticity. In the case of structure (5) these restrictions are not feasible, and the theory of wood creep, built on the basis of this principle, proves to be deeply erroneous.

In non-stationary properties of wood, the Boltzmann principle of superposition incorrectly builds the creep core, incorrectly describes the processes of change of instantaneous deformations and creep deformations due to incorrect determination of the full rate of deformation value corresponding to (5).

According to structure (5), the deformation rate is equal to:

$$v_\varepsilon = \dot{\sigma}(\tau)\delta[\omega(\tau),t,\tau] + \sigma(\tau)\frac{\partial\delta[\omega(\tau),t,\tau]}{\partial\omega(\tau)}\dot{\omega}(\tau) + \sigma(\tau)\frac{\partial\delta[\omega(\tau),t,\tau]}{\partial(\tau)} + \sigma(\tau)\frac{\partial\delta[\omega(\tau),t,\tau]}{\partial t}.$$

It can be seen from this expression that in integral equation (1) the last three components of the deformation rate, caused by the rate of change of the full unit deformation, are lost $\delta[\omega(\tau),t,\tau]$. These losses are as follows:

$$-\sigma(\tau)\frac{E'[\omega(\tau)]}{E^2}\dot{\omega}(\tau)\sigma(\tau)\frac{1}{E[\omega(\tau)]}\frac{\partial\delta[\omega(\tau),t,\tau]}{\partial\omega(\tau)}\dot{\omega}(\tau) + \sigma(\tau)\frac{1}{E[\omega(\tau)]}\frac{\partial\varphi[\omega(\tau),t,\tau]}{\partial(\tau)} +$$
$$\sigma(\tau)\frac{1}{E[\omega(\tau)]}\frac{\partial\varphi[\omega(\tau),t,\tau]}{\partial t} - \sigma(\tau)\varphi[\omega(\tau),t,\tau]\frac{E'[\omega(\tau)]}{E^2[\omega(\tau)]}\dot{\omega}(\tau).$$

They are comparable in order of importance to the remaining (1) component. These losses cause significant discrepancies between the theory and the experiment described in the scientific literature on wood creep.

In application of the Boltzmann principle, errors are made with (and without necessity) opposite to actions under the important result of classical theory (5): first is differentiation and then integration. In the process of differentiation, many components of the rate of full wood deformation are lost, after integration the losses pass to the deformation value and then into practical application.

The Boltzmann principle distorts the classic linear relationship causing three types of errors: I- incorrectly finds the values of short-term deformations; II – incorrectly defines the core of an integral equation; III – erroneously adds instantaneous strains to the notion of creep deformations.

Case I. The rate of elastic deformation equals:

$$\dot{\varepsilon}_y(\tau) = \dot{\sigma}(\tau)\frac{1}{E[\omega(\tau)]} + \sigma(\tau)\frac{\partial \quad 1}{\partial\tau E[\omega(\tau)]}.$$

Integrating, we have:

$$\varepsilon_y(t) - \varepsilon_y(\tau_1) = \int_{\tau_1}^{t}\frac{1}{E[\omega(\tau)]}\dot{\sigma}(\tau)d\tau + \int_{\tau_1}^{t}\sigma(\tau)\frac{\partial \quad 1}{\partial\tau E[\omega(\tau)]}d\tau.$$

Integrating here the first term in parts and reducing, we will find the value of short-term deformation:

$$\varepsilon_y(t) = \frac{\sigma(t)}{E[\omega(\tau)]}. \qquad (6)$$

It can be seen that the first term under the integral sign (2) is superfluous and usage in (1) and (2) the Boltzmann principle:

$$\varepsilon_y(t) = \frac{\sigma(\tau_1)}{E[\omega(\tau_1)]} + \int_{\tau_1}^{t}\frac{1}{E[\omega(\tau)]}d\sigma(\tau) = \frac{\sigma(t)}{E[\omega(t)]} - \int_{\tau_1}^{t}\sigma(\tau)\frac{\partial \quad 1}{\partial\tau E[\omega(\tau)]}d\tau. \qquad (7)$$

is deeply mistaken.

The Boltzmann principle of superposition erroneously reconstructs the real elastic model of wood with modulus of elasticity $E[\omega(\tau)]$; it adds to it nonexistent model of linear liquid viscous with coefficient of viscosity $k_y(\tau) = \frac{E^2[\omega(\tau)]\,1}{\dot{E}[\omega(\tau)]\dot{\omega}(\tau)}$,forming in such a way the Maxwell scheme.

Consider, the example, putting in (6), (7) $\sigma(t) = \sigma_0 = const$, we will get:

$$\varepsilon_y(t) = \frac{\sigma_0}{E[\omega(\tau)]} \text{ and } \varepsilon_y(\tau_1) = \frac{\sigma_0}{E[\omega(\tau\tau_1)]}.$$

Comparison of these deformations is shown in Figure 1.

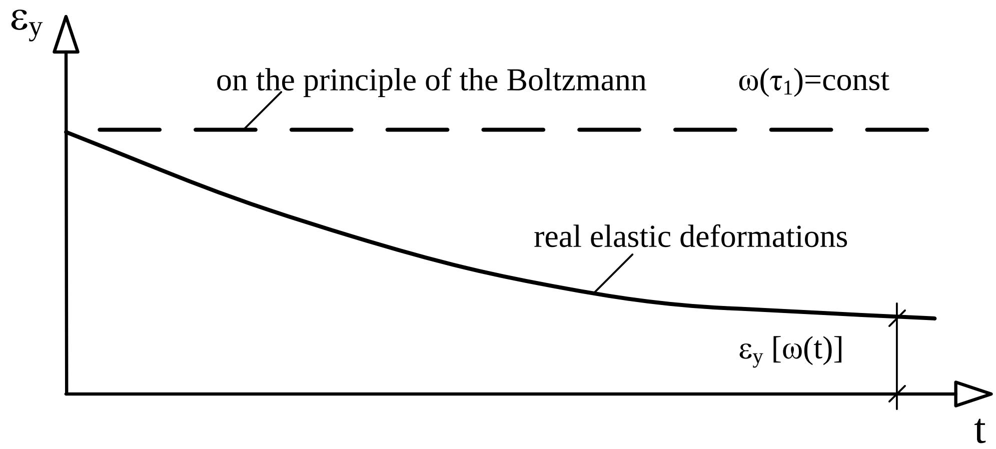

Figure 1. Comparison of deformation graphs

Case II. In the field of linear creep, the number of additional (fictitious) bodies arising due to the wrong scheme of creep core (inherited function of the 1-st kind) increases significantly. It depends on the type of function $\varphi[\omega(\tau), t, \tau]$, describing the non-stationary characteristic of wood creep. Let's write it down, for example, in the form (6).

In this case, the basic law (5) forms the set of redundant (non-existent) bodies connected in series with each other: two bodies of Voigt type and two viscous elements. The creep deformations of these bodies are equal:

$$\varepsilon_1(t) = \int_{\tau_1}^{t} \sigma(\tau) \frac{1}{k_1(\tau)} e^{-\beta(t-\tau)} d\tau, \quad k_1(\tau) = \frac{E[\omega(\tau)]}{\varphi'_\infty[\omega(\tau)]\dot{\omega}(\tau)}; \tag{8}$$

$$\varepsilon_2(t) = \int_{\tau_1}^{t} \sigma(\tau) \frac{1}{k_2(\tau)} d\tau, \quad k_2(\tau) = \frac{E^2[\omega(\tau)]}{E'[\omega(t)]\varphi_\infty[\omega(t)]\dot{\omega}(\tau)}; \tag{9}$$

$$\varepsilon_3(t) = \int_{\tau_1}^{t} \sigma(\tau) \frac{1}{k_3(\tau)} e^{-\beta(t-\tau)} d\tau, \quad k_3(\tau) = -\frac{E^2[\omega(\tau)]}{E'[\omega(t)]\varphi_\infty[\omega(t)]\dot{\omega}(\tau)}; \tag{10}$$

$$\varepsilon_4(t) = \int_{\tau_1}^{t} \sigma(\tau) \frac{1}{k_4(\tau)} d\tau, \quad k_4(\tau) = -\frac{E[\omega(\tau)]}{\varphi'_\infty[\omega(t)]\dot{\omega}(\tau)}; \tag{11}$$

where k_1,k_2,k_3,k_4 are viscosity factors or internal friction factors of fictitious bodies; moreover, Voigt body (10) and body (11) of the viscous element expand at compression.

Creep deformations (8)-(11) caused by the influence of the superposition principle are summed up with a short-term fictitious deformation:

$$\varepsilon_5(t) = \int_{\tau_1}^{t} \sigma(\tau) \frac{\partial \quad 1}{\partial\tau E[\omega(\tau)]} d\tau,$$

and introduce large errors in the value of full deformation determined by the creep law (3).

Essence and the secondariness of the Boltzmann scheme for the theory of wood creep are investigated on the rheological model described by Ugolevym B.N. [5] and Latishenko V.A. [6], in the record of Ishlinsky A.U.:

$$\frac{\partial\sigma}{\partial t} + r\sigma = b\frac{\partial\varepsilon}{\partial t} + bn\varepsilon,$$

what gives:

$$\varepsilon(t) = \frac{\sigma(t)}{b} - \int_{t_0}^{t} \sigma(t) \frac{1\partial\varphi(t-\tau)}{b \quad \partial(\tau)} d\tau \tag{12}$$

where $\varphi(t-\tau) = b\frac{r-n}{b}\left[1 - e^{-n(t-\tau)}\right]$.

The Boltzmann case is obtained from (12) by its series transformations, mathematically appropriate only in the conditions of stationary properties:

$$\varepsilon(t) = \sigma_0\left[\frac{1}{b} + \frac{1}{b}\varphi(t-t_0)\right] + \int_{t_0}^{t}\left[\frac{1}{b} + \frac{1}{b}\varphi(t-\tau)\right] d\sigma(\tau). \tag{13}$$

In the transformation (13), as opposed to (12), ductility function is used, which attracted attention of scientists. However, transformation (13) is possible only with very strong limitations. With the broad interpretation of ductility, in the conditions of non-stationary wood, these restrictions were not taken into account, and the theory of creep proves to be deeply erroneous.

First, instantaneous deformation with extremely simple physical meaning for arbitrary time t is imposed the property of the process that creates the temptation to expand the theory and turns into the above-mentioned gross error at non-stationary variables $E[\omega(t)]$.

Secondly, it is necessary to integrate (12) in parts, that at broad interpretation of ductility function in the conditions of aging (4) creates another temptation, traditionally leading to another blunder when finding the core of the integral equation; as it is known, at non-stationary properties of wood creep deformation is obtained from another solution of differential equation, solution, written in more complex form:

$$\varepsilon_n(t) = e^{-F(t)}\left[\varepsilon_{n0} + \int_{t_0}^{t}\sigma(t)\frac{1}{\eta(t)}e^{F(t)}dt\right]; F(t) = \int_{t_0}^{t} n(t)dt,$$

when parameters $b(t)$ and $n(t)$ are time functions.

In the wood of Ugolev B.A., Latishenko V.A. the deformation rate is degenerated due to the difference core. The Boltzmann principle distorts the essence of non-stationary wood model: one classical wood creep body it replaces by the chain model of sequentially connected bodies with set of erroneous properties.

In the theory of wood creep there is a case when at a difference core of integral equation, the broad interpretation of ductility function is unacceptable. For example, the creep core is represented as (second case):

$$k(t-\tau) = \frac{\mathrm{A}e^{-n(t-\tau)}}{(t-\tau)^{\propto -1}}.$$

This kinematic equation of motion, in relation with the expansion of the reverse problem of mechanics, corresponds to certain forces. Here, from the analysis of the differential equation of creep it is found out that in this core there is a force of resistance with coefficient of viscosity, equal $\eta(t,\tau') = A_1(t-\tau)^{\propto -1}$, that it is impossible within the meaning of the coefficient of viscosity in the frames of resistance forces of Newton mechanics.

Case III. Corresponds to the broad interpretation of the ductility function in the ' ' chain model ' '. It is presented in the theoretical rheology, and as a repetition in the theory of wood creep (Schanzlin J., Toratti T., Becker P., Martensson A., Hanhijarvi A. and others).

Let us pre-write the Boltzmann scheme for the Maxwell body:

$$\varepsilon(t) = \sigma_0\left[\frac{1}{E_0} + \frac{1}{\eta}(t-t_0)\right] + \int_{t_0}^{t}\left[\frac{1}{E_0} + \frac{1}{\eta}(t-\tau)\right]d\sigma(\tau), \tag{14}$$

where η is a stationary viscosity coefficient.

In the ' ' chain model ' ', by serial connection of the bodies (12) and (14), we have expansive record of ductility function:

$$\sigma(t-\tau) = \frac{1}{E_0} + \frac{1}{E_0}\varphi(t-\tau) + \frac{1}{\eta}(t-\tau)\ , \tag{15}$$

A pair of integral equations corresponding to the expansive scheme (15) and permitted either in relation to deformation or relative to strains in theoretical rheology are called ' ' equations of Boltzmann-Volterra ' '; It is also noted that this pair ' ' represents the full mathematical formulation of the principle of linear superposition ' '.

However, such ' ' chain model ' ' with its broad interpretation of the ductility coefficient, is essentially erroneous. This is demonstrated by bringing it to differential form:

$$\ddot{\varepsilon}(t)\frac{\eta}{n} + \dot{\varepsilon}(t)\eta = \ddot{\sigma}(t)\frac{\eta}{E_0 n} + \dot{\sigma}(t)\left(\frac{\eta}{E_0} + \frac{1}{n} + \frac{\eta}{c_0}\right) + \sigma(t), \tag{16}$$

where $c_0 = \frac{b}{r-n}$.

From (16) it can be seen that there is a force of resistance $\ddot{\varepsilon}(t)\frac{\eta}{n}$ proportional to acceleration which is incompatible with classical mechanics, and, in connection with article 5.1.1 (3) P EN 1990, ' ' chain model ' ' is inconsistent to calculation model.

Components of force of the calculation model can be function from position $\varepsilon(t)$, rate $\dot{\varepsilon}(t)$, time and other variables. If there is (among others) force proportional to acceleration $\ddot{\varepsilon}(t)$, the fundamental principle of mechanics about independence of actions of forces turns out to be violated. The well-known scientist Pars L. found unacceptability of such forces both in tasks of mechanics, and in applications [7].

3. Conclusion

' ' Chain models ' ' are often used to investigate the wood creep, beginning with the publication of Burgers [8], complemented the classic Kelvin scheme with a viscous element, which led to the occurrence of resistance forces proportional to acceleration. The same defect is found in investigations of wood, which complement classic Voigt scheme with viscous element.

In case of non-linear creep and short-term non-linearity of wood, the application of the Boltzmann scheme is also erroneous.

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