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Single population modelling on predicting global carbon dioxide concentration

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Abstract. Every year, the concentration of carbon dioxide is continuously increasing. In regard to this, it becomes a concern if this phenomena occurs repeatedly over time. When the concentration reaches at a certain point, it may have some bad effects on earth and will threaten life. If this event is ignored, then the earth and life on it will gradually be more effected and turned its condition to be worse. Regarding to this, it might be helpful to have a model that can predict the concentration of carbon dioxide. Since a preventive action can be taken before the concentration gets higher in the future based on the prediction. The prediction model can be formulated by a population modelling, where the concentration amount is perceived as a single population. In this case, carbon dioxide emission and its absorption by photosynthesis will be two considered factors. Where emission will represent the birth, and the absorption represents the death of the population. By this model, it is predicted that about 58 years later the concentration will reach 500ppm. It has been evaluated that the model has small error to predict the current data.

1. Introduction

It cannot be denied that the concentration level of carbon dioxide has increased over years. This fact is related with the release or the emission of carbon dioxide of human activities in daily basis. Such as industrial activity, the use of oil and coal, even from human respiration [1]. Moreover, the development of technological industry has been running so significantly. As mentioned in history that the industrial era has been started since the 18th and 19th century. Numerous number of industries keep growing with various sectors [2]. As industry keeps developing, the number of vehicles is also increasing for transportation and regular activities. It can also be stated that the number of vehicles has been sky-rocket in last decades.

Surely, there is implication from the carbon dioxide emission activities. The major impact is the air pollution. It cannot be denied that plants, manufacture, and machines in industry yield some chemical waste to the air in its process. The same case with the combustion of vehicles that release some chemical waste. One of some chemical release is carbon dioxide. It has been recorded that every year, the concentration level of carbon dioxide keeps increasing [3]. Certainly, this event may results some negative impact. The increasing of carbon dioxide may lead to global warming. In fact, when it gets higher concentration, then it will affect the radiation balance in the atmosphere. Such event is called as greenhouse effect. Besides, and change of climate that will surely affect the life on earth. Another dangerous effect is on health. When the level of carbon dioxide is too high, the air will not be good any longer. Some effects are like difficulty in breathing, poisoning, headache, rapid pulse, rate, etc. [4].

As additional information, this paper aims to analyze and generate a model that can predict the concentration of carbon dioxide in global area by differential equation. Two things that considered here are the emission of the chemical and its absorption. The emission here is taken as aggregate, and the absorption will only be taken from photosynthesis by plants, in which it will have relation with total of landmass area of forest. Both emission and absorption will cover global scale. The product of this paper will be an equation that can predict the concentration of carbon dioxide in annual period. This paper will give the basic knowledge and idea in section II, and the modelling process will be discussed in section III. As the result is obtained, it will be discussed and analyzed in section IV. Then, the section V will summarize and give suggestions related for the future improvement.

2. Literature review

Now, to model carbon dioxide, it is important to know the basic of modelling a single population. In general, a population can be modelled according its behavior. Normally, modelling population is about to see the dynamic growth of the population [5]. This growth is basically affected by 2 factors, birth and death of the population. Each birth and death has a certain value of rate, and it implies population to have a rate change. Then birth and death rate determine the growth of the population. The growth process of the population can be perceived as

$$\begin{Bmatrix} rate\ change \\ of\ population \\ size \end{Bmatrix} = \begin{Bmatrix} rate \\ of \\ birth \end{Bmatrix} - \begin{Bmatrix} rate \\ of \\ death \end{Bmatrix}$$

Then, each part can be set into mathematical term

$$\begin{Bmatrix} rate \\ of \\ birth \end{Bmatrix} = \beta X, \text{ and} \begin{Bmatrix} rate \\ of \\ death \end{Bmatrix} = \alpha X,$$

Where $\alpha, \beta \geq 0$.

Then the equation of the population can be written as

$$\frac{dX}{dt} = \beta X - \alpha X \tag{1}$$

Assume that $r = \beta - \alpha$. This tells the growth rate of the population, so

$$\frac{dX}{dt} = rX \tag{2}$$

The solution of this equation will be

$$X(t) = Ae^{rt} \tag{3}$$

When $t = 0$, it gives the number of population equals to A, and such value is called as initial number of population.

As the subject to model is carbon dioxide, then the behavior of its cycle has to be understood. The carbon dioxide is much related with carbon cycle. Basically, there is an exchange interaction between carbon dioxide in atmosphere with the activities on land. The carbon dioxide in atmosphere is absorbed by plants for the use of photosynthesis. As it is known that photosynthesis results oxygen that is used for respiration by living things. Then, respiration will produce carbon dioxide that will be accumulated again in atmosphere [6]. Beside this cycle, there is also increasing amount of carbon dioxide release from human activities. The major activity that contributes to the increasing of carbon dioxide is from burning of fossil fuels. Besides, it cannot be denied that the destruction of forest for reduces the area of plants to absorb carbon dioxide. This cycle keeps repeating, where there is reduction of carbon dioxide and there is release of carbon dioxide that adds its amount. Nevertheless, it will be considered that ocean takes 30% of the amount of carbon dioxide [7]. Therefore, there will be reduction of 30% in the emission.

When it comes to model the carbon dioxide, some concepts are applied. First, the population in this case is the population or the amount of carbon dioxide. Then, the birth of the carbon dioxide is obtained from emission in aggregate [8]. Surely the birth will have rate value. The death of the population is the absorption. This absorption will be influenced by the rate of photosynthesis and the area of forest where plants do photosynthesis. Both of emission and forest area are depend on time, since every year the number changes. One important thing that should not be forgotten is unit. The table 1 below shows the unit of each element.

Table 1. Units.

Name	Symbol	Unit
Carbon dioxide concentration	C(t)	ppm
Photosynthesis rate	P	$\mu molm^{-2}s^{-1}$
Emission function	E(t)	kt
Absorption function	A(t)	kt
Emission rate	ε	kt/year
Absorption rate	γ	kt/year
Time	t	year

3. Carbon dioxide modelling

Following the equation of population model, then it is adjusted that birth rate will be perceived as rate of carbon dioxide obtained from emission. Then, the death rate of the population will be discern as absorption of carbon dioxide through photosynthesis by plants. The basic idea of using a single population model for carbon dioxide is because of the suitability. The amount of carbon dioxide behaves like population, where there is birth and death. Now, before the model is made, several assumptions are made. The first assumption is to let that the photosynthesis rate will just be a constant. Second, is that the rate of emission and absorption will assumed to be constants as well. The rate change of carbon dioxide will be defined as the difference between emission rate, which acts as the birth rate, and the absorption rate that has role as death rate. The equation to model compartment of carbon dioxide is as follow

$$\begin{Bmatrix} rate\ change \\ of\ carbon \\ dioxide \end{Bmatrix} = \begin{Bmatrix} rate \\ of \\ emission \end{Bmatrix} - \begin{Bmatrix} rate \\ of \\ absorption \end{Bmatrix}$$

Implementing the same procedure, let that

$$\begin{Bmatrix} rate\ change \\ of\ carbon \\ dioxide \end{Bmatrix} = \frac{dC}{dt}$$

$$\begin{Bmatrix} rate \\ of \\ emission \end{Bmatrix} = \varepsilon C \text{ and } \begin{Bmatrix} rate \\ of \\ absorption \end{Bmatrix} = \gamma C.$$

So, the equation for carbon dioxide model can be written as

$$\frac{dC}{dt} = \varepsilon C - \gamma C \tag{4}$$

Now, the task is to find the value of each ε and γ.

3.1. The value of emission rate (ε)

The idea to find the value of ε is by taking the first derivative of emission function, where the emission function is done by linear regression. So, the first thing to do is to find the emission function. Taking year from 1994 to 2014, emission data is collected and regressed. This means that the time reference of $t = 0$ is in year of 1994. The data is taken from the World Bank in global level. It turns out that the equation is

$$E(t) = 21{,}233{,}501.25 + 750{,}187.1736t \tag{5}$$

With the value of R^2 is 0.971. Note that the unit of carbon dioxide emission is in kilotons (kt). It has to be concerned that the ocean absorbs 30% of the emission. Hence, each value of the terms in the equation has to be reduced to 70%, it becomes

$$E(t) = 14{,}863{,}450.88 + 525{,}131.0215t \tag{6}$$

In order to find the rate change of emission, then take the first derivative of $E(t)$. So,

$$\varepsilon = E'(t) = 525{,}131.0215$$

3.2. The value of absorption rate

The same concept is applied to get the rate of absorption. It is done by taking the absolute value of the first derivative of absorption function. The rate has to be taken as positive value since both birth and death rate has to be greater than or equals to 0. The absorption function will be the multiplication of photosynthesis rate and the area of forest landmass. However, it has to be noticed that each year the forest area has decreased. Then, the forest area will be regressed such that it becomes a function of time. As a consequence, the absorption rate will also be a function of time. Note that the unit of photosynthesis rate is in $\mu molm^{-2}s^{-1}$. This unit has to be converted into kt$m^{-2}Y^{-1}$ (kilotons per meter square per year). In order to do that, then the value of the function has to be multiplied by 6.939×10^{-7}. In this case, the same range of time is used and obtained from World Bank. By linear regression, it is obtained that the function of absorption is as follow

$$A(t) = P(111{,}568{,}073.9 - 128{,}132.9372t) \tag{7}$$

With $R^2 = 0.966$. Since the absorption rate is the absolute value of the first derivative of absorption function, then

$$\gamma = |A'(t)| = |-128{,}132.9372P| = 128{,}132.9372P.$$

Where P is the value of photosynthesis rate. The value will be determined from initial value problem (IVP) after the equation is solved. Since it is actually hard to determine the global level of photosynthesis rate. Besides, it has been assumed previously that this value will just be a constant.

Now, both value for emission and absorption have been known. So the equation can be evaluated

$$\frac{dC}{dt} = \varepsilon C - \gamma C$$

$$\frac{dC}{dt} = 525{,}131.0215C - 128{,}132.9372PC \tag{8}$$

By integrating the equation, it results

$$C(t) = C_0 e^{(525,131.0215-128,132.9372P)t} \tag{9}$$

Setting $t = 0$, which is in year of 1994, the value of initial value can be known. In 1994, the concentration of carbon dioxide is 358.83ppm. So, the value of C_0 is 358.83. The thing now is to find the value of P.

$$C(t) = 358.83e^{(525,131.0215-128,132.9372P)t}.$$

Set up that $t = 20$, which is at year 2014, then it is obtained that the value of P is about 4.098 $\mu molm^{-2}s^{-1}$. Hereby, the complete equation will be

$$C(t) = 358.83e^{0.00526175t} \tag{10}$$

This equation will then be tested by comparing with the past data.

4. Result and discussion

By comparing real data and predictive result from the equation, it is recorded that the predictive result comes close to the real data. The real data is obtained from NOAA that is accessible online. Taking year from 1994 to 2017, the biggest deviation is 1.54 unit. The comparison can be seen in the table 2 below.

Table 2. Comparison between real and predicted data.

Year	Real Data	Prediction	Residual
1994	358.83	358.83	0
1995	360.82	360.723	0.097
1996	362.61	362.626	0.016
1997	363.73	364.539	0.809
1998	366.7	366.462	0.238
1999	368.38	368.396	0.016
2000	369.55	370.339	0.789
2001	371.14	372.293	1.153
2002	373.28	374.257	0.977
2003	375.8	376.231	0.431
2004	377.52	378.216	0.696
2005	379.8	380.212	0.412
2006	381.9	382.217	0.317
2007	383.79	384.234	0.444
2008	385.6	386.261	0.661
2009	387.43	388.299	0.869
2010	389.9	390.347	0.447
2011	391.65	382.407	0.757
2012	393.85	394.477	0.627
2013	396.52	396.588	0.038
2014	398.65	398.65	0
2015	400.83	400.75	0.08
2016	404.21	402.87	1.34
2017	406.23	404.99	1.54

With the same range of year, it is measured that this mathematical model has error of 0.53%. Nevertheless, when it tries to predict the concentration of carbon dioxide in 1959 ($t = -35$), it deviates for about 12 unit. So, it can be said the model will have a bigger error when a far range of time is taken. In order to see visually how close the comparison between the real data value and the prediction is, observe the following graph.

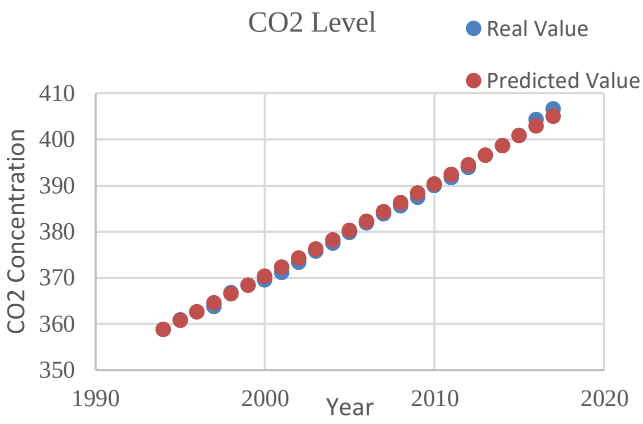

Figure 1. Carbon dioxide comparison.

Using the model obtained, it is predicted that 58 years from now, the carbon dioxide level will reach 500ppm. This level of carbon dioxide is no longer good for health. So, before the number touches 500ppm, some preventive actions can be taken. Some actions that can be implemented can be urban management, and considering public transport [9].

5. Conclusion and recommendations

As carbon dioxide becomes a concern, knowing the prediction of its concentration at a given time might be beneficial. Since preventive actions can be implemented before it reaches a certain concentration. The model that predicts carbon dioxide has a good accuracy for about the last two decades with a small error. Nevertheless, when it is tested for a far period of time, the residual gets bigger. This might be caused by the difference trend of data, since it is assumed that photosynthesis, emission, and absorption rate to be constant. Besides, this model is not valid for a very long period of time. Since linear regression is used to predict the function of forest area with a constant reduction, it does not make sense if after a very long time there will be no more area of forest or plants. To improve this model, it is suggested to add carrying capacity to the area of forest to keep the existence of it. Besides, this model does not account the factor that photosynthesis rate may change over time, that is affected by carbon dioxide concentration itself.

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