Identification of Projectile Structure Response Combined with Modal Analysis and Blind Source Separation

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Abstract. In order to study the frequency domain response of the projectile structure in the process of hard target penetration, a method of combining modal analysis and blind source separation is proposed to identify the response of projectile structures. Firstly, the natural frequency range of the projectile structure under the free state is given through the simulation analysis of the projectile structure, and the resonant frequency of the projectile structure is obtained when the periodic load is excited. Further, through the ensemble empirical mode decomposition, ascending dimension, recombination and blind source separation of the test signals, the penetration characteristics and high-frequency vibration signals of the projectile are obtained. Finally, comparing the simulated resonant frequency and the high-frequency vibration frequency after separation, the similar component is recognized as the structural response of projectile penetrating process.

1. Introduction

The process of projectile penetrating target is very complicated, and its impact signal includes the acceleration of the projectile penetrating the rigid body, the response acceleration of projectile structure by penetration stress wave and the external noise signal. The acceleration signal waveform of the rigid body reflects the impact force signal of the projectile, which is the main component of the projectile penetrating the target and reacts the motion characteristics of the projectile penetration process (acceleration, velocity and displacement)[1]. The acceleration of projectile structure is manifested as the high frequency vibration of projectile penetrating curve, coupled to rigid body overload in oscillating form. When the projectile penetrates the hard targets, such as steel target, concrete and rock, the vibration of stress wave in the body of the projectile directly affects the structure of the projectile, the stability of the internal charge and the reliability of the fuse. At the same time, the superposition of the stress wave on the rigid acceleration of the projectile is also not conducive to the study of the penetration characteristics of the projectile. Therefore, it is necessary to study the structural response of projectile in the process of projectile penetrating into target.

At present, the method of response analysis of missile body structure is mainly focused on two aspects: the finite element simulation of missile body structure, and the frequency domain characteristic analysis of overload signal of projectile penetrating. The modal simulation of the structure of the projectile is a method for the projectile structure by the finite element simulation software. Through the modal analysis of the structure of the projectile, the natural frequency and the vibration modes of the projectile in the free state can be obtained. However, when the finite element software is applied to the modal analysis of the test projectile, and the test projectile is usually analyzed according to the space free body due to the inability to obtain the real constraint conditions

of the projectile in the penetration process. This leads to the difference between the frequency of the projectile and the real value . So sometimes it is inaccurate to accord the natural frequency filtering [2]. Frequency domain characteristic analysis is another processing method for projectile penetrating overload signal. At present, there are many methods are used to analyze acceleration signals of penetration processing ,such as Classic Fast Fourier Transform, Wavelet Transform and Ensemble Empirical Mode Decomposition (EEMD). In the literature [1], EEMD decomposition is used, and the natural frequencies of missile body structure overload are obtained by comparing the BURG power spectrum of IMF component and original signal. Document [3] proposed a wavelet threshold filter for penetrating signals. Document [4] proposed a time-frequency analysis algorithm based on EEMD and Choi-Williams distribution, and the time-frequency distribution of the signal were obtained . The above frequency analysis method can obtain the vibration frequency of the projectile penetration process. But the frequency by time-frequency analysis is smaller than the actual value . Document [5] adopted the method of combining modal analysis with measured data spectrum analysis to test projectile. EEMD algorithm is used to decompose the measured penetration signal, and the frequency consistent with the harmonic response is obtained, and the corresponding frequency of the projectile structure response is estimated.

In summary, it is found that the finite element analysis of single projectile structure and the time-frequency analysis method of the penetration overload signal can obtain the approximate response of the projectile structure, but the analysis results using the combined methods are more closer to the experimental results. In this paper, a Blind Source Separation (BBS) technique is introduced, and a frequency domain response analysis method for penetrating projectile structures is proposed based on integrated modal analysis and blind source separation. Through the simulation analysis of the structure of projectile, the method gives the natural frequency range of the body structure under the free state, and excuses the vibration by periodic load to find the resonant frequency of the structure of the projectile. The test signal is further decomposed, raised and reassembled, and the Second Order Blind Identification(SOBI) algorithm is used to separate the projectile penetrating rigid body features and high frequency vibration signals. Finally, by comparing the resonant frequency and the high-frequency vibration frequency after separation, the consistent component is considered as the structural response of projectile penetrating process.

2. Mode Analysis and Blind Source Separation Theory

2.1 Modal Analysis Theory

The process of projectile penetration is more complicated. The motion of a projectile belongs to a multi degree of freedom vibration system, whose vibration characteristics include natural frequencies, modes of vibration, etc. According to the theory of mechanical vibration, the vibration equation of the projectile is as follows:

$$M\ \ddot{s} +\ \ C\ \ \dot{s} +\ \ K\ \ s =\ \ F\ (t\ ) \qquad (1)$$

In the formula, M is the mass matrix of the projectile system, C is the system damping matrix, the K is the stiffness matrix, and the F (t) is the simple harmonic load matrix of the system, and the $\ddot{s}$ 、 $\dot{s}$ 、 s are the acceleration, velocity and displacement of the system respectively.

Before penetration test, the projectile is neither restrained nor subjected to any external excitation. At this point, F (T) = 0. The eigenvalues and eigenvectors of the formula (1) are the solutions of the natural frequencies and corresponding main modes of the system in free state[6].

In order to get the resonant frequency of the projectile under test resistance, F（t）=Fsin θ t，θ is the exciting force frequencies. By defining the harmonic response file of the projectile node in the finite element simulation analysis software, the resonant frequency of the missile body structure under excitation state can be obtained.

2.2 Blind Source Separation Theory

The blind source separation (BSS) method can recover the source signal from the observed signal when the transmission channel characteristics are unknown or the input information is unknown[7]. At present, BSS method has been widely used in speech recognition, image processing, biomedical signal processing and mobile communication, and other fields [7].The common blind source separation algorithms have Independent component analysis (ICA) based on characteristic matrix, Joint Approximate Diagonalization of Eigenmatrices (JADE), se-order blind identification (SOBI), and four order blind identification (FOBI). Their common feature is to use the diagonal matrix to solve the separation matrix, and then separate the source signals.

In this paper, the SOBI algorithm is applied to blind source separation of X (t) of test signals. The separation process is shown in Figure 1. The basic steps are as follows:
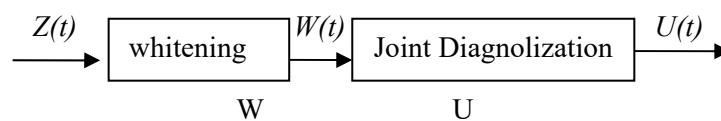
Z(t) → [whitening] W(t) → [Joint Diagnolization] U(t) →
           W   U

Figure 1. SOBI method flow chart

(1) The centering of the separation signal Z(t);
(2) Centralized signal whitening, Z (t) ➜ W (t);
(3) W(t) joint diagonalization, estimating the mixing matrix U(t), W(t)➜U(t).
Through the above steps, vibration source estimation of the source signal can be obtained

$$U(t) = UWZ(t) \qquad\qquad (2)$$

3. Mode Experimental Analysis

The test projectile has a full-length 640mm and a maximum diameter of 98mm, with aluminium filler and testing device inside. In the modal analysis, the finite element model of the projectile is built according to the physical structure, and its structure is shown in Figure 2. Material properties of the body and filling material such as table 1.
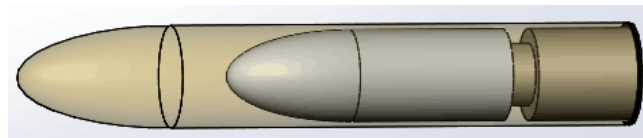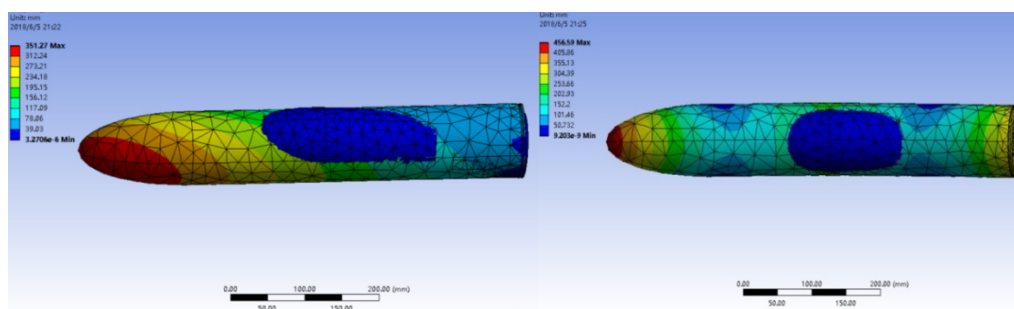
Figure 2. Finite element model of projectile

Table 1. Material properties of the model materials

Material	E /GPa	NUXY	DENS/ （g/cm3）
Steel	210	0.33	7.85
Aluminium	70	0.3	2.70
Testing device	200	0.3	4.8

Modal analysis of the finite element model of the projectile is carried out by Analysis Workbench, and the natural frequencies and the main vibration modes of the projectile in free state are obtained. It can be seen from table 1 that the first 6 order modes of the projectile are mainly rigid body vibration, and the natural frequencies are all close to 0. From the seventh order mode, the bending, torsion and tensile compression of the projectile are not rigid body vibration, and the bending vibration mode usually appears in pairs, and the frequency is almost equal. This paper mainly studies the axial impact and axial compression of projectile, and shows the axial tensile and compression modes of the projectile as shown in Figure 3.

Table 2 . Modal analysis result of test projectile

Order	Natural frequency /Hz	Modal shape
1	0	rigid vibration
2	0	rigid vibration
6	0	rigid vibration
7	612.32	bending vibration
8	614.45	bending vibration
9	936.27	torsional vibration
10	1037.37	tensile compression vibration
11	1037.26	bending vibration
12	1039.42	bending vibration
13	1108.32	torsional vibration
14	3789.35	tensile compression vibration

（a）the 10th vibration mode   （b）the 14th vibration mode

Figure 3. Axial tension and compression model of the projectile

In order to simulate the resonant frequency of the projectile subjected to penetration, the harmonic response of the projectile is applied at the bottom of the projectile, and the projectile end is constrained by the actual penetration scene, and the axial overload frequency curve of the projectile fuze position is obtained. It is found that the resonant frequency is about 1045Hz and 3801HZ respectively, and near 1045Hz, the axial tension compression deformation of the projectile is the largest.

Through the static structural modal analysis and harmonic response analysis of the projectile, we know that the maximum vibration amplitude of the projectile is the corresponding frequency of 1045 Hz.

4. Blind Source Separation for Penetration Overload Signal

The penetration test of projectile is very complicated. Generally, a special test device is placed at the bottom of the projectile to get the overload signal. The classical blind source separation algorithm is applied when the number of observation signals is not less than the number of source signals. The penetration test of the projectile is limited by the volume of the projectile, the measured instrument is not too much, the measured signal is a single observation signal, and the penetrating source signal includes the projectile penetration of the rigid body acceleration, the structure response acceleration and the external noise signal formed by the stress wave of the projectile, which belongs to the underdetermined blind source of the typical observation signal which is less than the source signal. The problem of separation. In this paper, the total empirical mode decomposition (EMD) is used to complete the decomposition of the signal and the reconstruction of the signal. Then the two order

blind identification algorithm is used to separate the reconstructed signal, and then the rigid body features and the response of the projectile structure are extracted.

The specific process is as follows:

1) The penetration overload test signal S(t) is EEMD-decomposed and the obtained IMF component. The test signal S (t) is shown in Figure 4 , and the decomposed IMF component is shown in Figure 5.
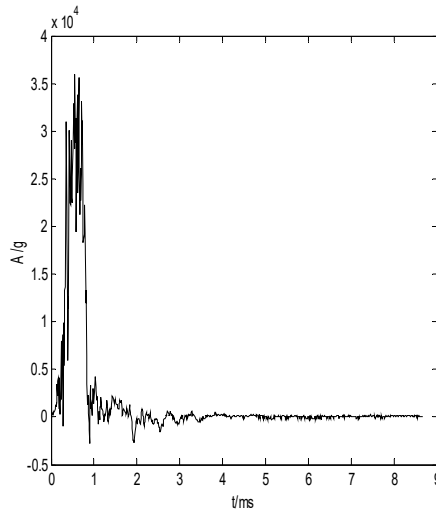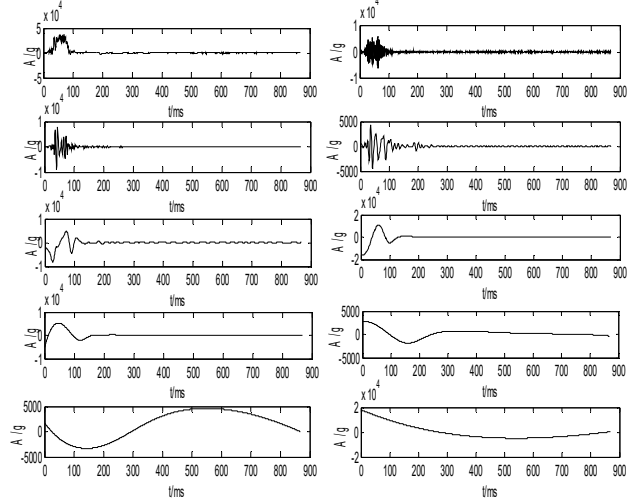

Figure 4. The penetration overload signal S (t)

Figure 5. The IMF component after the decomposition of signal S (t)

2）the decomposed intrinsic mode function IMFi and the original signal S(t) constitute a new multi-dimensional observation signal Y(t).

$$Y(t) = [s(t), IMF_1, IMF_2, IMF_3, IMF_4, IMF_5, IMF_6, IMF_7, IMF_8, IMF_9, IMF_{10}]^{\mathbf{T}} \quad （3）$$

3）Singular value decomposition is performed on the multi-dimensional observation signal Y(t) autocorrelation matrix Ryy, and the number of vibration sources of the test signal is determined according to dominance ratio of the first K eigenvalues.

The autocorrelation matrix of the recombinant multi-dimensional observation signal Y(t) can be expressed as:

$$R_{yy} = \mathbf{E}[Y(t)\ Y^{\mathbf{H}}(t)] \tag{4}$$

Where, $\cdot^{\mathbf{H}}$ indicates complex conjugate transformation.

Singular value decomposition of R_{yy} ,

$$\begin{aligned} R_{yy} &= \mathbf{E}[Y(t)\ Y^{\mathbf{H}}(t)] \\ &= V_S \mathbf{\Lambda}_S \mathbf{V}_S^{\mathbf{T}} + V_N \mathbf{\Lambda}_N \mathbf{V}_N^{\mathbf{T}} \end{aligned} \tag{5}$$

In the formula, V_S is the main component arranged in descending order after the signal decomposition, V_N corresponds to the decomposed noise features, $\Lambda_S = diag\{\lambda_1 \geq \lambda_2 \dots \geq \lambda_n\}$ and Λ_N is the corresponding feature value of the signal main component V_S and noise signal V_N.

At this point, estimation of the number of signal sources is completed for the principal components that contain most of the characteristics of the signal by using dominance ratio λ_K of the first K eigenvalues.

$$\lambda_K = (\delta_1 + \delta_2 + \dots \delta_k) / (\delta_1 + \delta_2 + \dots + \delta_n) \tag{6}$$

When i takes a certain value k, λ_K is a value close to 1, indicating that the information amount of the previous i decomposed singular values has been dominant. If we continue to take i= m+1, m+2, ..., etc. to reconstruct the signal, λ_K will change slowly due to the closeness of the decomposed singular value. At this time, it is considered that the signal reconstruction dimension is optimal. It does not make any sense if i value continues to be increased, which will also increase the computing amount of signal processing.

Table 3. Eigenvalue ΛS of multi-dimensional observation signal Y(t)

singular value	λ1	λ2	λ3	λ4	λ5	λ6	λ7	λ8	λ9
value	7.955	2.6549	0.8562	0.1086	0.0753	0.0609	0.0416	0.0252	0.0110

Dominance ratio of the first K eigenvalues of signal Y(t) is calculated as shown in Table 4.

Table 4 . Dominance ratio of the first K eigenvalues of signal Y(t)

No.	eigenvalue	Ratio of eigenvalue	Dominance ratio of the first *K* eigenvalues
1	7.9550	67.50%	67.50%
2	2.6549	22.52%	90.02%
3	0.8562	7.26%	97.28%
4	0.1086	0.92%	98.20%
5	0.0753	0.64%	98.84%
6	0.0609	0.52%	99.36%
7	0.0416	0.35%	99.71%
8	0.0252	0.21%	99.92%
9	0.0110	0.08%	1

From Table 4, it can be seen that when K = 3, the dominance ratio λ_K of the first K eigenvalues of the signal is 97.28%, indicating that the first three principal components have been dominant after the signal decomposition. When K value continues to increase, λ_K changes little. At this point, it can be determined that the number of primary vibration sources of the observation signal is 3, which is consistent with the hypothesis.

4) According to the estimated number of vibration sources, take the corresponding number of intrinsic modal functions and the original signal to form separation signal Z(t), and use second-order blind identification algorithm to achieve blind separation of the signal.

According to the above estimated number 3 of signal vibration sources, take the intrinsic modal functions IMF1, IMF2 with the largest correlation coefficient with observation signal and penetration overload signal S(t) to constitute the separation signal $Z(t) = [s(t), IMF_1, IMF_2]^T$. Then, adopt the second-order blind identification algorithm for signal separation. The signal components after separation are shown in Figure 6.
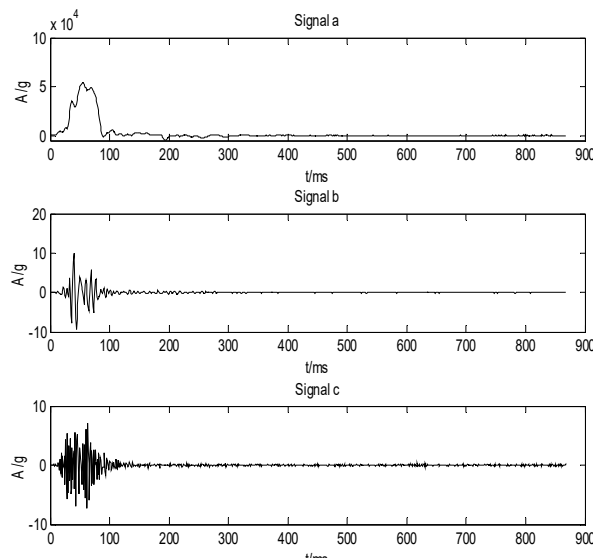
Figure 6. Signal components after $Z(t)$ blind source separation

It can be seen from Figure 6 that after the projectile penetration overload signal is processed by the method described herein, three independent vibration components a, b, and c can be obtained. Compared with the penetration time curve S(t) in Figure 4, it is found that the separated signal component a contains most of the characteristics of signal S(t), and the curve is relatively smooth; signal components b and c are two high-frequency vibration signals with low amplitude. The spectrum analysis of test signal S(t) and separated signal components b and c continues as shown in Figures 7 and 8. The frequency domain of the separated signals b and c is found to be 1071 Hz and 3871 Hz, respectively, which is very close to the frequency spectrum of the test signal S(t).

At this point, it can be considered that after separation of the test signal S(t) by EEMD-BSS method, not only the rigid body acceleration characteristics (signal a) of projectile penetrating target body is acquired, but also structural response(signal b, and signal c) of the projectile during the penetration is extracted. The corresponding frequency spectrum 1071Hz of signal b is the natural vibration frequency of the projectile itself during penetration, while corresponding frequency spectrum 3813Hz of signal c is secondary or multiple resonant frequency of the projectile.
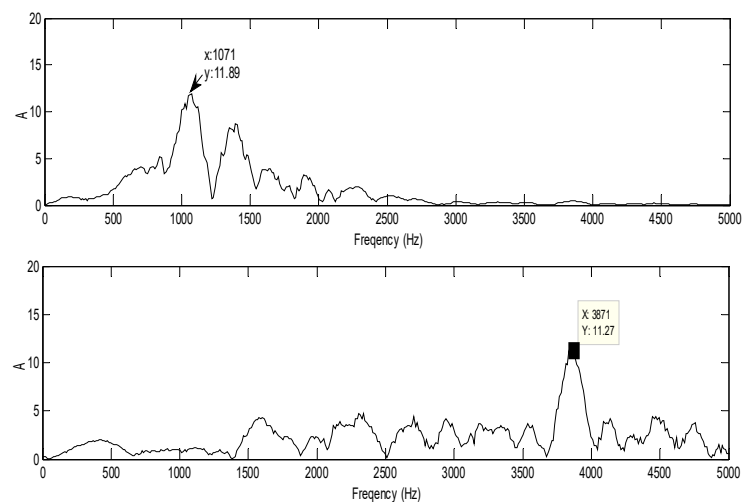

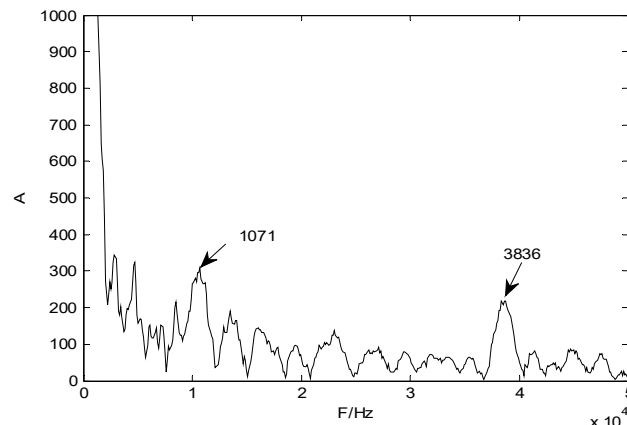Figure 7. Signal component b, c spectrum

Figure 8. Frequency domain analysis of penetrating signal

　　Comparing the natural frequencies of each order and the frequency of the penetrating separation signal from the finite element analysis of the test projectile, we can find that the separated signal B corresponds to the spectrum 1071Hz, the corresponding spectrum 3813H of the signal C is very close to the 10 order and 14 order natural frequencies of the axial tensile compression of the projectile. It is verified that the corresponding spectrum of the separated signal B and the signal C is the structural response of the projectile penetration process, and the structure response of the projectile is mainly composed of the low order axial tensile compression mode, and the amplitude is maximum at the 1 order axial tensile compression 1071HZ frequency, reflecting the axial structure mode of the projectile penetrated by the projectile.

5. Conclusion
Through modal analysis and harmonic response analysis of the projectile, the corresponding frequency of the projectile in the maximum axial amplitude is 1045Hz. At the same time, the two order blind identification algorithm is used to separate the high frequency vibration signal of the projectile, and the inherent vibration frequency 1071Hz of the body is obtained. The frequency is very close to the frequency of the modal analysis and the analysis of the harmonic response. The relative error of the two is 3.3%. It can be considered that the inherent frequency of the structure of the projectile is close to 1071Hz. In this study, the method of combining modal analysis and undetermined blind source separation of projectile structure parameter identification method can be used to estimate the frequency range of the projectile structure more accurately.

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