Locally Optimal Detection of Signals in Underwater Acoustic Noise with Student's t-distribution

Y Yousif Al-Aboosi, H Abdulrahem Taha, H Ali Abdualnabi
Faculty of Engineering, University of Mustansiriyah, Baghdad, Iraq.

Abstract: Signal detection is imperative in underwater signal processing and digital communication, and based on a knowledge of noise statistics, optimum signal detection in underwater acoustic noise (UWAN) can be more effectively realised. The hypothesis of normal (Gaussian) noise allows the use of matched filter (MF) detectors; accordingly, a locally optimal detector (LO) is designed in this study to improve detection probability (P_D) based on the knowledge of noise probability density function. The underwater noise used for validation is real data collected from the sea using broadband hydrophones at the beach of Desaru on the eastern seashore of Johor, Malaysia. The performance of the LO detector is then compared with a conventional MF detector and these are evaluated according to their P_D values. For a time-varying signal, a false alarm probability specified as 0.01, and a P_D value of 90%, the energy-to-noise ratios (ENR) of the LO are better than those of the MF by 4.2 dB and for fixed frequency signals, the LO is better than the MF by 5.2 dB.

Keywords: Underwater Acoustic Noise ・ Detection theory ・ Student's t-distribution ・ non-Gaussian signal detection.

1. Introduction

The detection of signals in the presence of noise is a significant problem that arises in various signal processing applications, including radar and sonar systems. Previous studies on detection [1, 2], assumed that signals are embedded in additive white Gaussian noise, and that receivers are designed accordingly. However, various practical noise sources, such as atmospheric noise detected with radar systems and underwater acoustic noise (UWAN) detected with sonar systems, are non-Gaussian and show highly impulsive characteristics. Where these noises' statistics are known, the matched filter (MF) detector is regarded as the optimum detector when the noise is Gaussian [1, 3]. The MF detector becomes less than ideal when the noise is non-Gaussian, however, because of degradation in its performance [4]. In spite of this disadvantage, and because of its simple implementation and the absence of complete statistical data about underwater noise, the MF detector remains widely used for signal detection where noise does not follow a Gaussian pdf [4, 5]. UWAN in shallow waters with biological noise is non-Gaussian distributed, and features accentuated impulsive behavior [6-8]. The suboptimum performance of MF detectors in UWAN thus creates major potential for enhancing performance in underwater conditions [3, 5].

In this study, an experimental model for noise in an acoustic underwater channel was developed based on field data measurements, and through Monte Carlo simulation, the performances of locally optimum detectors (LO) in UWAN were compared with conventional MF detectors. The paper is organised as follows. Section 2 offers a summarized introduction of the signal model and the data collection and analysis techniques used to define the properties of UWAN. Section 3 describes the signal detection in t-distribution noise using LO. Section 4 presents the results, and Section 5 briefly discusses the conclusions.

2. Signal Detection Problem

In this section, a shared problem in digital communication using radar and sonar systems is presented, where a known signal is to be detected in a non-Gaussian additive noise channel.

2.1 Signal Model

The signals used are a fixed frequency sinusoidal signal and linear frequency modulated (LFM) signal. These are used to represent the single frequency signals and time-varying signals that could be encountered in practical situations. An arbitrary sinusoidal signal can be defined as follows:

$$s(n) = A cos(\theta(n)) \qquad 0 \leq n \leq N-1$$
$$= 0 \qquad \text{elsewhere} \tag{1}$$

where N is the signal duration in the samples, A is the signal amplitude, and $\theta(n)$ is the instantaneous phase. For a fixed-frequency signal, the instantaneous phase is defined as

$$\theta(n) = 2\pi f_m n T_s \tag{2}$$

where f_m is the signal frequency and T_s is the sampling period. The instantaneous phase for the LFM signal is

$$\theta(n) = 2\pi(f_m + \frac{\varphi}{2} n T_s) n T_s \tag{3}$$

where $\varphi$is the frequency defined as $\varphi = f_{BW}/NT_s$, where f_{BW} is the bandwidth of the signal. The received signal can be defined as follows:

$$x(n) = s(n) + v(n) \tag{4}$$

where $s(n)$is the signal of interest and $v(n)$ is the UWAN.

The main idea of detection is to determine the presence of a signal in the underwater noise. Given an observation vector x and several hypotheses, H_i, the goal is to discover the set of data that matches a hypothesis. Although the number of hypotheses could be random, the situation of having two hypotheses, H_0 and H_1, is considered valid for most communication, radar, and sonar systems [1]. The hypothesis-testing is thus expressed as follows:

$$H_0 \text{ (Null hypothesis): } y(n) = v(n) \qquad n = 0, 1, . . . ,N - 1 \tag{5}$$

$$H_1 \text{ (Alternative hypothesis) :} x(n) = s(n) + v(n) \qquad n = 0, 1, . . . ,N - 1 \tag{6}$$

Neyman–Pearson (NP) and Bayesian methods are primarily used for hypothesis testing. Method selection depends on the availability of the prior probability, as although digital communication and pattern recognition systems use Bayes risk [9], the NP criterion is employed for radar and sonar systems. Furthermore, the derivation of the optimal detectors depends on the assumptions made about the noise [1]. Given that UWAN is dependent on the frequency [6, 10], the AWGN assumption is invalid, and UWAN is more suitably modelled as coloured noise [7, 8, 11].

2.2 Data Collection and Non-Gaussian Noise Model

Field trials were conducted at Desaru beach (1° 35.169′ N, 104° 21.027′ E) to collect a noise data set and to study the statistical properties of underwater noise (Figure (1)). The signals were received at a frequency range of 7 Hz to 22 KHz using a broadband hydrophone (Dolphin EAR 100 Series). The measurements were obtained at depths from 1 m to 7m from the sea surface, which was at a depth of 8 m. The wind speed was about 7 kn, and the surface temperature was around 27 °C [12].

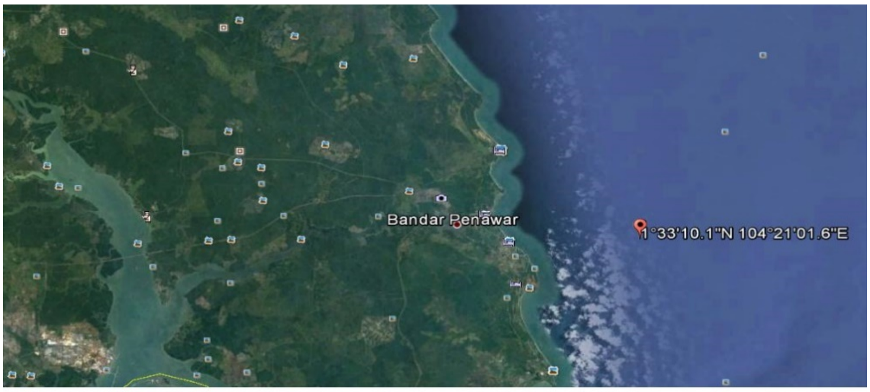
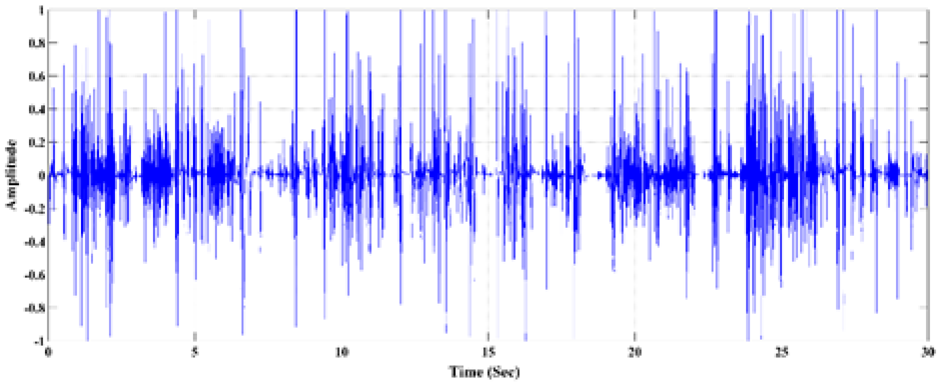
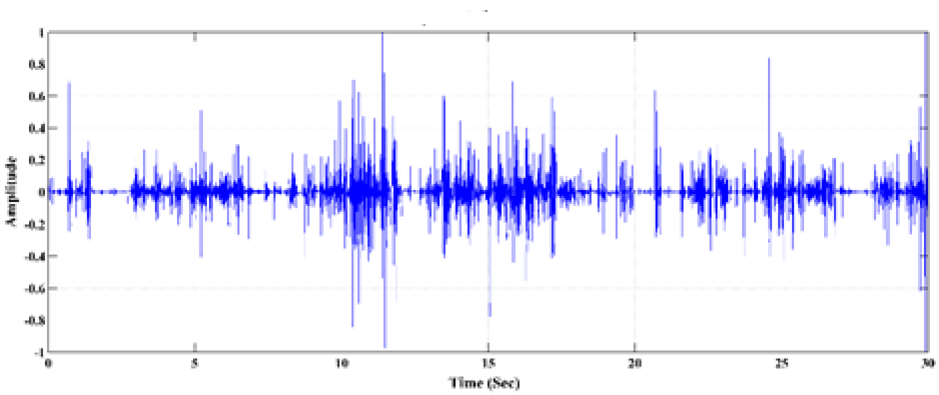


Figure.1. Experiment test site conducted at Tanjung Balau, Johor, Malaysia.

Figure (2) illustrates the time representation of the collected data at depths of 5 meters and 7 meters, and the impulsive nature of the noise can be clearly observed.



(a) Time representation at 3 meters depth.



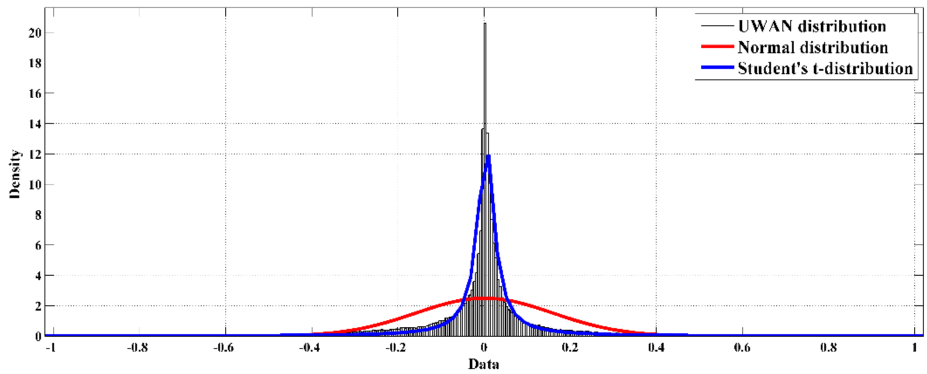
(b) Time representation at 7 meters depth.

Figure 2. Time representation of the UWAN at depths of 3 meters and 7 meters.

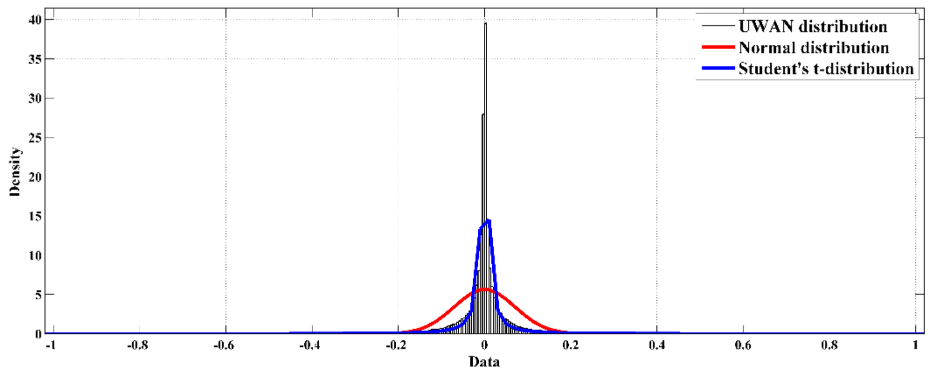
The amplitude distributions found from the collected data were compared with the Student's t distribution and Gaussian distribution using the distribution fitting tool in MATLAB. As shown in Figure (3), the comparison results show that the pdf of the underwater noise generally follows the Student's t distribution. Therefore, the UWAN does not validate an assumption of Gaussian distribution, and, clearly, the noise pdf distribution must be fitted with the t distribution. The Student's t pdf is expressed as [13]

$$\rho_{v,d}(v,d) = \frac{\Gamma\left[{(d+1)}/{2}\right]}{\sqrt{\pi v}\,\Gamma({d}/{2})}\left(1+\frac{v^2}{d}\right)^{-{(d+1)}/{2}} \tag{7}$$

where $\Gamma(\cdot)$ is the gamma function and d is the degree of freedom that controls the dispersion of the distribution. The pdf represented in equation (7) has a zero mean and a variance equal to ${d}/{(d-2)}$ for $d \geq 2$.



(a) 3 meters depth.



(b) 7 meters depth.

Figure 3. Comparison of the amplitude distribution of the UWAN with the Gaussian distribution and t-distribution.

Table (1) specifies the degrees of freedom for different depths. The UWAN can be assumed to be stationary [17] for a short period of time, about a few seconds.

Table 1. Degree of freedom for different depth

Depth (m)	Analysis period (Sec)	Degree of freedom (v)
1	1.85	2.94
3	1.26	2.91
5	1.55	2.82
7	1.12	2.8

From Table (1), the degree of freedom is around 3. Analysis of the UWAN shows that its characteristics are not the same as for AWGN. The pdf of the UWAN follows a Student's t distribution, in contrast to the assumption of Gaussian pdf proposed in a previous study [14].

3. Signal Detection in Non-Gaussian Distribution Noise

For optimum detector and near-optimum detection in non-Gaussian noise distributions, nonlinear detectors should be used. A locally optimal detector (LO) was thus designed to obtain such performance, and this was compared with conventional Matched Filter (MF).

3.1 Matched Filter (MF)

In the presence of a Gaussian noise, the MF detector is optimal for detecting a known signal. Many communication systems thus use this detector as a matched filter. The test statistic for the MF is specified by [1]

$$T(x) = \sum_{n=0}^{N-1} x[n]s[n] \tag{8}$$

where $s\,(n)$ is the reference signal and $x\,(n)$ denotes observed data. The expected value ($E\{T;H_i\}$ for i=0, 1) and the variance of the test statistic (Var $\{T;H_i\}$ for i=0, 1) are

$$T(x) = \begin{cases} N(0\,, \sigma_v^2.E_s) & under\ H_0 \\ N(E_s\,, \sigma_v^2.E_s) & under\ H_1 \end{cases} \tag{9}$$

where E_s is the signal energy and $var(v)$ is the noise variance that follows the t distribution as defined in Eq. 7. The false alarm probability (P_{FA}) is defined as

$$\mathrm{P}_{FA} = \mathrm{P}(\mathrm{H}_1;\mathrm{H}_0) = P_r\{x[0] > \gamma; H_0\} = Q\left(\frac{\gamma}{(\sigma_v^2.E_s\,)^{1/2}}\right) \tag{10}$$

where γ is the threshold value for a given P_{FA}, and this threshold value is determined using

$$\gamma = Q^{-1}(P_{FA}).\,(\sigma_v^2.E_s\,)^{1/2} \tag{11}$$

The probability of detection (P_D) is defined as

$$P_D = P(H_1;H_1) = P_r\{x[0] > \gamma; H_1\} = Q\left(\frac{\gamma - E_s}{(\sigma_v^2.E_s\,)^{1/2}}\right) \tag{12}$$

By using equation (9) and equation (11) in equation (12), the following equation emerges [1]:

$$P_D = Q\left[Q^{-1}(P_{FA}) - \sqrt{\frac{s}{\sigma_v^2}}\right] \tag{13}$$

3.2 Locally Optimal Detector (LO)

The locally optimal (LO) detector is used for signal detection in presence of non-Gaussian noise [16 ,15] . This detector can be used for weak signals detection by using a nonlinear transfer function (NLTF) prior to an MF detector as illustrated in Figure (4).

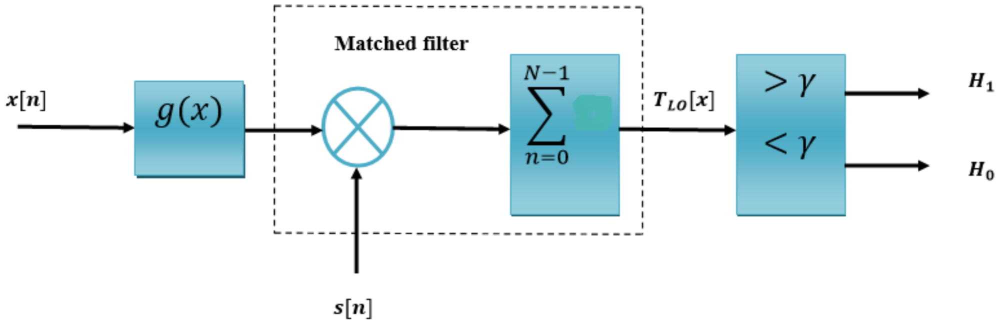


Figure 4. Schematic diagram of the LO detector for a known signal in non-Gaussian noise.

The test statistic for the LO detector is assumed by [1, 3-5]

$$T(x) = \sum_{n=0}^{N-1} g(x[n])s[n] \tag{14}$$

where $g(x[n])$ is the NLTF that can be calculated from the pdf of the noise. Thus,

$$g(x) = -\frac{1}{\rho(x)}\frac{d\rho(x)}{dx} \tag{15}$$

where $\rho(x)$ is the Student's t-distribution pdf as defined in Equation (7). The transfer function is:

$$g(x) = \frac{(d+1)x}{(d+x^2)} \tag{16}$$

for $d = 3$:

$$g(x) = \frac{4x}{(3+x^2)} \tag{17}$$

A characteristic transfer function is illustrated in Figure (5).

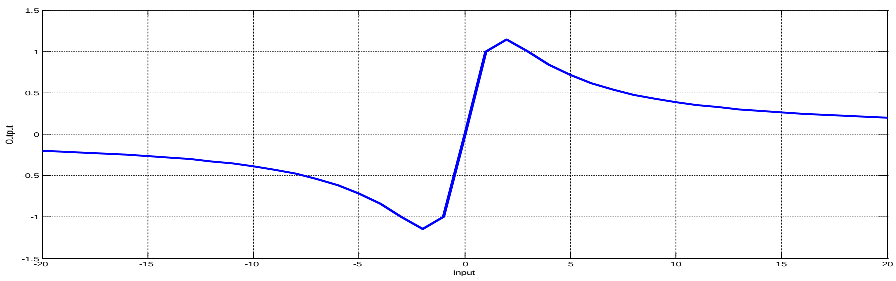


Figure 5. Nonlinear transfer function for a locally optimal detector in t-distribution noise with $\boldsymbol{d = 3}$.

The mean value and variance of $T(x)$under H_i are [1, 5]

$$T(x) = \begin{cases} N(0\,, IE_s) & under\ H_0 \\ N(I\,E_s\,, IE_s) & under\ H_1 \end{cases} \tag{18}$$

where I is [1]

$$I = \int_{-\infty}^{\infty} \frac{\left(\frac{d\rho(x)}{dx}\right)^2}{\rho(x)} dx \tag{19}$$

The value of I specified in equation (18) is determined mathematically. For $d = 3$,

$$I = 52.9338 \int_{-\infty}^{\infty} v^2.\left(1 + \frac{v^2}{3}\right)^{-4} dv \tag{20}$$

$$I = 0.6667 \tag{21}$$

For any given P_{FA}, the P_D of the LO detector can be stated as [5]

$$P_D = Q\left(Q^{-1}(P_{FA}) - \sqrt{I.E_s}\right) = \left(Q^{-1}(P_{FA}) - \sqrt{0.6667E_s}\right) \tag{22}$$

4. Results

The performance of the LO detector in detecting a signal in additive UWAN was tested and compared with the detection performance of MF using Monte Carlo simulations with ten thousand repetitions for each energy to noise ratio (ENR). Throughout each repetition, the signals defined by Equation 1 to Equation 4 were added to the underwater noise for both types of signals, time-varying and time-invariant. These signals are used in simulation as follows:

- Fixed frequency signal with 500 Hz frequency
- LFM signal with 400 Hz starting and 1500 Hz ending frequency

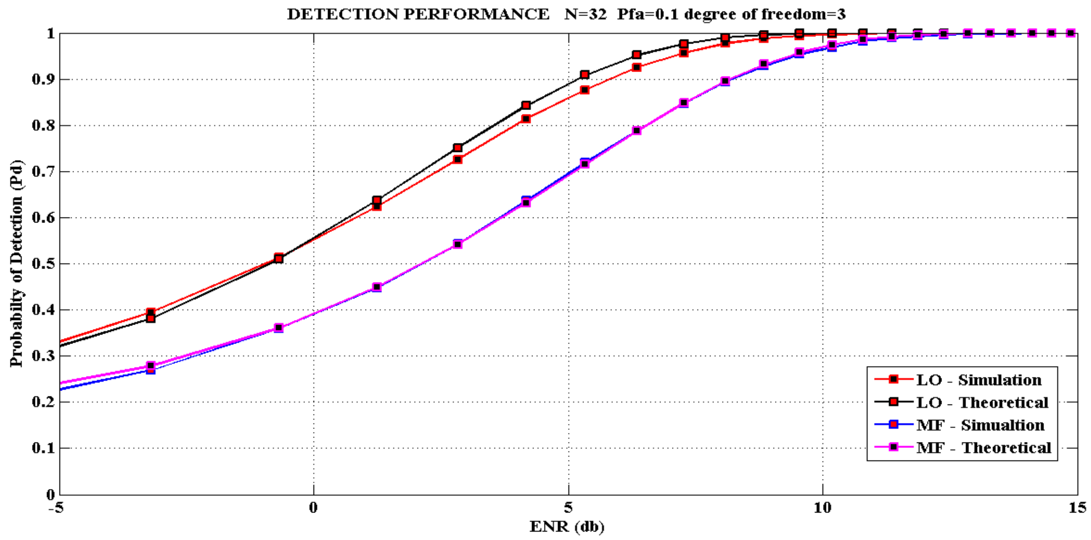
For different ENRs, the simulations are repeated by changing the signal energy while keeping the noise power constant. In general, the ENR is defined as

$$ENR(db) = 10 \log_{10}\left(\frac{\mathrm{NA}^2}{2\sigma_v^2}\right) \tag{23}$$

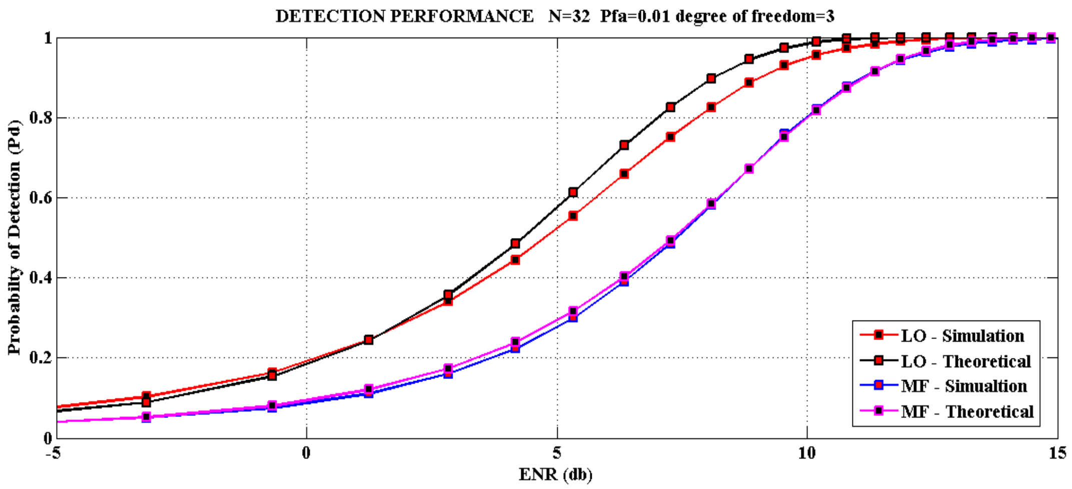
Figure (6) illustrates the two detectors' performances over the range of ENRs of minus five to fifteen dB for a signal with a fixed frequency of 500 Hz with P_{FA} of 10^{-1} and 10^{-2} [5, 14]. The results show that the LO detectors are clearly better than the MF detectors. The ENRs of the two detection methods used and the P_{FA} given a P_D of 90 percent are recorded in Table 2. Clearly, the ENR of the LO is better than that of the MF, by 5.2 dB.

TABLE 2. ENRs for various detection methods given a P_D of 90 percent for a single tone signal with frequency 400 Hz.

P_{FA}	LO	MF
0.1	5.8dB	8.1dB
0.01	7.4dB	12.5dB



$(a)P_{FA} = 0.1$



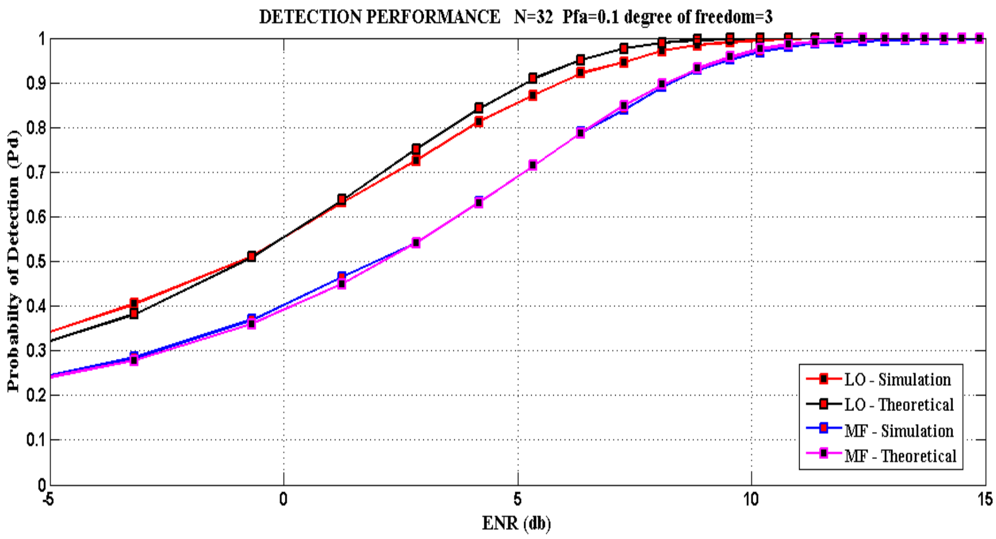
$(b)P_{FA} = 0.01$

Figure 6. Performances of the LO and MF detectors for the single-tone signal with 500 Hz frequency. $(a)P_{FA} = 0.1$. $(b)P_{FA} = 0.01$.

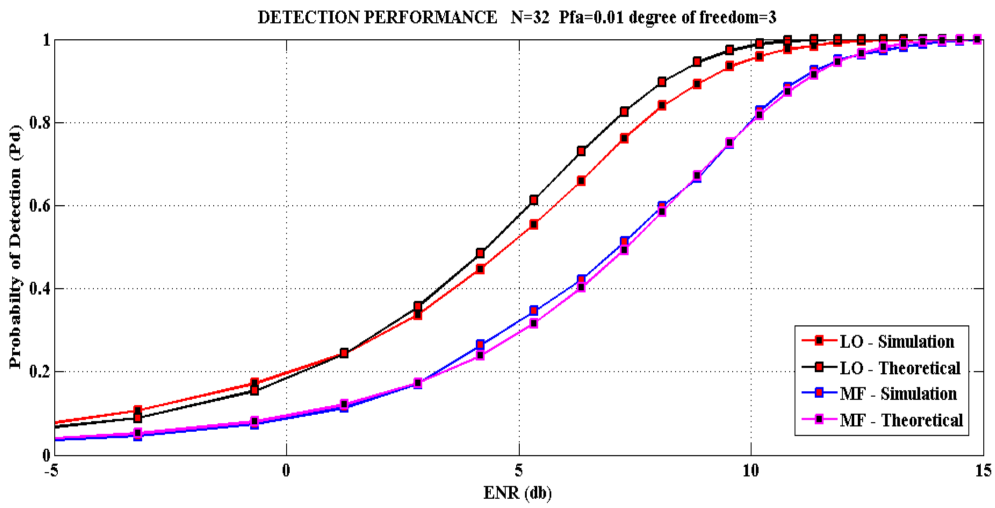
Figure (7) illustrates the two detectors' performances over the range of ENR from minus five to fifteen dB for the LFM signal at a fixed frequency of 500 Hz with P_{FA} of 10^{-1} and 10^{-2} [5, 14]. The results show that the LO detectors are clearly better than the MF detectors. The ENRs of the two detection methods used and the P_{FA} given a P_D of 90 percent are recorded in Table 3. Clearly, the ENR of the LO is better than that of the MF, by 4.2 dB.

TABLE 3. ENRs for various detection methods given a P_D of 90 percent for LFM signal

P_{FA}	LO	MF
0.1	5.3dB	7.8dB
0.01	8.1dB	12.3dB



$(a) P_{FA} = 0.1$



$(b) P_{FA} = 0.01$

Figure 7. The performance of the LO and MF detectors for the LFM signal.

$(a) P_{FA} = 0.1$. $(b) P_{FA} = 0.01$.

5. Conclusion

UWAN in tropical shallow waters demonstrates emphasised impulsive behavior and thus does not track a normal distribution. The analysis of field data measurements shows that the probability density function of noise successfully fits the Student's t distribution with three degrees of freedom. A knowledge of noise statistics assisted in the design and improvement of a suitable LO detector, which detector performed better than the conventional MF detector, as indicated by the detection probability(P_D). For a time-varying signal, specifying a false alarm probability of 0.01 and a P_D value of 90%, the energy-to-noise ratios (ENR) of the LO were better than the MF by 4.2 dB, and for fixed frequency signals, the LO was better than the MF by 5.2 dB. The near-optimal performance of the LO detector makes it an attractive tool for sonar and underwater digital communication.

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