

# Modeling and Analysis of Four-phase Interleaved Synchronous Rectifier

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**Abstract.** Because of the efficiency and heating problem, the single-phase BUCK converter cannot be applied to the situation of low voltage and large current. Under the same stress conditions, the multiphase parallel BUCK converter cannot only reduce the output ripple, improve the converter efficiency, but also reduce the volume of the converter. The four-phase interleaved synchronization BUCK converter is used as the research object, and the corresponding mathematical model was established to describe the instantaneous ripple of the converter. Finally, the model was built with Simulink to verify the simulation. The simulation results show that the four phase synchronous rectifier improves the output current and reduces ripple compared with single-phase.

## 1. Introduction

The single-phase BUCK converter is limited by the stress of the switch and cannot be applied to low-voltage and high-current applications. With the multiphase parallel topology, a larger output current can be obtained under the same stress condition of the switch device. At the same time, the output ripple can be reduced, the efficiency of the converter can be improved and the volume of the converter can be reduced by using the principle of ripple elimination of the interleaved converter<sup>[1-2]</sup>. In the process of solving the ripple, the average current method is mostly used, its disadvantage is that it cannot accurately describe the instantaneous ripple of the converter, but in practical use, we need to describe the instantaneous ripple<sup>[3-5]</sup>. In this paper, a four-phase interleaved synchronization BUCK converter is taken as the research object. By establishing the corresponding mathematical model, the instantaneous value of the inductance current of the converter is described. Finally, the Simulink is used to verify it. It has a certain reference value for the research of multiphase parallel topology and the improvement and compensation of ripple.

## 2. Theoretical analysis

In this paper, a four-phase interleaved synchronous BUCK converter is taken as an example to discuss, and the main circuit as shown in Figure 1.  $Q_{11}$ ,  $Q_{21}$ ,  $Q_{31}$ ,  $Q_{41}$  represents the main control switch of the first phase to the fourth phase, respectively, the basic assumption is as follows:

(1) The switch device MOSFET in the circuit is an ideal device without inertia, so switching state is completed in an instant;

(2) Switch off the device has the same on-resistance, and is  $R_{on}$ , the filter inductor DC equivalent internal resistance of  $R_L$ , the corresponding equivalent DC resistance  $R_0 = R_{on} + R_L$ ;



(3)  $C_1$  is large enough, that is, the DC power supply has no ripple, and the distributed inductance of the DC bus is  $L_S = 0$ ;

(4) The inductance is large enough to make the inductance current approximately linear;

(5) There is no circulation between phases.

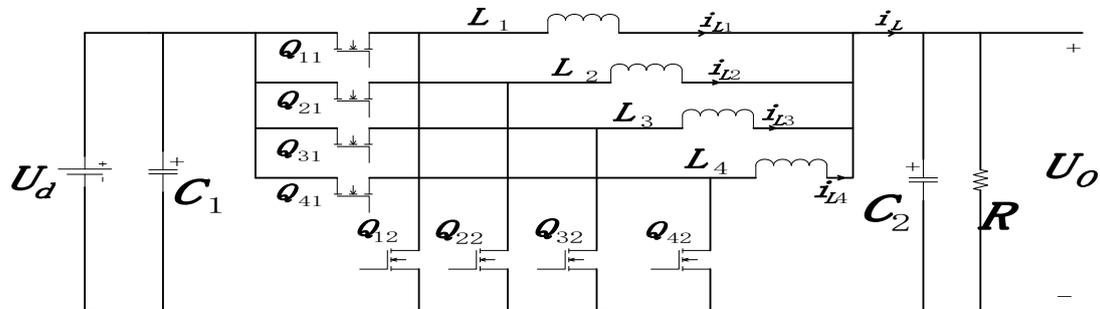


Figure 1. Main circuit topology

The duty ratio of this paper is 33.3%. At this time, the control pulse and the phase current waveform, as shown in Figure 2, the main control switch of the first phase to the fourth phase is interlaced by 90 degrees in turn. Combined with control pulse and phase current waveform, it can be found that in a switching cycle, the converter is divided into eight working states.

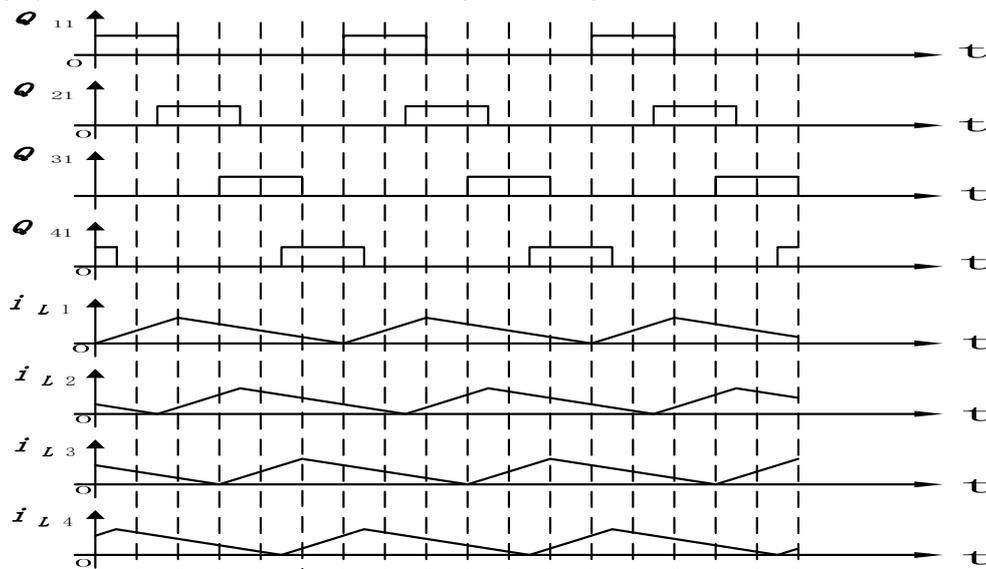


Figure 2. Waveform

### 3. Ripple reflection

The following example is taken to calculate the ripple, and the corresponding equivalent circuit is shown in Figure 3.

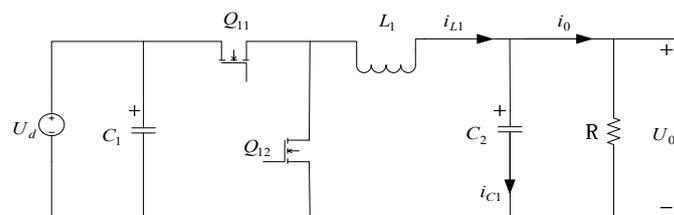


Figure 3. Equivalent circuit

According to the relationship between capacitance voltage and current, combined with the KCL theorem can be obtained:

$$\begin{cases} i_{C1} = i_{L1} - i_0 \\ i_c = \frac{dU_c}{dt} \end{cases} \tag{1}$$

The equivalent transformation can be obtained:

$$\begin{aligned} \Delta U &= \frac{1}{C} \int_0^T i_c dt \\ &= \frac{1}{C1} \int_0^T (i_{L1} - i_0) dt = \frac{1}{C1} \int_0^T i_{L1} dt - \frac{1}{C1} * I_0 * T \end{aligned} \tag{2}$$

As a result, it can be obtained:

$$U_C = U_0 + \frac{1}{C1} \int_0^T i_{L1} dt - \frac{I_0 T}{C1} \tag{3}$$

According to formula (3), it can be found that the amount of fluctuation depends on the variation of the inductance current or the total current variation.

#### 4. The establishment of mathematical models

##### 4.1. The establishment of expressions

Taking a phase as an example, when the main switch Q is turned on, the equivalent circuit diagram is shown in Figure 4.

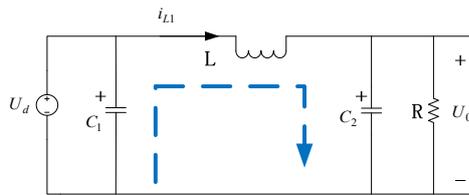


Figure 4. Switch tube conduction

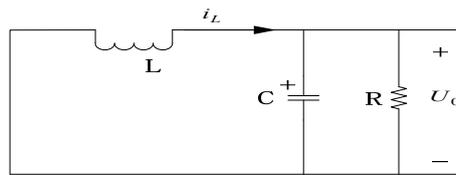


Figure 5. Switch off

The corresponding voltage and current relation can be obtained:

$$L \frac{di_L}{dt} = U_d - U_0 \quad , \quad \Delta I = \frac{1}{L} (U_d - U_0) t \tag{4}$$

Thus the instantaneous current value can be expressed as:

$$i_L = i_0 + \frac{1}{L} (U_d - U_0) t, \quad 0 \leq t \leq t_{on} \tag{5}$$

Where  $i_0$  is the initial value, that is, when  $t = 0$ ,  $i_0 = i_L$ .

When the main switching transistor Q is turned off, the equivalent circuit is shown in Figure 5, and the corresponding voltage-current relationship is available:

$$L \frac{di_L}{dt} = -U_0 \quad , \quad \Delta I = -\frac{1}{L} U_0 t \quad 0 \leq t \leq t_{off} \tag{6}$$

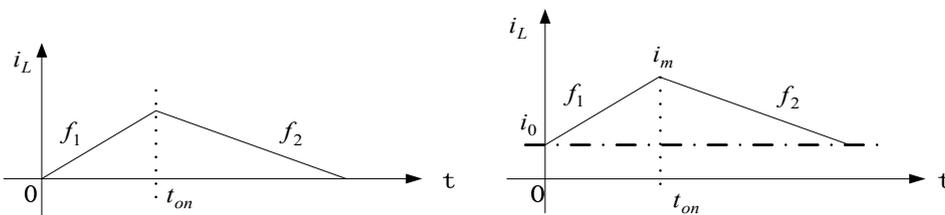


Figure 6. Critical and continuous modes

As shown in Fig. 6, a critical state mathematical model is established. The slopes of the  $f_1$  and  $f_2$  segments can be expressed as:

$$K_{f_1} = \frac{1}{L} (U_d - U_0) \quad , \quad K_{f_2} = -\frac{1}{L} U_0 \tag{7}$$

Correspondingly, in continuous mode,  $f_1$  and  $f_2$  can be expressed as:

$$\begin{cases} f_1 = \frac{1}{L}(U_d - U_0)t + i_0, 0 \leq t \leq t_{on} \\ f_2 = \frac{-(t - t_{on})}{L}U_0 + k, t_{on} \leq t \leq T \end{cases} \quad (8)$$

When  $t = T$ ,  $f_2 = i_0$ , can be obtained  $k = i_0 + \frac{T - t_{on}}{L}U_0$ . Thus, the state of each phase can be mathematically described in a single cycle.

$$\text{First phase: } f_{11} = \frac{1}{L}(U_d - U_0)t + i_0, \quad f_{12} = \frac{-U_0}{L}t + \frac{T}{L}U_0 + i_0 \quad (9)$$

$$\text{Second phase: } f_{21} = \frac{1}{L}(U_d - U_0)(t - \frac{T}{4}) + i_0, \quad f_{22} = \frac{-U_0}{L}(t - \frac{T}{4}) + \frac{T}{L}U_0 + i_0 \quad (10)$$

$$\text{Third phase: } f_{31} = \frac{1}{L}(U_d - U_0)(t - \frac{T}{2}) + i_0, \quad f_{32} = \frac{-U_0}{L}(t - \frac{T}{2}) + \frac{T}{L}U_0 + i_0 \quad (11)$$

$$\text{Fourth phase: } f_{41} = \frac{1}{L}(U_d - U_0)(t - \frac{3T}{4}) + i_0, \quad f_{42} = \frac{-U_0}{L}(t - \frac{3T}{4}) + \frac{T}{L}U_0 + i_0 \quad (12)$$

#### 4.2. Status Analysis

First, the state is discussed and the corresponding mathematical description is carried out in a single cycle, with the fourth phase inductance current as the base, the duty ratio is 33.3%, thus the eight states are obtained. At this time,  $\frac{3T}{4} \leq t \leq \frac{7T}{4}$ . The specific description is as follows:

State 1: The main control switch  $Q_{31}$  and  $Q_{41}$  are turn on,  $Q_{11}$  and  $Q_{21}$  are turn off, the current instantaneous value expression in this state is:

$$\begin{aligned} i_1 &= f_{12} + f_{22} + f_{31} + f_{41} \\ &= \frac{-U_0}{L}t + \frac{T}{L}U_0 + i_0 + \frac{-U_0}{L}(t - \frac{T}{4}) + \frac{T}{L}U_0 + i_0 + \frac{1}{L}(U_d - U_0)(t - \frac{T}{2}) + i_0 + \frac{1}{L}(U_d - U_0)(t - \frac{3T}{4}) + i_0 \\ &= \frac{2U_d - 4U_0}{L}t + \frac{14U_0 - 5U_d}{4L}T + 4i_0 \end{aligned} \quad (13)$$

State 2: The main control switch  $Q_{41}$  is turned on,  $Q_{11}$ ,  $Q_{21}$ ,  $Q_{31}$  is turned off, and the current instantaneous value expression in this state is:

$$\begin{aligned} i_2 &= f_{12} + f_{22} + f_{32} + f_{41} \\ &= \frac{-U_0}{L}t + \frac{T}{L}U_0 + i_0 + \frac{-U_0}{L}(t - \frac{T}{4}) + \frac{T}{L}U_0 + i_0 + \frac{-U_0}{L}(t - \frac{T}{2}) + \frac{T}{L}U_0 + i_0 + \frac{1}{L}(U_d - U_0)(t - \frac{3T}{4}) + i_0 \\ &= \frac{U_d - 4U_0}{L}t + \frac{18U_0 - 3U_d}{4L}T + 4i_0 \end{aligned} \quad (14)$$

State 3: The main control switch  $Q_{11}$  and  $Q_{41}$  are turn on,  $Q_{21}$  and  $Q_{31}$  are turn off, the current instantaneous value expression in this state is:

$$\begin{aligned} i_3 &= f_{11} + f_{22} + f_{32} + f_{41} \\ &= \frac{1}{L}(U_d - U_0)t + i_0 + \frac{-U_0}{L}(t - \frac{T}{4}) + \frac{T}{L}U_0 + i_0 + \frac{-U_0}{L}(t - \frac{T}{2}) + \frac{T}{L}U_0 + i_0 + \frac{1}{L}(U_d - U_0)(t - \frac{3T}{4}) + i_0 \\ &= \frac{2U_d - 4U_0}{L}t + \frac{14U_0 - 3U_d}{4L}T + 4i_0 \end{aligned} \quad (15)$$

State 4: The main control switch  $Q_{11}$  is turned on,  $Q_{21}$ ,  $Q_{31}$ ,  $Q_{41}$  is turned off, and the current instantaneous value expression in this state is:

$$\begin{aligned} i_4 &= f_{11} + f_{22} + f_{32} + f_{42} \\ &= \frac{1}{L}(U_d - U_0)t + i_0 + \frac{-U_0}{L}(t - \frac{T}{4}) + \frac{T}{L}U_0 + i_0 + \frac{-U_0}{L}(t - \frac{T}{2}) + \frac{T}{L}U_0 + i_0 + \frac{-U_0}{L}(t - \frac{3T}{4}) + \frac{T}{L}U_0 + i_0 \\ &= \frac{U_d - 4U_0}{L}t + \frac{9U_0}{2L}T + 4i_0 \end{aligned} \quad (16)$$

State 5: The main control switch  $Q_{11}$  and  $Q_{21}$  are turn on,  $Q_{31}$  and  $Q_{41}$  are turn off, the current instantaneous value expression in this state is:

$$\begin{aligned} i_5 &= f_{11} + f_{21} + f_{32} + f_{42} \\ &= \frac{1}{L}(U_d - U_0)t + i_0 + \frac{1}{L}(U_d - U_0)(t - \frac{T}{4}) + i_0 + \frac{-U_0}{L}(t - \frac{T}{2}) + \frac{T}{L}U_0 + i_0 + \frac{-U_0}{L}(t - \frac{3T}{4}) + \frac{T}{L}U_0 + i_0 \\ &= \frac{2U_d - 4U_0}{L}t + \frac{14U_0 - U_d}{4L}T + 4i_0 \end{aligned} \quad (17)$$

State 6: The main control switch  $Q_{21}$  is turned on,  $Q_{11}$ ,  $Q_{31}$ ,  $Q_{41}$  is turned off, and the current instantaneous value expression in this state is:

$$\begin{aligned} i_6 &= f_{12} + f_{21} + f_{32} + f_{42} \\ &= \frac{-U_0}{L}t + \frac{T}{L}U_0 + i_0 + \frac{1}{L}(U_d - U_0)(t - \frac{T}{4}) + i_0 + \frac{-U_0}{L}(t - \frac{T}{2}) + \frac{T}{L}U_0 + i_0 + \frac{-U_0}{L}(t - \frac{3T}{4}) + \frac{T}{L}U_0 + i_0 \\ &= \frac{U_d - 4U_0}{L}t + \frac{18U_0 - U_d}{4L}T + 4i_0 \end{aligned} \quad (18)$$

State 7: The main control switch  $Q_{21}$  and  $Q_{31}$  are turn on,  $Q_{11}$  and  $Q_{41}$  are turn off, the current instantaneous value expression in this state is:

$$\begin{aligned} i_7 &= f_{12} + f_{21} + f_{31} + f_{42} \\ &= \frac{-U_0}{L}t + \frac{T}{L}U_0 + i_0 + \frac{1}{L}(U_d - U_0)(t - \frac{T}{4}) + i_0 + \frac{1}{L}(U_d - U_0)(t - \frac{T}{2}) + i_0 + \frac{-U_0}{L}(t - \frac{3T}{4}) + \frac{T}{L}U_0 + i_0 \\ &= \frac{2U_d - 4U_0}{L}t + \frac{14U_0 - 3U_d}{4L}T + 4i_0 \end{aligned} \quad (19)$$

State 8: The main control switch  $Q_{31}$  is turned on,  $Q_{11}$ ,  $Q_{21}$ ,  $Q_{41}$  is turned off, and the current instantaneous value expression in this state is:

$$\begin{aligned} i_8 &= f_{12} + f_{22} + f_{31} + f_{42} \\ &= \frac{-U_0}{L}t + \frac{T}{L}U_0 + i_0 + \frac{-U_0}{L}(t - \frac{T}{4}) + \frac{T}{L}U_0 + i_0 + \frac{1}{L}(U_d - U_0)(t - \frac{T}{2}) + i_0 + \frac{-U_0}{L}(t - \frac{3T}{4}) + \frac{T}{L}U_0 + i_0 \\ &= \frac{U_d - 4U_0}{L}t + \frac{9U_0 - U_d}{2L}T + 4i_0 \end{aligned} \quad (20)$$

Through the above analysis we can find:

(1)The slopes of  $i_1, i_3, i_5, i_7$  are the same and all are  $\frac{2U_d - 4U_0}{L}$ ; the slopes of  $i_2, i_4, i_6, i_8$  are the same and all are  $\frac{U_d - 4U_0}{L}$ .

(2)When  $t = \frac{3T}{4}$ , can be obtained  $i_1 = \frac{5U_0}{4L}$ ; when  $t = \frac{7T}{4}$ , can be obtained  $i_8 = \frac{5U_0}{4L}$ , that is to satisfy the conservation of energy.

## 5. Simulation analysis

Simulink is used to build the four phase synchronous rectifier model. The waveforms of each phase inductor current and the synthetic current are observed, and the mathematical model is verified by simulation. The input voltage of the model is 24V, the output voltage is 8V, duty cycle is 33.3%, switching frequency is 10KHz, and inductance is  $100\mu H$ . The inductor current waveform of Figure 7 is obtained by simulation. Waveforms 1-4 are the inductor current waveforms for the first to fourth phases, respectively, and the fifth waveform is the synthetic inductance current. By synthesizing the inductor current waveforms, we can find that in one cycle:

- (1) The inductance current is composed of eight segments, corresponding to eight states respectively.
- (2) The eight segment curve is composed of four charging and discharging intervals. The charge slope of each charge and discharge interval is equal and the discharge slope is the same.
- (3) The inductor current is continuous;
- (4) The ripple of the four phase four phase is obviously lower than the single-phase ripple, and the output current is obviously increased.

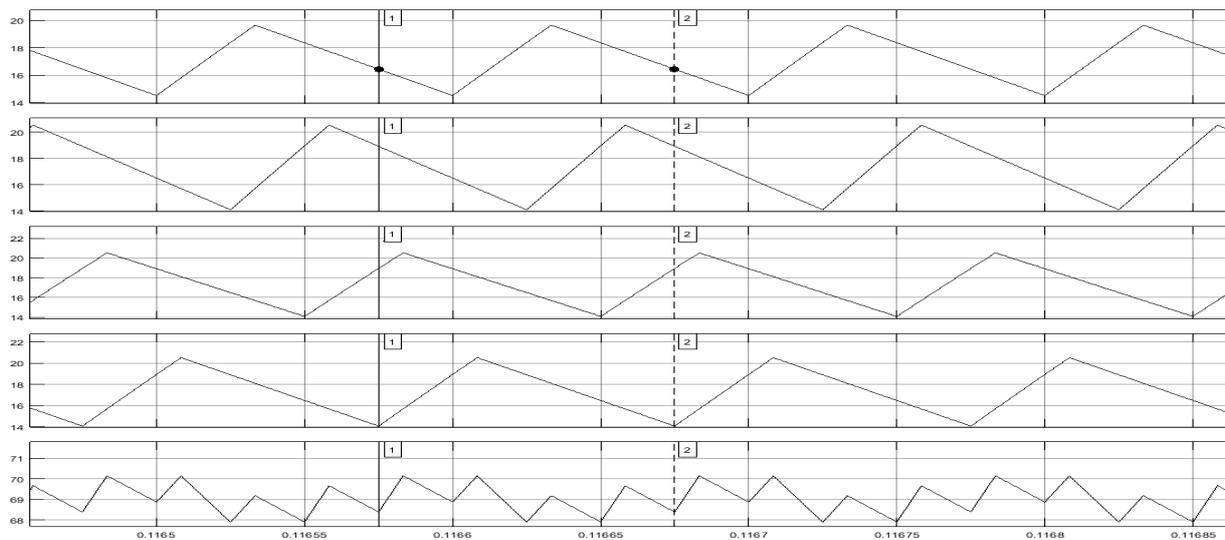


Figure 7. Inductor current waveform

## 6. Conclusion

Taking the four-phase interleaved synchronous BUCK converter as the research object, and the instantaneous ripple of the converter is described by establishing the corresponding mathematical model. Then the simulation model is built using Simulink to verify the conclusion. The simulation results show that compared to the single-phase BUCK converter, the ripple of the four-phase interleaved synchronous rectifier converter is obviously reduced, the output current is increased, and the demand of the low voltage and large current situation is satisfied.

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## References

- [1] Yang Bo. Research of interlaced Synchronous BUCK Tracking source[D].Harbin Institute of Technology,2014.
- [2] Gao Qing.Research on Multiphase Interleaving Buck Converters with Low Ripple[D].Harbin Institute of Technology,2017.
- [3] Paula Cervellini;Pablo Antoszczuk;Rogelio García Retegui;Marcos Funes.Current Ripple Amplitude Measurement in Multiphase Power Converters[J].IEEE Transactions on Power Electronics.2017,Vol.32(No.9): 6684-6688.
- [4] Lu, Weiguo;Li, Shaoling;Chen, Weiming.Current-Ripple Compensation Control Technique for Switching Power Converters[J].IEEE Transactions on Industrial Electronics.2018,Vol.65(No.5): 4197-4206.
- [5] Yang, Yugang;Guan, Tingting;Zhang, Shuqi;Jiang, Wei;Huang, Weiyi.More Symmetric Four-Phase Inverse Coupled Inductor for Low Current Ripples & High-Efficiency Interleaved Bidirectional Buck/Boost Converter[J].IEEE Transactions on Power Electronics.2018,Vol.33(No.3): 1952-1966.