

Bovine bone natural frequencies evaluation by using transfer matrix method based upon DICOM images

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Abstract

This study presents the use of the transfer matrix method to calculate the natural frequencies for bovine bone. The bovine bone was considered as continuous system. This continuous system was transferred to discrete system by discretizing it using the Digital Imaging and Communications in Medicine (DICOM) images obtained from CT-Scan test. Those images were used to evaluate the geometrical properties of the bone segments. Geometrical properties variation along the bone length were evaluated by using image-processing technique (Binary Image). Each slice was considered as beam and the total number of beam segments is equal to the number of DICOM images. The bone was divided into 226 segments (1.25 mm slice thickness). Euler Bernoulli equation of motion was used to formulate the transfer matrix. Finite elements method based on ANSYS mechanical APDL version 14.5 was used to calculate the natural frequency. This method was used to check the discrepancy of the results obtained from the transfer matrix method. This method was done by building the three dimensional model based upon the DICOM images and solving the modal analysis by using ANSYS software. The deviation in the results obtained from the two methods was less than 15%. The simplicity and efficiency of transfer matrix method recommends further improvement by using the gray images instead of binary images to evaluate bone geometrical, physical, and mechanical properties based upon the grayscale value of each pixel for both cortical and compact bone.

Keywords: -

Bovine Bones, Finite Elements, Natural Frequencies, Transfer Matrix Method, CT-Scan, DICOM

Introduction:-

Bones are made from living tissues and they have several vital functions. Therefore monitoring these functions to distinguish between normal and abnormal health condition is extremely important. The known diagnosis procedures for the bone integrity is radioactive, very expensive, high test cost, etc. Therefore, researchers are investigating an alternative test device to eliminate the disadvantages of the known diagnosis devices. The frequency analysis is one of those fields of study investigated by the researchers. It investigates the change in the structural dynamics properties of the bone due to bone diseases. Transfer matrix method was used in the earlier studies of the frequency analysis for long bones. Viano et al. [1] used the transfer matrix method to extract theoretically the natural frequency of femur bone after dispatching the ends. The method was based on the solution of Timoshenko beam theory and 16 sections were made to describe the variation in the cross sectional geometry. The lack of devices to



identify the mode of vibration experimentally, forced the researchers to develop mathematical model and by matching the results obtained from the theoretical with the experimental to identify the modes of vibration. Khalil et al. [2] presented a study to evaluate the natural frequencies of femur bone theoretically and experimentally. The full length femur was considered without dispatching any part from it. Fifty-nine sections was made to evaluate the cross sectional geometrical properties. Transfer matrix was used to calculate the natural frequency to evaluate the experimental results and to identify the mode of vibration. The beam segment used at that time was cylindrical. Viano [3] tried to improve the model used by considering taper beam segments. The study considered the evaluation of the used of the taper beam segment in the transfer matrix method based upon the Timoshenko beam theory. The model presented in this study showed great potential to be used in biomechanics applications.

The interest in this method was decreased when the finite element method was presented as powerful tool to solve the most complicated engineering problem efficiently and the computer development increases the interest in the finite element method. The transfer matrix method was used in other application such as beams and pipes with variable cross section applications. Mihail B. et al. [4] presented a study to evaluate the natural frequency of beams with variable cross section based upon Euler Bernoulli equation of motion. The equation was solved by using Bessel function and he used two type of segments (cylindrical and taper). The results showed that the accuracy of the results was increased by the use of taper segments and Bessel function.

All the earlier studies consider the bone as homogeneous and isotropic materials. Therefore, the physical and mechanical properties were considered for the entire bone. While, in this study these properties were evaluated for each beam segments based upon the gray scale value. Transfer matrix method is very efficient, easy to program, relatively short run time, and taking into account the shape and material irregularity. The application of this method in the biomechanical field is very efficient. The number of section can be increased by using the DICOM images obtained from the CT-Scan test device. The accuracy of the method will increase with the increase of the number of beam segments to have better description for the variation in the geometrical properties.

Theoretical Analysis:-

The transfer matrix method will be derived based upon Euler Bernoulli equation of motion shown below [5]:

$$\frac{\partial^2}{\partial^2 x} \left[EI(x) \frac{\partial^2 Y(x,t)}{\partial^2 x} \right] + \rho A(x) \frac{\partial^2 Y(x,t)}{\partial^2 t} = f(x, t) \quad (1)$$

The solution of equation (1) for free vibration and uniform beam is shown below-

$$Y(x) = C_1 \cos \beta x + C_2 \sin \beta x + C_3 \cosh \beta x + C_4 \sinh \beta x \quad (2)$$

Where,

$$\beta^4 = \frac{\rho A \omega^2}{EI} \quad (3)$$

Where,

ω is the natural frequency.

E is the modulus of elasticity.

I is the moment of inertia.

ρ is the density.

It is well-known that the first derivative of the displacement represents the slop, the second derivative will represent the bending moments, and the third derivative will represent the shear force. Thus,

$$\phi(x, t) = EI \frac{\partial Y}{\partial x} = EI \{-\beta C_1 \sin \beta x + \beta C_2 \cos \beta x + \beta C_3 \sinh \beta x + \beta C_4 \cosh \beta x\} \quad (4)$$

$$M(x, t) = EI \frac{\partial^2 Y}{\partial x^2} = EI \{-\beta^2 C_1 \cos \beta x - \beta^2 C_2 \sin \beta x + \beta^2 C_3 \cosh \beta x + \beta^2 C_4 \sinh \beta x\} \quad (5)$$

$$V(x, t) = EI \frac{\partial^3 Y}{\partial x^3} = EI \{\beta^3 C_1 \sin \beta x - \beta^3 C_2 \cos \beta x + \beta^3 C_3 \sinh \beta x + \beta^3 C_4 \cosh \beta x\} \quad (6)$$

Where,

$Y(x, t)$ is the Deflection in the y direction along the beam at any time.

$\phi(x, t)$ is the Slope in the y direction along the beam at any time.

$M(x, t)$ is the Bending moment in the y direction along the beam at any time.

$V(x, t)$ is the Shear force in the y direction along the beam at any time.

Equations (3) to (6) can be written in matrix form, as shown below:

$$[q] = \begin{bmatrix} Y \\ \phi \\ M \\ F \end{bmatrix} = \begin{bmatrix} \cos \beta x & \sin \beta x & \cosh \beta x & \sinh \beta x \\ -\beta \sin \beta x & \beta \cos \beta x & \beta \sinh \beta x & \beta \cosh \beta x \\ -\beta^2 \cos \beta x & -\beta^2 \sin \beta x & \beta^2 \cosh \beta x & \beta^2 \sinh \beta x \\ \beta^3 \sin \beta x & -\beta^3 \cos \beta x & \beta^3 \sinh \beta x & \beta^3 \cosh \beta x \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \quad (7)$$

Transfer Matrix Method:-

The bone is continues system which will be discretized by using the transfer matrix method. The following procedure will be used to insure the continuity of the deflection, slop, bending moment, and shear force. The step beam shown in figure (1) will be discretized in n segments and each segment has cylindrical shape.

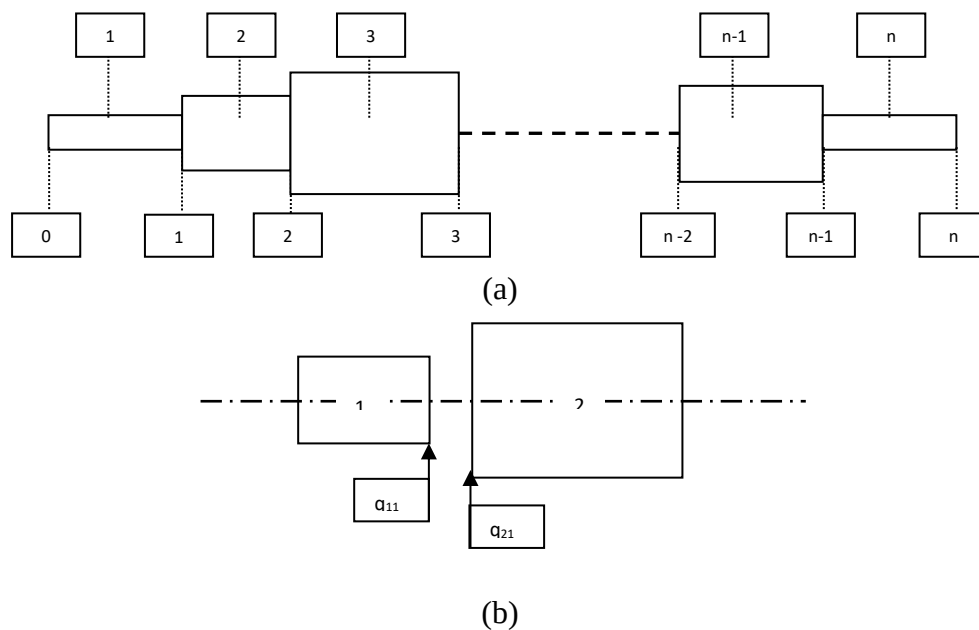


Figure (1): Continuous beam. **(a)** discretized the beam into nth number small beams. **(b)** Interface between two adjacent beams.

Let $\mathbf{q}_{mn}$ be the state vector. where m is the segment number and n is the section number.

Let $[H_m]$ be the nominator matrix.

Let $[C_m]$ be the constants matrix.

The state vector for segment 1 and 2 are showing in the following equations.

$$\mathbf{q}_{11} = [H1] [C1] \quad (8)$$

$$\mathbf{q}_{21} = [H2] [C2] \quad (9)$$

The deflection, slope, bending moments, and shear force continuity should be kept. Therefore

$$\mathbf{q}_{11} = \mathbf{q}_{21} \quad (10)$$

$$[H1] [C1] = [H2] [C2] \quad (11)$$

Therefore,

$$[C2] = \frac{[H1] [C1]}{[H2]} \quad (12)$$

$$\mathbf{q}_{21} = [H1] [H2]^{-1} [C1] \quad (13)$$

If we have nth number of section, then the transfer matrix can be written as follow:

$$\mathbf{q}_n = [H1] [H2]^{-1} [H3] [H4]^{-1} [H5] [H6]^{-1} \dots [Hn] [Cn] \quad (14)$$

Equation (14) will be used to calculate the frequency characteristic equation by the aid of the boundary conditions. In this study, the boundary conditions were assumed to be free-free.

Finite Elements Method:-

Bones has very complicated shape and the apparent density vary from location to another. This problem was solved by using CT-Scan test as shown in figure (2). The images of the bones were collected and stored in DICOM format (Digital Imaging and Communications in Medicine). Those images has the size of 512X512 pixels. CT-scan device type GE Medical Systems\Discovery CT 750 HD was used and the images were taken with the same setting (KVP 120, slice thickness 1.25 mm, and tube current 120 mA). The DICOM images are exported to MIMICS program to construct the three dimensional model. Program named 3-matic was used to generate the surface meshing. The quality of the surface meshing was tested by using height to base ratio. The volumetric meshing was generated by using elements type TET 4 (3-matic program has two option for the volumetric meshing TET 4 and TET 8). The meshed modal is exported to MIMICS program again for the material assignment. Based on the grayscale of each pixel in the region of interest and the apparent density was calculated [6]. The finite element modal is exported to ANSYS mechanical APDL for the modal analysis as displaced in figure (3). Modal analysis is used for the calculation of the natural frequency for different modes of vibration. Figure (4) below shows the process of construction the three dimensional model.



Figure (2): CT-Scan test for the bones.

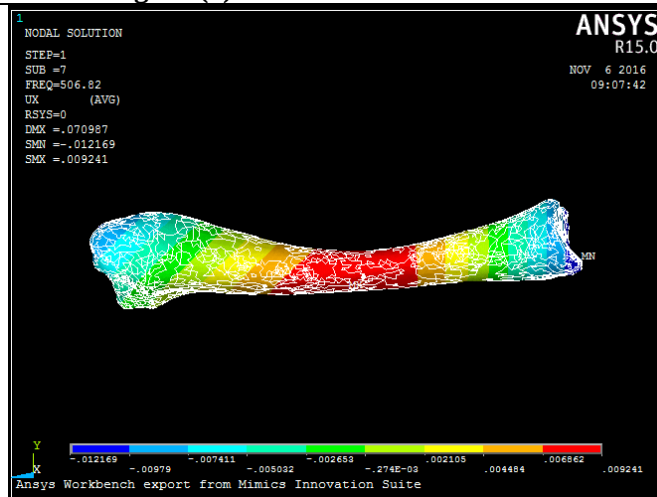


Figure (3): FEA Modal solution

3-D Model and FEA

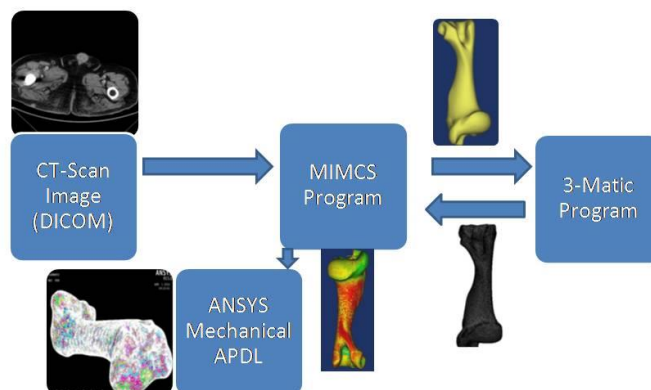


Figure (4): Three dimensional modal and FEA modal

Materials:-

The test specimen was ulna bone collected from cadaver cow. The cow did not suffer any bone disease and the bone was extracted carefully to prevent any damage to the bone structure. The bone length was 282.5 mm. Soft tissues were removed manually and a great care was taken to prevent the initiation of cracks in the bone surface.

Results and Discussion:-

The bovine bone was subjected to CT-Scan test to create the raw data required for the calculation of the geometrical properties and the construction of the three dimensional model for the finite element analysis.

Geometrical properties:-

The geometrical properties of the ulna bone was evaluated by using image processing technique [8]. The gray images obtained from the CT-Scan were converted to binary images by the aide of J-Image program. The geometrical properties of the bone was calculated by using Mat-Lab program . Figures 5 8 will illustrate the variation of the area, centroid, maximum moment of inertia, and minimum moment of inertia along the bone length.

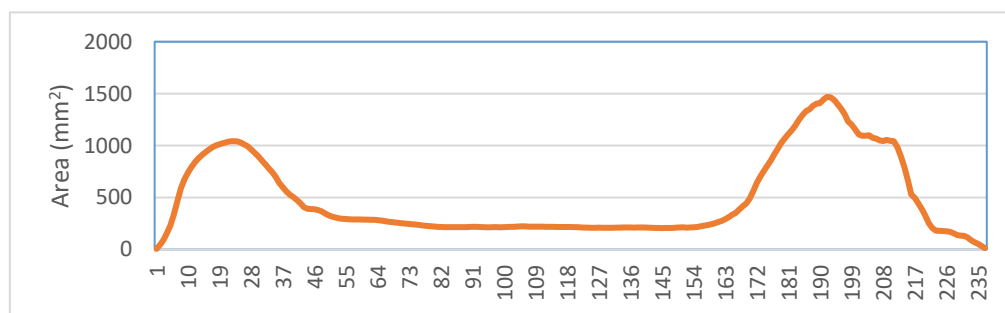


Figure (5): Area variation of bone cross-section.

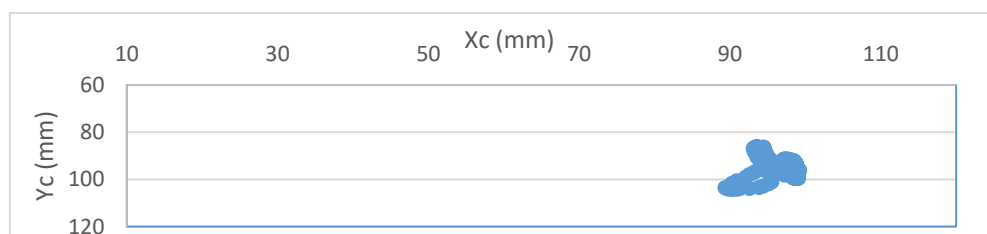


Figure (6): Trajectory of the centroid of each section along the bone

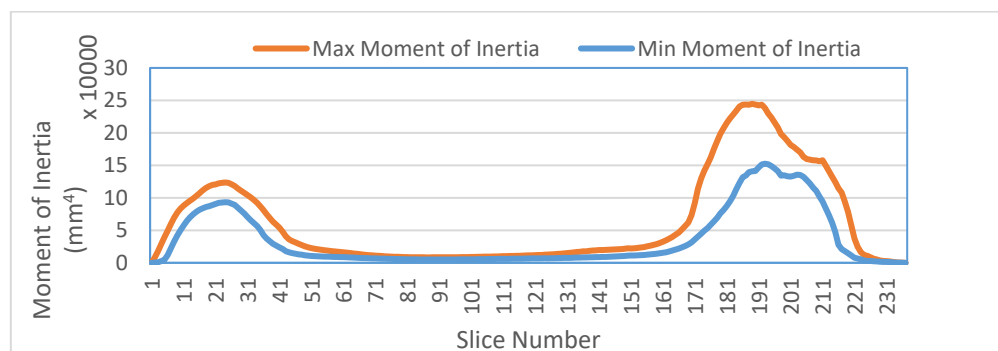


Figure (7): Max and Min moment of inertia variation of bone cross-section.

Transfer Matrix Method Results:-

The transfer matrix method is applied to the bovine bone sample to evaluate the natural frequencies for different modes of vibration at the two principle planes. The frequency characteristic equation obtained for the ulna bone in free-free boundary condition has a very complicated form and it's impossible to obtain the analytical solution. Therefore, the residual of that equation was plotted. Each location the residual curve cross the frequency axis will identify a natural frequency. Figure (8) and (9) shows the plot of the residual of the frequency characteristic equation at the two principle planes.

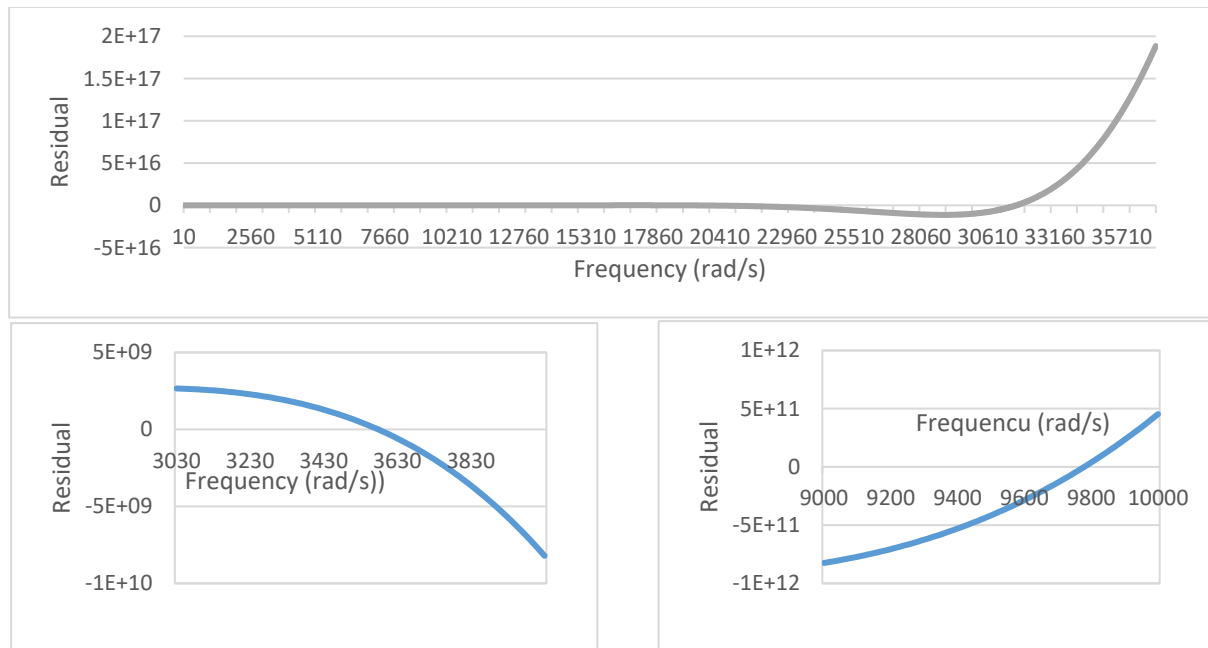


Figure (8): The residual of the frequency characteristic equation (Min. Inertia).

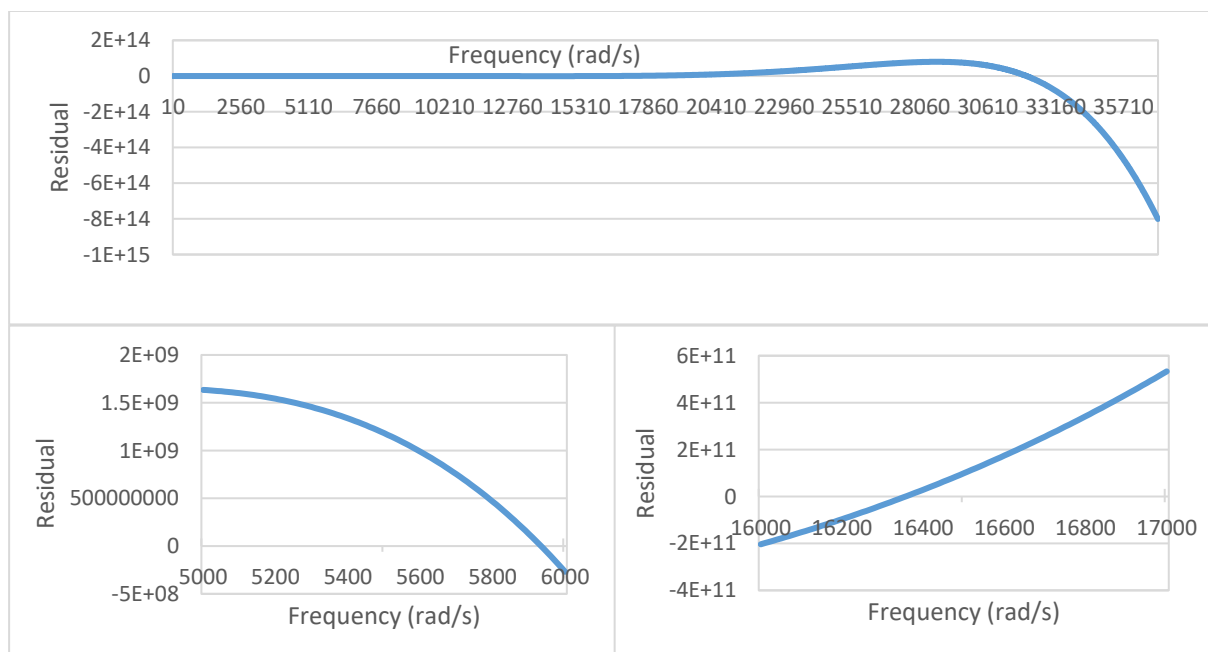


Figure (9): The residual of the frequency characteristic equation (Max. Inertia).

The results obtained from the transfer matrix method and finite element analysis are listed in table (1).

Table (1): The natural frequencies of bovine tibia bone.

NO	MODE	FEA (HZ)	TRANSFER MATRIX (HZ)	DISCREPANCY %
1	Transverse	565	506	10%
2	Transverse	810	945	15%
3	Transverse	1642	1556	5%
4	Transverse	2246	2600	15%
5	Transverse	3254	3449	6%

The calculation of the natural frequencies using transfer matrix method have several challenges because of dealing with the multiplication of at least 600 matrices and calculation of at least 300 inverse matrices. The singularity prevents the program from determination of higher modes of vibrations. Also, long time was consumed in preparation of the data for the calculation of the natural frequency of the bone. The main plot was subdivided into small regions to identify the cross of the residual with the frequency axis. This cross will identify the natural frequency. This procedure was applied to achieve the natural frequency of the bone in the maximum and minimum principle planes. The results are in good agreement as shown in the table (3). A further improvement needs to be done for the method to determine the physical and mechanical properties of each pixel instead of averaging these properties for the entire section. The round error affects the accuracy of the methods due to the large number of matrices and the mathematical operations conducted to achieve the results. The solution is not always achieved and in many cases it diverges.

Conclusions:-

The transfer matrix method are very effective and useful in evaluating the dynamic properties of the bones and other biomedical applications. The method take into consideration the shape irregularity, materials assignments, and various boundary conditions. This method are easy to program and the run time is relatively short. In addition, this method does not require the building of three-dimensional models, which needs special soft wares such as MIMICS and IP Scan. The results of this study encourage further developments by considering more complicated equation of motion and the use of gray images instead of binary images. The use of the gray images will enable the evaluation of the physical and mechanical properties based upon empirical equations for each pixel and by using theories of composite materials a more accurate results will be obtained.

Conflict of interest

No outside funding or grants in support of our research for or preparation of the work is received. In addition, no personal or institutional financial support is related to the study. Furthermore, we have had full control of all primary data and we agree to allow the journal to review the data if recommended.

Acknowledgements:-

The Authors are grateful for the University of Technology for their help. Special thanks for Al-Kafeel Hospital for their assistance in providing access to the CT-Scan devices and special thanks for the staff of the hospital for their assistance and cooperation.

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