

Collaborative Consistency Control Algorithm Research of Unmanned Aerial Vehicle Formation

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Abstract. In this paper, the cooperative consistency control algorithm of unmanned aerial vehicle is studied. The formation of unmanned aerial vehicles has the characteristics of controlling uncertainty. This makes the controller have certain uncertainty which may cause the multi-agent system to be difficult to agree. In view of this feature, the intelligent algorithm for collaborative consistency control of unmanned aerial vehicle formation is designed with the properties of the Laplace matrix of the multi-agent network which is composed of a series of intelligent bodies with autonomous capability and the interaction between them. The simulation results verify the feasibility of the intelligent algorithm.

1. Raise of problem

Unmanned aerial vehicle formation flight has a broad application prospect[1-3]. Its consistency and collaborative control technology has always been the focus of research, and it can provide a strong support control algorithm for formation and track planning. In this paper, the collaborative control algorithm is based on multi-agent system. This system is composed of a series of intelligent bodies with autonomous capability and the interaction between them, and it can also be described as a multi-agent network. In this system, a single agent is a node in a network, and the interaction between the intelligent bodies (information interaction) is the connection between network nodes. Intelligent body can be unmanned aerial vehicle, sensor node and so on[4-8].

At present, in the research on the collaborative control of unmanned aerial vehicles, all the cooperative controllers designed by unmanned aerial vehicles can be executed. In practice, the controller has some uncertainty due to hardware (such as A/D, D/A conversion), software (such as the calculation phase error) and other reasons [9]. This makes the controller have certain uncertainty which may cause the multi-agent system to be difficult to agree. Therefore, this paper studies the control problem of unmanned vehicle with control uncertainty and gives the control model of the controller. In order to facilitate the discussion and consider the universality of the model, the intelligent body is replaced by the unmanned aerial vehicle for analysis modeling and simulation calculation.

2. Model building

Considering the following N linear time-invariant smart bodies composed of a multi-agent system, the dynamic model of the intelligent body is described as follows:



$$\begin{aligned}\dot{x}_i &= Ax_i + Bu_i \\ y_i &= Cx_i \quad i = 1, 2, \dots, N\end{aligned}\quad (1)$$

Among them, $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^p$, $y_i \in \mathbb{R}^q$ are the state of the i th intelligence, controlling the input and output. A , B and C are a constant matrix of dimensions.

For a given multi-agent system, a system agreement means that for any finite initial value $x_i(0)$, if there is a controller which makes:

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i, j = 1, 2, \dots, N \quad (2)$$

In this case, the communication topology between the intelligent body is described by a directed graph containing a directed spanning tree [10].

Based on the neighbor relative status information, a distributed collaborative controller with uncertainty is constructed:

$$u_i = \sum_{j=1}^N a_{ij} (K + \Delta K_i)(x_j - x_i), \quad i = 1, 2, \dots, N \quad (3)$$

Where $K \in \mathbb{R}^{p \times n}$ is the feedback control matrix to be designed, c is the coupling strength parameter to be designed, and a_{ij} is the element of the adjacency matrix \mathcal{A} of communication topological \mathcal{G} . ΔK_i represents the uncertainty in the controller of the intelligent body i , which has the following form:

$$\Delta A_i = DF_iH \quad (4)$$

In this case, the matrix D and H are the constant matrices with the appropriate dimension, which represent the uncertain structure.

$$F_i^T F_i \leq \rho_i^2 I \quad (5)$$

Where, $i = 1, 2, \dots, N$ is the normal number giver.

It can be seen from the above that different intelligent bodies have the same nominal feedback matrix in the multi-agent system, but the controllers with different norm are uncertain. The main research of this paper is to design the appropriate feedback matrix (2) so that the multi-agent system can reach agreement under the controller.

According to the above analysis, the closed-loop dynamic equation of the intelligent body system composed of N linear time-invariant intelligent bodies is as follows:

$$\dot{x} = (I_N \otimes A - (I_N \otimes B)(I_N \otimes K + \tilde{K})(\mathcal{L} \otimes I_n))x \quad (6)$$

Where, $\Delta \tilde{K} = \text{diag}\{\Delta K_1, \Delta K_2, \dots, \Delta K_N\}$, \mathcal{L} is the Laplacian matrix of figure \mathcal{G} [11-13].
 $x = [x_1^T, x_2^T, \dots, x_N^T]^T$

According to the literature, there is a matrix $M \in \mathbb{R}^{N \times N-1}$ for the Laplacian matrix $\mathcal{L} \in \mathbb{R}^{N \times N}$ of the graph, which makes $\mathcal{L} = ME$, among which, $E \in \mathbb{R}^{N-1 \times N}$ is as follows:

$$E = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (7)$$

Further, if the graph contain a directed spanning tree, the matrix M is column full rank, and the eigenvalue of matrix EM is the non-zero eigenvalue of \mathcal{L} , $\text{Re}(\lambda(EM)) > 0$. For matrix EM , if figure G contains a directed spanning tree, there is a symmetric positive definite matrix Q and a positive scalar parameter α , which makes:

$$(EM)^T Q + QEM > \alpha Q \quad (8)$$

Where, $0 < \alpha < 2 \min \{ \text{Re}(\lambda(EM)) \}$.

Make $\xi = (E \otimes I_n)x$, $\xi_i = x_i - x_{i+1}$, $i = 1, 2, \dots, N-1$. The system closed-loop dynamic equation can be reconstituted in the following form.

$$\dot{\xi} = (I_{N-1} \otimes A - EM \otimes BK - (E \otimes B)\Delta\tilde{K}(M \otimes I_n))\xi \quad (9)$$

Where, b is the matrix that satisfies c . When $\xi \rightarrow 0$, $x_1 = x_2 = \dots = x_N$, the system agrees.

Therefore, the system consistency and cooperative control problem is transformed into system stability problem. Based on this, the following algorithm is introduced to design the appropriate feedback matrix K to make the system agree. The topology G contains a directed spanning tree. The collaborative controller (2) can be designed as follows:

(1) Given the parameters $M = \mathcal{L}E^T(EE^T)^{-1}$ and $0 < \alpha < 2 \min \{ \text{Re}(\lambda(EM)) \}$, the solution can get a feasible solution $Q > 0$;

(2) For a given parameter $c > 0$, the following linear matrix inequalities are solved to obtain a feasible solution $P > 0$.

$$\begin{bmatrix} A^T P + PA - c\alpha PBB^T P & PBD & H^T \\ D^T B^T P & \frac{1}{\beta\rho_0^2\varphi_1} I & 0 \\ H^T & 0 & \frac{\varphi_2}{\phi} \end{bmatrix} < 0 \quad (10)$$

Where, $\beta = \max \{ \lambda(EE^T) \}$, $\phi = \max \{ \lambda(M^T M) \}$, $\varphi_1 = \max \{ \lambda(Q) \}$, $\varphi_2 = \min \{ \lambda(Q) \}$, $\rho_0 = \max \{ \rho_1, \rho_2, \dots, \rho_N \}$;

(3) The design feedback matrix is $K = cB^T P$.

Assume that the topology diagram contains a directed a spanning tree, for a given parameter $c > 0$, linear matrix inequality (5) is true, the design of coordinated controller (2) to agree on the multi-agent system.

3. The simulation result

Simulation and verification of the above models and methods are carried out. Assuming a multi-intelligence system with four intelligent bodies, the matrix of the intelligent body system with controller uncertainties is described as follows.

$$x_i = \begin{bmatrix} x_{i1} \\ x_{i2} \end{bmatrix}, A = \begin{bmatrix} -3 & -9 \\ 9 & 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 5 & 1 \end{bmatrix}, D = H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (11)$$

$F_i, i = 1, 2, 3, 4$ is randomly selected in $(0, 1]$.

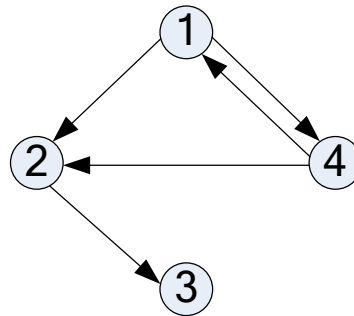


Figure 1. Communication topology.

The communication topology of the intelligent body is shown in figure 1. As can be seen from the figure, the communication topology contains a directed spanning tree. The Laplacian matrix of the topology diagram is described as follows.

$$\mathcal{L} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 2 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

You have a column full rank matrix M that makes $\mathcal{L} = ME$. Where

$$E = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \quad M = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 0 \\ -1 & -1 & -1 \end{bmatrix} \quad (13)$$

So you get $\beta = 3.4142$, $\phi = 7.0340$.

Make $\alpha = 3.5$, a feasible solution of the linear matrix inequality (3) is

$$Q = \begin{bmatrix} 0.9406 & -0.0006 & -0.4304 \\ -0.0006 & 0.3064 & 0.2037 \\ -0.4304 & 0.2037 & 0.7363 \end{bmatrix} \quad (14)$$

So you get $\varphi_1 = 1.2975$, $\varphi_2 = 0.1762$.

Make $c = 10$, $\rho_0 = 9063$. To solve matrix inequality (5), we can find a feasible solution for P.

$$P = \begin{bmatrix} 7.7553 & 0.1602 \\ -0.1602 & 1.1270 \end{bmatrix} \quad (15)$$

According to the algorithm, the feedback matrix can be designed as follows:

$$K = \begin{bmatrix} -29.9477 & 20.2827 \\ -0.5608 & 3.9444 \end{bmatrix} \quad (16)$$

The state trajectories of each agent are shown in figure 2 and figure 3. The trajectories of consensus error of each agent are shown in figure 4 and figure 5. The simulation results show that the multi-agent system is in agreement.

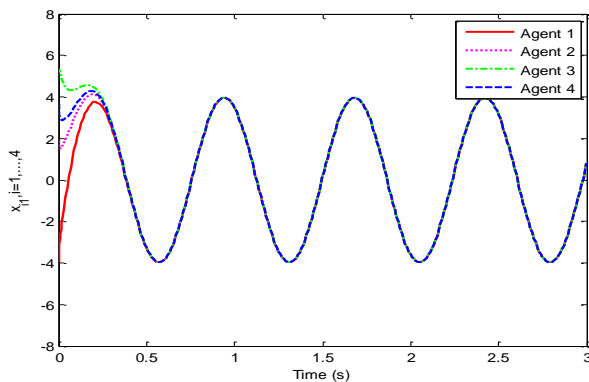


Figure 2. Trajectories of $x_{i1}, i=1,...,4$

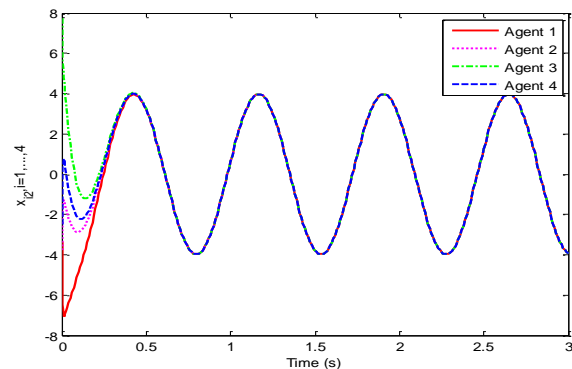


Figure 3. Trajectories of $x_{i2}, i=1,...,4$

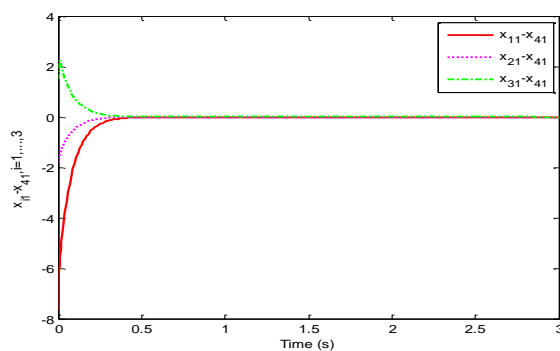


Figure 4. Trajectories of consensus error

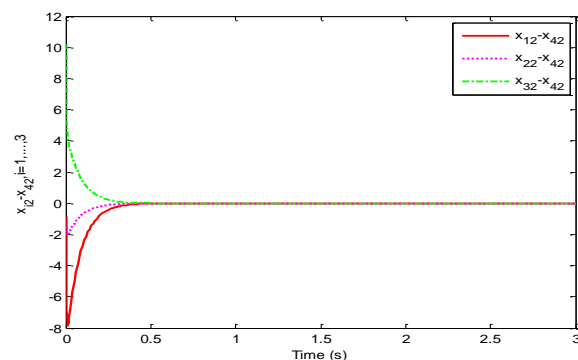


Figure 5. Trajectories of consensus error

4. The conclusion

In this paper, a class of unmanned aerial vehicle formation coordination problem with controller uncertainty is solved by using the special property of topology diagram and linear matrix inequality method. The design algorithm of formation cooperative controller is given, and the simulation results prove the correctness of the design method.

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